

Course Title:	Content Area:	Grade Level:	Credit (if applicable)					
Precalculus ACA	Mathematics	11-12	1.0					
Course Description:								
This course is the fourth course in the college preparatory mathematics sequence. It is a prerequisite for the analytic geometry and calculus course offered by colleges. Topics studied are algebraic functions, logarithms and exponential functions, trigonometric functions and conics. All topics studied will involve the use of a TI-83+ or TI-84+ graphing calculator. Students enrolled in this class must take the common mid-term and final assessment for Pre-Calculus Academic.								
Aligned Core Resources:		Connection to the BPS Vision of the Graduate						
<i>Precalculus</i> , 7th Edition (2022) by Robert F. Blitzer Published by Pearson		CRITICAL THINKING AND PROBLEM SOLVING <ul style="list-style-type: none"> ● Collect, assess and analyze relevant information ● Reason effectively. Use systems thinking ● Make sound judgments and decisions. ● Identify, define and solve authentic problems and essential questions. ● Reflect critically on learning experience, processes and solutions ● Transfer knowledge to other situations CONTENT MASTERY <ul style="list-style-type: none"> ● Develop and draw from a baseline understanding of knowledge in academic disciplines from our Bristol curriculum. 						
Additional Course Information: Knowledge/Skill Dependent courses/prerequisites		Link to Completed Equity Audit						
Algebra 2 prerequisite course		Equity Curriculum Review- PreCalculus						
Standard Matrix								
	Standard	Unit P	Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6
	A.APR.7	X						
	A.APR.A.1			X				
	A.APR.B.2			X				
	A.APR.B.3			X				
	A.CED.1	X						
	A.CED.4	X						
	A.CED.A.2				X			
	A.REI.11	X			X			X
	A.REI.2	X						
	A.REI.3	X						
	A.REI.4	X						
	A.REI.D.11				X			X
	A.SSE.3	X			X			
	A.SSE.A.1			X	X			
	F.BF.4		X					
	F.BF.A.1		X		X			
	F.BF.B.3		X					
	F.BF.B.4		X					
	F.BF.B.5				X			
	F.IF.A.1		X					
	F.IF.A.2		X					

F.IF.B.4		X					
F.IF.B.5		X					
F.IF.C.7		X	X	X			
F.IF.C.8			X	X			
F.IF.C.9			X				
F.LE.A.1				X			
F.LE.A.2				X			
F.LE.A.3				X			
F.LE.A.4				X			
F.LE.B.5				X			
FTF.A.1					X		
FTF.A.2					X		
FTF.A.3					X		
FTF.A.4					X		
FTF.B.6					X		
FTF.C.8						X	
FTF.C.9						X	
G.SRT.C.6					X		
G.SRT.C.7					X		
G.SRT.C.8					X		
G.SRT.D.10						X	
G.SRT.D.11						X	
G.SRT.D.9						X	
HSA.CED.A.3							X
HSA.REI.C.6							X
HSA.REI.C.7							X
HSA.REI.D.12							X
N.CN.3			X				
N.CN.A.1			X				
N.CN.A.2			X				
N.CN.C.7			X				
N.CN.C.8			X				
N.CN.C.9			X				
S.ID.7	X						

See each unit for the standard language

Unit Links

[Unit P: Fundamental Concepts of Algebra](#)

[Unit 1: Functions and Graphs](#)

[Unit 2: Polynomials and Rational functions](#)

[Unit 3: Exponential and Logarithmic Functions](#)

[Unit 4: Trigonometric Functions](#)

[Unit 5: Analytic Trigonometry](#)

[Unit 6: Systems of Equations and Inequalities](#)

Unit Title	
Unit P: Fundamental Concepts of Algebra	
Relevant Standards: Bold indicates priority	
<p>A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.</p> <ul style="list-style-type: none"> • A.SSE.3a. Factor a quadratic expression to reveal the zeros of the function it defines. • A.SSE.3b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. • A.SSE.3c. Use the properties of exponents to transform expressions for exponential functions <p>A.APR.7 (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.</p> <p>A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple (rational) and exponential functions.</p> <p>A.REI.2 Solve simple (rational) and radical equations in one variable, and give examples showing how extraneous solutions may arise.</p> <p>A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.</p> <p>A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.</p> <p>A.REI.4 Solve quadratic equation in one variable</p> <ul style="list-style-type: none"> • A.REI.3a Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. $2 = q$ • A.REI.3b Solve quadratic equations by inspection, taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b. <p>A.REI.11 Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solution of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and $g(x)$ are linear, polynomial, (rational), absolute value, exponential, and logarithmic functions.</p> <p>S.ID.7 Interpret the slope (rate of change) and the intercept of a linear model in the context of data</p>	
Essential Question(s)	Enduring Understanding(s)
<ul style="list-style-type: none"> • Why do we structure expressions in different ways? • In what ways can the problem be solved, and why should one method be chosen over another? • How can the properties of the real number system be useful when working with polynomials and rational expressions? • Which function is being modeled in specific real world applications and how can I use acquired knowledge to solve the problem? 	<ul style="list-style-type: none"> • Expressions can be written in multiple ways using the rules of algebra; each version of the expression tells something about the problem it represents. • There is often an optimal method of manipulating equations to solve a mathematical problem; however other methods, which may not be as efficient, can still provide insight to the problem. • Algebraic expressions such as polynomials and rational expressions, symbolize numerical relationships and can be manipulated in much the same way as numbers.
Demonstration of Learning	Pacing for Unit
Homework Class Practice Readiness Quizzes Skills Check Quizzes Mid-Unit Checkpoint End of Unit Assessment	13 blocks
Family Overview (link below)	Integration of Technology:
Precalculus ACA - Family Overview (2024-25) Precalculus ACA - Family Overview (2024-25) SPANISH	TI-84 graphing calculator Desmos calculator MyMathLab interactive homework, quizzes and tests

	MyMathLab video explanations
Unit-specific Vocabulary	Aligned Unit Materials, Resources, and Technology (beyond core resources)
Linear Equations, Slope, Y-intercept, Radical Equation, Rational Exponent, Rational Expression, Rational Equation, Extraneous Solution, Literal Equation, Quadratic Equation, Polynomial, Factoring, Square root method, Completing the square, Quadratic Formula, Radicand, Complex Number, Imaginary, Solution	TI-84 graphing calculator Desmos calculator Mini white boards Teacher created activities Vertical surfaces
Opportunities for Interdisciplinary Connections	Anticipated misconceptions:
<ul style="list-style-type: none"> Quadratic expressions frequently appear in physics, especially in kinematics, where equations of motion describe how objects move under gravity. Quadratic equations model motion under gravity, where the height or position of an object is a function of time. Exponential functions model financial growth (compound interest) and depreciation (exponential decay). Exponential functions are crucial in biology for modeling population growth, radioactive decay, and disease spread. 	<ul style="list-style-type: none"> Every quadratic equation can be factored easily. Completing the square is useful only to find the vertex of a quadratic function. The rules for simplifying polynomials apply to exponential expressions in the same way. Rational expressions can always be added, subtracted, multiplied, or divided without considering restrictions. Rearranging an equation always preserves all original solutions. The quadratic formula should always be used when solving quadratics. Every rational or radical equation has a valid solution. When solving $f(x)=g(x)$ there is always a single intersection. Exponential growth can be analyzed using the concept of slope. A line's slope is always positive or negative.
Connections to Prior Units	Connections to Future Units
These standards served as a bridge from Algebra 1 to Algebra 2 by establishing essential algebraic skills. Algebra 2 extended these ideas by introducing higher-degree polynomials, logarithms, complex numbers, advanced transformations, and real-world applications of these concepts.	These Algebra standards lay the foundation for more advanced topics in Precalculus, including polynomials, exponentials/logs, complex numbers, nonlinear systems, and limits. Each concept builds towards a deeper understanding of functions and prepares students for Calculus applications.
Differentiation through Universal Design for Learning	
UDL Indicator	Teacher Actions
<p>Multiple Means of Representation (Principle I) Consideration 2.5 - Illustrate through multiple media: Use graphing technology (Desmos, GeoGebra) to show how different forms of an expression impact the graph.</p> <ul style="list-style-type: none"> Show how factoring a quadratic highlights the zeros, while vertex form highlights the maximum/minimum. Have students manipulate sliders to see real-time changes in quadratic and exponential expressions. <p>Consideration 3.2 - Highlight patterns, critical features, big ideas, and relationships: Use step-by-step examples with color-coding to highlight transformations and relationships.</p> <ul style="list-style-type: none"> Color different parts of an expression when factoring, completing the square, or using exponent rules. Provide scaffolded notes where students fill in missing steps to highlight key algebraic patterns. <p>Multiple Means of Action & Expression (Principle II) Consideration 5.1 - Use multiple media for communication: Encourage students to verbally explain why they choose a particular algebraic method.</p> <ul style="list-style-type: none"> Have students record voice memos or videos explaining how different forms of an expression provide insights. Use math journaling to reflect on how algebraic manipulations reveal different properties of functions. <p>Multiple Means of Engagement (Principle III)</p>	

Consideration 7.1 - Optimize individual choice and autonomy:

Provide students with a choice of problem-solving strategies.

- Example: Solve a quadratic equation using three different methods (factoring, completing the square, quadratic formula) and discuss which is most efficient.

Consideration 8.3 - Foster collaboration and community:

Encourage students to defend different algebraic methods.

- Pose a problem and have students debate the best method for solving it.
- Assign roles where some students advocate for factoring, while others argue for completing the square.

Supporting Multilingual/English Learners

Related CELP standards

Learning Targets

CELP Standard 2:

Participate in grade-appropriate oral and written exchanges of information, ideas, and analyses, responding to peer or teacher feedback.

- I can discuss different methods for solving algebraic equations and respond to feedback from my peers.
- I can explain my mathematical thinking to others and revise my explanation based on new insights.
- I can participate in discussions where I compare strategies and justify my reasoning using evidence.

CELP Standard 3:

Speak and write about grade-appropriate complex mathematical ideas using evidence.

- I can explain why different algebraic forms of an expression reveal different information about a problem.
- I can justify my choice of method (factoring, completing the square, or exponent properties) when solving an equation.
- I can write and speak about how algebraic expressions represent numerical relationships.

CELP Standard 4:

Construct grade-appropriate oral and written arguments supported by reasoning and evidence.

- I can compare different ways to manipulate algebraic expressions and explain when each is useful.
- I can represent algebraic expressions in different ways (symbolic, graphical, and verbal) and explain how they are related.
- I can use mathematical language to describe how polynomials and rational expressions follow similar rules as numbers.

Lesson Sequence	Learning Target	Success Criteria/ Assessment	Resources
P.2 Exponents and Scientific Notation	I can identify and apply the key rules of exponents to simplify expressions with exponents	<ul style="list-style-type: none"> • I can apply the product rule, quotient rule, power of a power rule, negative exponent rule, and zero exponent rule to simplify expressions with exponents • I can demonstrate a clear understanding of how to manipulate exponents with the same base when multiplying, dividing, or raising to a power. 	
P.4 Polynomials	I can perform operations on polynomials	<ul style="list-style-type: none"> • I can add, subtract, and multiply polynomials 	
P.5 Factoring Polynomials	I can factor polynomials	<ul style="list-style-type: none"> • I can identify which type of factoring is needed. • I can factor by greatest common factor, grouping, trinomials, difference of squares. 	
P.6 Rational Expressions	I can use perform operations on rational expressions	<ul style="list-style-type: none"> • I can simplify rational expressions • I can multiply, add and subtract rational expressions 	
P.7 Equations	I can solve polynomial equations using various methods	<ul style="list-style-type: none"> • I can solve radical and rational equations • I can solve quadratic equations <ul style="list-style-type: none"> ○ Quadratic formula (no complex numbers) ○ completing the square ○ Factoring ○ Square root property 	

Unit Title

Unit 1: Functions and Graphs

Relevant Standards: Bold indicates priority

F.IF.A.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x . The graph of f is the graph of the equation $y = f(x)$.

F.IF.A.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

F.IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

F.IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

F.IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

- F.IF.C.7.B Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

F.BF.A.1 Write a function that describes a relationship between two quantities.

- F.BF.A.1.A Determine an explicit expression, a recursive process, or steps for calculation from a context.
- F.BF.A.1.C (+) Compose functions. For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.
- F.BF.B.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
- F.BF.B.4 Find inverse functions.
 - F.BF.4a Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x) = 2(x^3)$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$ (x not equal to 1).
 - F.BF.4b (+) Verify by composition that one function is the inverse of another.
 - F.BF.4c (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.
 - F.BF.4d (+) Produce an invertible function from a non-invertible function by restricting the domain.

Essential Question(s)	Enduring Understanding(s)
<ul style="list-style-type: none"> • What is a function and what are the different ways they can be represented? • How are functions used in the real world? • What are the key characteristics of a graph and why are they important? • How does composing functions affect the domain? • What is the relationship between a function and its inverse? 	<ul style="list-style-type: none"> • A function is a specific type of relation where each input is associated with exactly one output. This concept is fundamental in describing how variables interact and is a building block for analyzing more complex mathematical relationships. • Functions can be transformed through translations, reflections, stretches, and compressions. These transformations affect the graph of the function and help in understanding how changes in the function's formula impact its graph. • Functions can be combined through addition, subtraction, multiplication, and division, and can be composed with each other. Understanding these operations is essential for building more complex functions and analyzing their behavior. • An inverse function essentially reverses the effect of the original function. Understanding how to find and use inverse functions is important for solving equations and understanding the relationship between functions and their inverses. • Functions are used to model and solve real-world problems in various fields such as physics, economics, biology, and engineering. Applying functions to practical situations helps in interpreting data and making predictions.
Demonstration of Learning	Pacing for Unit
Homework Class Practice Readiness Quizzes Skills Check Quizzes Mid-Unit Checkpoint End of Unit Assessment	20 blocks
Family Overview (link below)	Integration of Technology
Precalculus ACA - Family Overview (2024-25) Precalculus ACA - Family Overview (2024-25) SPANISH	TI-84 graphing calculator Desmos calculator MyMathLab interactive homework, quizzes and tests MyMathLab video explanations
Unit-specific Vocabulary	Aligned Unit Materials, Resources, and Technology (beyond core resources)
Function, Input, Output, Independent variable, Dependent variable, Domain, Range, Function notation: $f(x)$, Parent function, Transformation, Translations, Vertical stretch, Vertical compression, Vertical reflection, Horizontal reflection, Piecewise function, Vertical line test, One-to-one function, Inverse function, Domain, Restricted domain, Composition	TI-84 graphing calculator Desmos calculator Mini white boards Teacher created activities Vertical surfaces
Opportunities for Interdisciplinary Connections	Anticipated misconceptions
<ul style="list-style-type: none"> • Functions are used to model relationships between variables in science, such as the decay of radioactive substances, population growth, and the motion of objects. • Functions are the foundation of programming and computational thinking, used in defining algorithms, 	<ul style="list-style-type: none"> • Every equation represents a function. • A function must always be a linear equation. • If a function has the same output for two different inputs, it is not a function. • The vertical line test determines if a graph is a function based on its shape alone.

<p>recursive processes, and data transformations.</p> <ul style="list-style-type: none"> • Functions describe relationships in economics, such as cost, revenue, and profit models, as well as statistical trends in business. 	<ul style="list-style-type: none"> • Function notation $f(x) = y$ means multiplication between f and x. • The domain of a function is always all real numbers. • Graph transformations (shifts, stretches, reflections) change the function's properties unpredictably. • The inverse of a function always exists. • Composing two functions always produces another function. • The x-values where two graphs intersect are only approximate solutions.
<p>Connections to Prior Units</p>	<p>Connections to Future Units:</p>
<p>Algebraic Manipulation & Functions – Factoring, completing the square, and working with rational expressions (A.SSE.3, A.APR.7) help reveal key features of functions like zeros, domain restrictions, and asymptotic behavior.</p> <p>Function Transformations & Equation Solving – Graphing transformations (F.BF.B.3) align with solving and rearranging equations (A.REI.3, A.CED.4), while finding inverse functions (F.BF.4) requires algebraic solving (A.REI.2).</p> <p>Data Interpretation & Functions – Understanding slopes, intercepts, and key features of graphs (F.IF.B.4, S.ID.7) connects functions to real-world data trends.</p>	<p>The Interpreting and Building Functions standards connect to future units by...</p> <ul style="list-style-type: none"> • Preparing students for function transformations, composition, inverses, graphing, and real-world applications. • Understanding domain, range, and key features leads to advanced functions like trigonometric, logarithmic, and exponential functions. • Skills in composition and inverse functions help with nested functions and transformations. • Graphing concepts extend to polynomials, rational functions, and asymptotic behavior. • Modeling functions supports sequences, series, and calculus readiness.
<p>Differentiation through Universal Design for Learning</p>	
<p>UDL Indicator</p>	<p>Teacher Actions:</p>
<p>Multiple Means of Representation (Principle I) Consideration 1.2: Support multiple ways to perceive information – Use graphs, tables, equations, and real-world scenarios to show the same function in multiple ways.</p> <p>Consideration 3.2: Highlight patterns, critical features, big ideas, and relationships – Help students recognize transformations and connections between functions through color coding, annotations, and guided questions.</p> <p>Multiple Means of Action & Expression (Principle II) Consideration 2.5: Illustrate through multiple media– Allow students to explain functions using writing, drawings, verbal descriptions, or digital tools.</p> <p>Consideration 6.3: Organize information and resources– Guide students in breaking down multi-step function problems through problem-solving scaffolds, peer discussions, and structured templates.</p>	<p>Multiple Means of Representation (Principle I)</p> <ul style="list-style-type: none"> • Interactive Technology: Use Desmos, GeoGebra, or graphing calculators to visually manipulate function transformations. Let students drag sliders to see how coefficients affect graphs dynamically. • Color-Coded Notes: Assign different colors for transformations (e.g., green for shifts, blue for reflections) to help students track changes in equations vs. graphs. • Real-World Analogy Mapping: Compare function transformations to real-world scenarios (e.g., stretching a rubber band = vertical stretch, flipping a pancake = reflection). <p>Multiple Means of Action & Expression (Principle II)</p> <ul style="list-style-type: none"> • Function Storytelling: Have students personify functions (e.g., "I am a quadratic function. My vertex is my home, and I always curve symmetrically!"). • Function Transformations Dance: Assign different movements for function transformations (e.g., stepping forward = horizontal shift, jumping = vertical shift, turning = reflection). • Peer Teaching with Choice: Let students choose between creating a tutorial video, writing a math comic strip, or presenting a mini-lesson on function transformations.

Supporting Multilingual/English Learners			
Related <u>CELP standards:</u>		Learning Targets:	
<p>CELP Standard 1: Construct meaning from oral presentations and literary and informational text through grade-appropriate listening, reading, and viewing.</p> <p>CELP Standard 3: Speak and write about grade-appropriate complex literary and informational texts and topics.</p> <p>CELP Standard 6: Analyze and critique the arguments of others orally and in writing.</p>		<p>CELP Standard 1:</p> <ul style="list-style-type: none"> I can determine the main idea and key details of a text or presentation to show my understanding. I can use context clues to determine the meaning of unfamiliar words in spoken and written texts. I can make connections between what I hear, read, and view to build my understanding of a topic. <p>CELP Standard 3:</p> <ul style="list-style-type: none"> I can use details and evidence from texts to support my ideas in discussions and writing. I can organize my thoughts clearly when speaking or writing about complex topics. I can revise and improve my writing to make my ideas clearer and stronger. <p>CELP Standard 6:</p> <ul style="list-style-type: none"> I can identify the claim and supporting evidence in an argument. I can evaluate whether an argument is strong or weak based on the evidence provided. I can respectfully respond to and challenge the arguments of others using logical reasoning and evidence. 	
Lesson Sequence	Learning Target	Success Criteria/ Assessment	Resources
1.2 Basics of functions and their graphs	<ul style="list-style-type: none"> I can determine whether a relation is a function or not I can determine the domain and range of a function 	<ul style="list-style-type: none"> I can determine if a relation is a function using <ul style="list-style-type: none"> A table of values Vertical line test I can determine the domain and range given a graph 	
1.3 More on functions and their graphs	<ul style="list-style-type: none"> I can identify intervals on which a function increases or decreases I can locate relative extrema 	<ul style="list-style-type: none"> I can identify intervals on which a function increases or decreases given a graph I can locate relative minima and maxima given a graph 	
1.4 Linear functions and slope	I can analyze linear functions	<ul style="list-style-type: none"> I can calculate slope I can write point slope form of equation I can write and graph slope-intercept form of an equation I can graph horizontal and vertical lines I can recognize and use standard form of an equation I can use intercepts to graph an equation 	
1.6 Transformations of Functions	I can identify transformations of Functions	<ul style="list-style-type: none"> I can use vertical shifts to graph functions I can use horizontal shifts to graph functions. I can use reflections to graph functions. I can use vertical stretching and shrinking to graph functions I can use horizontal stretching and shrinking to graph functions. 	
1.7 Combinations of functions, composite functions	I can combine Functions	<ul style="list-style-type: none"> I can combine functions using the algebra of functions. I can form composite functions 	

1.8 Inverse functions	I can analyze the inverse of a function	<ul style="list-style-type: none">• I can verify that functions are inverses.• I can find the inverse of a function.• I can use the horizontal line test to determine if a function has an inverse function.
1.10 Modeling with functions	I can use functions to model real world situations	<ul style="list-style-type: none">• I can construct functions from verbal descriptions.• I can construct functions from formulas.

Unit Title:

Unit 2: Polynomials and Rational Functions

Relevant Standards: Bold indicates priority

N.CN.A.1 Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.

N.CN.A.2 Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

N.CN.3 Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

N.CN.C.7 Solve quadratic equations with real coefficients that have complex solutions.

N.CN.C.8 Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$.

N.CN.C.9 Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

A.SSE.A.1 Interpret expressions that represent a quantity in terms of its context.

A.SSE.B.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

A.APR.A.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

A.APR.B.2 Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a , the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.

A.APR.B.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

F.IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

- F.IF.C.7.C Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

F.IF.C.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

F.IF.C.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

Essential Question(s):

- What are the defining characteristics of polynomial functions and how do their degrees and coefficients affect their behavior?
- Why are there different methods of finding the zeros of a polynomial and how do we decide which method is appropriate?
- How can polynomial functions represent real-life situations?
- What is the Fundamental Theorem of Algebra, and how does it relate to the existence of complex roots?
- How can you use the Factor Theorem and synthetic division to factorize polynomials?
- How do different types of variation apply to real-world problems, and how can understanding these relationships help solve those problems?

Enduring Understanding(s):

- Polynomial functions can be classified by their degree, which determines their general shape and the maximum number of roots they can have.
- The degree and leading coefficient determine the end behavior of a polynomial function, which helps in sketching and analyzing graphs.
- The zeros/roots of a polynomial function correspond to the x-intercepts of its graph. The multiplicity of a zero affects the behavior of the graph at that intercept, such as whether it touches or crosses the x-axis.
- Polynomial functions model various real-world phenomena, from projectile motion to economics.
- The complex number set is composed of the real numbers and the imaginary numbers.
- Complex solutions will not be x-intercepts.
- The intermediate value theorem can be used to determine the zeros of a function.
- Recognizing symmetry in polynomial functions (even, odd, or neither) helps in graphing and analyzing these functions.
- The remainder theorem tells us that if a polynomial $f(x)$ is divided by $x - c$, then the remainder is $f(c)$.

	<ul style="list-style-type: none"> The factor theorem tells us that if $f(c)=0$, then $x - c$ is a factor of $f(x)$. Graphs of rational functions have asymptotes and have many forms. Many real-life quantities can be modeled by types of variation including: direct, inverse, and joint.
Demonstration of Learning:	Pacing for Unit
Homework Class Practice Readiness Quizzes Skills Check Quizzes Mid-Unit Checkpoint End of Unit Assessment	12 blocks
Family Overview (link below)	Integration of Technology:
Precalculus ACA - Family Overview (2024-25) Precalculus ACA - Family Overview (2024-25) SPANISH	TI-84 graphing calculator Desmos calculator MyMathLab interactive homework, quizzes and tests MyMathLab video explanations
Unit-specific Vocabulary:	Aligned Unit Materials, Resources, and Technology (beyond core resources):
Complex Number, Complex Conjugate, Polynomial Function, Leading coefficient test, End Behavior, Multiplicities, Intermediate Value Theorem, Turning Points, Synthetic Division, Remainder Theorem, Factor Theorem, Rational Zero Theorem, Vertical Asymptote, Horizontal Asymptote, Slant Asymptote, Direct Variation, Inverse Variation, Joint Variation	TI-84 graphing calculator Desmos calculator Mini white boards Teacher created activities Vertical surfaces
Opportunities for Interdisciplinary Connections:	Anticipated misconceptions:
Science (Physics and Biology) <ul style="list-style-type: none"> Quadratic Expressions in Motion: Completing the square helps analyze projectile motion by revealing the maximum height (vertex) of a thrown object. Exponential Functions in Growth & Decay: Transforming exponential expressions models radioactive decay, bacterial growth, and population dynamics. Finance & Economics <ul style="list-style-type: none"> Interest & Investment Models: Exponential transformations explain compound interest and financial forecasting. Quadratic Cost & Revenue Functions: Factoring quadratics can help find break-even points in business models. Computer Science & Engineering <ul style="list-style-type: none"> Algorithm Efficiency (Big-O Notation): Exponential and polynomial expressions describe algorithm complexity. Structural Engineering: Quadratic equations help model forces and stress in materials. 	<ul style="list-style-type: none"> Factoring always works for quadratics. Completing the square is only for finding the vertex. Exponential expressions can be transformed like polynomials. Rational expressions can always be added, subtracted, multiplied, or divided without considering restrictions. Rearranging an equation always preserves all original solutions. The quadratic formula should always be used when solving quadratics. Every rational or radical equation has a valid solution. When solving $f(x)=g(x)$, there is always a single intersection. Exponential growth can be analyzed using the concept of slope. A line's slope is always positive or negative.
Connections to Prior Units:	Connections to Future Units:
<ul style="list-style-type: none"> Factoring quadratic expressions helps determine the x-intercepts (zeros) of a function. This is 	<ul style="list-style-type: none"> Using exponent properties (e.g., rewriting $2^x = 8$ as $x = \log_2(8)$) allows for solving exponential

<p>essential for graphing parabolas and understanding solutions to equations.</p> <ul style="list-style-type: none"> • Rewriting a quadratic function in vertex form by completing the square reveals the maximum or minimum point, aiding in graphing and real-world applications like projectile motion. • Using exponent properties to transform expressions reveals key features of exponential functions, helping in modeling financial growth, population changes, and radioactive decay. • Adding, subtracting, multiplying, and dividing rational expressions mirrors operations with rational numbers. These functions often have asymptotes and discontinuities, crucial for graphing. • Understanding where two function graphs intersect helps solve systems of equations, interpret business models (e.g., break-even analysis), and analyze competing trends in data. 	<p>equations, essential in financial modeling, population growth, and radioactive decay.</p> <ul style="list-style-type: none"> • Rearranging formulas (e.g., converting $A = Pe^{rt}$ into $t = \frac{\ln(A/P)}{r}$) helps analyze interest rates, half-life problems, and inverse exponential relationships in scientific contexts. • Applying factoring, completing the square, or algebraic methods to trigonometric functions (e.g., solving $2\sin^2(x) - \sin(x) - 1 = 0$ by factoring) supports work in physics, engineering, and wave motion. • Factoring and transforming expressions simplify trigonometric identities, such as rewriting $\cos^2(x) - \sin^2(x)$ using polynomial techniques to reveal properties of sine and cosine graphs. • Finding points of intersection between exponential and logarithmic functions or trigonometric and polynomial functions is crucial in modeling business trends, sound waves, and electrical circuits.
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Differentiation through *Universal Design for Learning*

UDL Indicator	Teacher Actions
<p>Multiple Means of Representation (Principle I) Consideration 1.3 Offer alternatives for visual information – Using dynamic graphing tools, interactive simulations, or color-coded diagrams can help students visualize polynomial functions, their end behavior, and how factors affect the graph.</p> <p>Consideration 3.2: Highlight patterns, critical features, big ideas, and relationships – Emphasizing key connections, such as the relationship between degree, roots, and graph shape, helps students generalize these mathematical concepts.</p>	<p>Visual and Interactive Tools – Use graphing calculators, Desmos, or GeoGebra to explore polynomial and rational function behaviors dynamically.</p> <p>Real-World Applications – Show how polynomial functions model real-life phenomena like physics (projectile motion) and economics (profit models) to improve relevance.</p> <p>Scaffolded Problem-Solving – Use guided questioning and step-by-step problem-solving to help students analyze polynomials and apply theorems (Intermediate Value Theorem, Factor Theorem).</p> <p>Multiple Forms of Representation – Present concepts using verbal explanations, symbolic notation, graphical representations, and real-world contexts to reach diverse learners.</p>

Supporting Multilingual/English Learners

Related <i>CELP standards:</i>	Learning Targets
<p>CELP Standard 1: Construct meaning from oral presentations and literary and informational text through grade-appropriate listening, reading, and viewing.</p> <ul style="list-style-type: none"> • I can determine the main idea and key details of a text or presentation to show my understanding. • I can use context clues to determine the meaning of unfamiliar words in spoken and written texts. • I can make connections between what I hear, read, and view to build my understanding of a topic. <p>CELP Standard 3: Speak and write about grade-appropriate complex literary and informational texts and topics.</p> <ul style="list-style-type: none"> • I can use details and evidence from texts to support my ideas in discussions and writing. • I can organize my thoughts clearly when speaking or writing about complex topics. • I can revise and improve my writing to make my ideas clearer and stronger. <p>CELP Standard 6: Analyze and critique the arguments of others orally and in writing.</p> <ul style="list-style-type: none"> • I can identify the claim and supporting evidence in an argument. • I can evaluate whether an argument is strong or weak based on the evidence provided. 	

- I can respectfully respond to and challenge the arguments of others using logical reasoning and evidence.

Lesson Sequence	Learning Target	Success Criteria/ Assessment	Resources
2.1 Complex numbers	I can identify complex numbers and use them to solve problems	<ul style="list-style-type: none"> • I can add and subtract complex numbers. • I can multiply complex numbers. • I can perform operations with square roots of negative numbers. • I can solve quadratic equations with complex solutions. 	
2.2 Quadratic functions	I can analyze quadratic functions	<ul style="list-style-type: none"> • I can Recognize characteristics of parabolas. • I can graph parabolas. • I can determine a quadratic functions' max or min value. • I can solve problems involving a quadratic function's min or max value. 	
2.3 Polynomial Functions and Their Graphs	I can analyze polynomial functions	<ul style="list-style-type: none"> • I can identify polynomial functions • I can recognize characteristics of graphs of polynomial functions. • I can determine end behavior. • I can Use factoring to find zeros of polynomial functions • I can identify zeros and their multiplicities • I can graph polynomial functions. 	
2.4 Dividing Polynomials Remainder and Factor Theorems.	I can divide polynomials	<ul style="list-style-type: none"> • I can use long division to divide polynomials • I can use synthetic division to divide polynomials. • I can evaluate a polynomial using the remainder theorem. • I can use the factor theorem to solve a polynomial equation. 	
2.5 Zeros of polynomial functions	I can find zeros of a polynomial function.	<ul style="list-style-type: none"> • I can solve polynomial equations. • I can use the linear factorization theorem to find polynomials with given zeros 	
2.6 Rational Functions and Their Graphs	I can analyze rational functions	<ul style="list-style-type: none"> • I can find the domains of rational functions. • I can use arrow notation. • I can identify vertical asymptotes. • I can identify horizontal asymptotes • I can use transformations to graph rational functions. • I can graph rational functions. 	

Unit Title

Unit 3: Exponential and Logarithmic Functions

Relevant Standards: Bold indicates priority**A.SSE.A.1** Interpret expressions that represent a quantity in terms of its context.

- A.SSE.A.1.B Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1 + r)^n$ as the product of P and a factor not depending on P.

A.SSE.B.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

- A.SSE.B.3.C Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15^t can be rewritten as $1.15^{1/12} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.

A.CED.A.2 Create equations that describe numbers or relationships. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.**A.REI.D.11** Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.**F.IF.C.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

- F.IF.C.7.E Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

F.IF.C.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

- F.IF.C.8.B Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = 1.02^t$, $y = 0.97^t$, $y = 1.01^{12t}$, $y = 1.2^{t/10}$, and classify them as representing exponential growth and decay.

F.BF.A.1 Write a function that describes a relationship between two quantities.**F.BF.B.5** Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.**F.LE.A.1** Distinguish between situations that can be modeled with linear functions and with exponential functions.

- F.LE.A.1.C Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

F.LE.A.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).**F.LE.A.3** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.**F.LE.A.4** For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.**F.LE.B.5** Interpret the parameters in a linear or exponential function in terms of a context.**S.ID.B.6** Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

- S.ID.B.6.A Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.

Essential Question(s)

- How can exponential functions be used to model growth and decay?
- What is the purpose of logarithms and exponential functions and how are they related to each other?
- How do you graph exponential and logarithmic functions by analyzing intercepts and end behavior?
- How are exponential and logarithmic functions used to model real-world phenomena?

Enduring Understanding(s):

- Exponential functions model situations which grow or decline at a constant percent rate.
- Graphing exponential functions and logarithmic functions will result in asymptotic behavior. Transformations (shifts, stretches, reflections) will affect the graphs.
- Logarithms can be used to solve exponential functions; and conversely, exponents are used to solve logarithmic equations.

	<ul style="list-style-type: none"> Logarithms can be used to solve equations for which no other algebraic method exists. Exponential functions model situations involving growth or decay, such as population growth, radioactive decay, and interest calculations, while logarithmic functions are used to model phenomena like the Richter scale for earthquakes and pH in chemistry. Exponential and logarithmic functions are inverse functions.
Demonstration of Learning	Pacing for Unit
Homework Class Practice Readiness Quizzes Skills Check Quizzes Mid-Unit Checkpoint End of Unit Assessment	10 blocks
Family Overview (link below)	Integration of Technology
Precalculus ACA - Family Overview (2024-25) Precalculus ACA - Family Overview (2024-25) SPANISH	TI-84 graphing calculator Desmos calculator MyMathLab interactive homework, quizzes and tests MyMathLab video explanations
Unit-specific Vocabulary	Aligned Unit Materials, Resources, and Technology (beyond core resources)
Exponential Growth, Exponential Decay, Exponential Function, Exponential Regression, Growth Factor, Decay Factor, Compounded monthly, quarterly, etc. Asymptote, Base, Natural base e, Compounded continuously, Logarithm, Logarithmic Regression, Common Logarithm, Natural Logarithm, Power Function, Logarithmic Equation, Extraneous Solution, Inverse functions	TI-84 graphing calculator Desmos calculator Mini white boards Teacher created activities Vertical surfaces
Opportunities for Interdisciplinary Connections	Anticipated misconceptions
Science (Physics, Biology, and Chemistry) <ul style="list-style-type: none"> Radioactive Decay & Half-Life (Physics & Chemistry): Exponential decay models how radioactive substances break down over time. The half-life formula uses logarithms to determine how long it takes for a substance to decay to a certain amount. pH Scale (Chemistry): Logarithmic functions are used to measure acidity and alkalinity, where pH is the negative logarithm of the hydrogen ion concentration. Population Growth (Biology & Ecology): Exponential functions model how populations grow under ideal conditions, and logistic growth models (which incorporate carrying capacity) adjust the exponential model for limited resources. Economics & Finance <ul style="list-style-type: none"> Compound Interest: Exponential functions describe how money grows in savings accounts, investments, and loans. The compound interest formula uses exponents, and logarithms are used 	<ul style="list-style-type: none"> Misunderstanding the structure of expressions- seeing each term in an algebraic expression separately rather than recognizing the expression's overall structure. Believing that equivalent expressions are always identical in appearance. incorrectly apply exponent rules, such as thinking $(a^b)^c = a^{b+c}$ instead of a^{bc}. Assuming exponential growth means a constant increase in the function's output, like linear growth. Treating logarithms as unrelated to exponents instead of understanding them as inverses. Believing that every function graph must cross the x-axis (e.g., assuming all polynomials have real roots). Automatically applying linear models, even when an exponential or quadratic model fits better. Thinking that an asymptote is a physical boundary the graph cannot cross. Thinking the b in $y = ab^x$ represents the total amount rather than the growth/decay factor per

<p>to solve for unknown time periods or rates.</p> <ul style="list-style-type: none"> ● Inflation & Depreciation: Economic trends like inflation (increasing prices over time) and depreciation (value loss of assets) are modeled using exponential functions. <p>Earth Science & Natural Disasters</p> <ul style="list-style-type: none"> ● Richter Scale for Earthquakes: The Richter scale is logarithmic, meaning that an earthquake of magnitude 6 is 10 times stronger than a magnitude 5. ● Sound Intensity (Decibels): Logarithmic functions are used in measuring sound levels in decibels, where an increase of 10 dB represents a tenfold increase in intensity. ● Carbon Dating: Scientists use exponential decay equations to estimate the age of fossils and ancient artifacts by analyzing the remaining Carbon-14 in the sample. 	<p>unit interval.</p> <ul style="list-style-type: none"> ● Believing transformations only affect shape but not key features like intercepts, end behavior, and asymptotes.
<p>Connections to Prior Units</p>	<p>Connections to Future Units</p>
<ul style="list-style-type: none"> ● Exponential functions model constant percent growth or decay, distinguishing them from linear and polynomial functions that grow at a constant rate or degree-based rate. ● Just like polynomial and rational functions, exponential and logarithmic functions can undergo transformations such as shifts, reflections, and stretches. ● Logarithms and exponentials are inverse functions, similar to how square and square root functions relate. ● Rational functions and logarithmic functions both model asymptotic behavior, meaning they never cross a certain boundary. 	<ul style="list-style-type: none"> ● Solving Trigonometric Equations Involving Exponents and Logarithms: In physics and engineering, exponential functions often appear in damping or wave equations. ● Logarithms and Periodic Growth: Logarithms are used in measuring decibels (sound intensity), earthquake magnitudes (Richter scale), and pH levels (acidity), all of which relate to oscillatory or wave-based phenomena. ● The Fourier Transform, a key tool in signal processing, represents periodic wave functions (trigonometry) as sums of exponentials (Euler's formula). ● The logarithmic spiral, which appears in nature (e.g., shells, hurricanes), is modeled using both exponential and trigonometric functions $r = ae^{b\theta}$. This function combines exponential growth (size increases) with circular motion (angle-based progression).
<p>Differentiation through <i>Universal Design for Learning</i></p>	
<p>UDL Indicator</p>	<p>Teacher Actions</p>
<p>Multiple Means of Representation (Principle 1) Consideration 1.1 Perception: Provide options for perception by offering multiple formats (e.g., visual, auditory, kinesthetic) for understanding exponential growth, decay, and transformations in graphs.</p> <p>Consideration 1.2 Language & Symbols: Clarify vocabulary and symbols by providing definitions, explanations, and examples of terms like growth/decay, logarithms, and transformations.</p>	<p>Multiple Means of Representation (Principle 1)</p> <ul style="list-style-type: none"> ● Present the exponential function $y = P(1 + r)^t$ using multiple formats, such as a table, graph, and verbal explanation. Use graphing software (like Desmos or GeoGebra) to show the curve's shape and asymptotic behavior. ● Offer a visual representation alongside a text-based explanation for students with different learning preferences. ● Provide animated videos showing the behavior of exponential growth (such as population growth) and decay (such as radioactive decay), so students can see how these concepts evolve over time.

- Simplify complex expressions by breaking them down into smaller, more digestible pieces.

Supporting Multilingual/English Learners

Related CELP standards:

Learning Targets

CELP Standard 1:

Construct meaning from oral presentations and literary and informational text through grade-appropriate listening, reading, and viewing.

- I can determine the main idea and key details of a text or presentation to show my understanding.
- I can use context clues to determine the meaning of unfamiliar words in spoken and written texts.
- I can make connections between what I hear, read, and view to build my understanding of a topic.

CELP Standard 3:

Speak and write about grade-appropriate complex literary and informational texts and topics.

- I can use details and evidence from texts to support my ideas in discussions and writing.
- I can organize my thoughts clearly when speaking or writing about complex topics.
- I can revise and improve my writing to make my ideas clearer and stronger.

CELP Standard 6:

Analyze and critique the arguments of others orally and in writing.

- I can identify the claim and supporting evidence in an argument.
- I can evaluate whether an argument is strong or weak based on the evidence provided.
- I can respectfully respond to and challenge the arguments of others using logical reasoning and evidence.

Lesson Sequence	Learning Target	Success Criteria/ Assessment	Resources
3.1 Exponential Functions	I can analyze exponential functions	<ul style="list-style-type: none"> • I can evaluate exponential functions • I can graph exponential functions • I can evaluate functions with base e. • I can use compound interest formula. 	
3.2 Logarithmic Functions	I can analyze logarithmic functions	<ul style="list-style-type: none"> • I can change from log to exponential form. • I can change from exponential to log form. • I can evaluate logarithms. • I can use basic logarithmic properties. • I can use common and natural logarithms 	
3.3 Properties of Logarithms	I can use properties of logarithms to find equivalent expressions	<ul style="list-style-type: none"> • I can use the product rule. • I can use the quotient rule. • I can use the power rule. • I can expand logarithmic expressions. • I can condense logarithmic expressions. • I can use the the change of base property 	
3.4 Exponential and Logarithmic Equations	I can solve exponential and logarithmic equations.	<ul style="list-style-type: none"> • I can use like bases to solve exponential equations. • I can use logarithmic to solve exponential equations • I can use the definition of a logarithm to solve logarithmic equations. 	
3.5 Exponential Growth and Decay	I can solve real world problems using logarithms.	<ul style="list-style-type: none"> • I can model exponential growth and decay. • I can use logistic growth models. • I can express and exponential model in base e. 	

Unit Title	
Unit 4: Trigonometric Functions	
Relevant Standards: Bold indicates priority	
<p>F.TF.A.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.</p> <p>F.TF.A.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.</p> <p>F.TF.A.3 Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.</p> <p>F.TF.A.4 Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.</p> <p>F.TF.B.6 Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.</p> <p>G.SRT.C.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.</p> <p>G.SRT.C.7 Explain and use the relationship between the sine and cosine of complementary angles.</p> <p>G.SRT.C.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.</p>	
Essential Question(s)	Enduring Understanding(s)
<ul style="list-style-type: none"> • How are sine, cosine, and tangent defined using the sides of a right triangle? • What are the reciprocal trigonometric ratios (cosecant, secant, cotangent)? • How do the coordinates of points on the unit circle relate to trigonometric functions? • What are the radian and degree measures, and how do they relate to each other? • How do amplitude, period, phase shift, and vertical shift affect the graphs of trigonometric functions? • What are the key features of the graphs of sine, cosine, tangent, cosecant, secant, and cotangent functions? • What are the definitions and domains of the inverse sine, cosine, and tangent functions? • How are the inverse functions used to find angles given a trigonometric ratio? 	<ul style="list-style-type: none"> • The trigonometric functions (sine, cosine, tangent, and their reciprocals) represent specific relationships between the angles and side lengths of a right triangle. Understanding these functions as ratios helps students connect geometric intuition with algebraic representation. • The unit circle is a foundational concept in trigonometry that connects the angle measures in radians with the coordinates of points on the circle. This understanding leads to the concept of periodicity in trigonometric functions, where patterns repeat over regular intervals. • The graphs of sine, cosine, and tangent functions exhibit unique characteristics such as amplitude, period, phase shift, and vertical shift. Recognizing these properties allows students to model real-world phenomena and understand oscillatory behavior. • Inverse trigonometric functions, such as arcsine, arccosine, and arctangent, are essential for finding angles when given a trigonometric ratio. Understanding their domains, ranges, and interpretations is crucial for solving trigonometric equations and modeling. • Trigonometric functions can be represented analytically (using equations) and graphically (using graphs). Understanding both forms and how they interrelate is important for solving problems and interpreting data. • Understanding different units of angle measurement, such as degrees and radians, and how to convert between them is crucial for working with trigonometric functions and applying them in various contexts.
Demonstration of Learning	Pacing for Unit
Homework Class Practice	16 blocks

Readiness Quizzes Skills Check Quizzes Mid-Unit Checkpoint End of Unit Assessment	
Family Overview (link below)	Integration of Technology
Precalculus ACA - Family Overview (2024-25) Precalculus ACA - Family Overview (2024-25) SPANISH	TI-84 graphing calculator Desmos calculator MyMathLab interactive homework, quizzes and tests MyMathLab video explanations
Unit-specific Vocabulary	Aligned Unit Materials, Resources, and Technology (beyond core resources)
Radian, Degree, Coterminal Angles, Complement, Supplement, Sine, Cosine, Tangent, Secant, Cosecant, Cotangent, Reference Angle, Unit Circle, Amplitude, period	TI-84 graphing calculator Desmos calculator Mini white boards Teacher created activities Vertical surfaces
Opportunities for Interdisciplinary Connections:	Anticipated misconceptions
<p>Physics: Wave Motion and Harmonics Trigonometric functions such as sine and cosine are commonly used to model wave motion, which is a fundamental concept in physics. The oscillations of waves, such as sound waves, light waves, and mechanical vibrations, can be described using trigonometric functions. The concepts of amplitude, period, and phase shift are used to understand wave behavior and harmonic motion.</p> <p>Engineering: Signal Processing In electrical engineering, trigonometric functions are integral to analyzing and designing circuits, particularly in signal processing. The concept of periodicity in trigonometric functions is crucial for understanding alternating current (AC) signals and their behaviors over time.</p> <p>Astronomy: Orbital Mechanics Trigonometry plays a vital role in astronomy and space science, especially in understanding the orbits of planets and satellites. The unit circle and trigonometric functions are used to calculate the positions of celestial bodies and to model orbital motion, such as the movement of planets around the sun or artificial satellites orbiting Earth.</p> <p>Computer Science: Graphics and Animation Trigonometric functions are foundational in computer graphics, particularly in the rendering of 3D graphics and animations. Sine and cosine functions help model rotations and scaling transformations of objects in a virtual space.</p> <p>Economics: Cyclical Behavior and Seasonal Trends Trigonometric functions can be used to model cyclical or periodic behaviors in economics, such as seasonal demand, temperature fluctuations, or economic cycles. Understanding how periodic functions like sine and cosine apply to real-world data can help economists predict trends and make forecasts.</p>	<ul style="list-style-type: none"> Students often confuse radians and degrees as two equivalent systems of measuring angles, without fully understanding the concept of radians as the ratio of the arc length to the radius on the unit circle. Thinking that the unit circle and trigonometric functions only apply to angles within one full rotation (0 to 360 degrees or 0 to 2π radians). Students may not fully grasp that trigonometric functions (like sine, cosine, and tangent) repeat at regular intervals (i.e., they are periodic functions). Thinking that all angles on the unit circle are positive (in a counterclockwise direction), neglecting the fact that angles can also be negative, which correspond to clockwise motion. Incorrectly applying the unit circle by misunderstanding how to find the sine, cosine, and tangent values based on the coordinates on the unit circle. Mistaking special triangle values for all angles
Connections to Prior Units:	Connections to Future Units:

<ul style="list-style-type: none"> • Just as algebraic functions (polynomials, rational functions) undergo transformations such as shifts, reflections, and stretches, so do trigonometric functions. These transformations (e.g., amplitude changes, phase shifts, vertical shifts) help students understand how functions behave in a coordinate plane, similar to the transformations applied to polynomial or rational graphs. • Exponential functions and trigonometric functions are deeply connected through Euler’s formula, $e^{ix} = \cos(x) + i\sin(x)$. This formula connects exponential growth or decay to trigonometric functions, demonstrating how trigonometric functions can model oscillatory behavior (e.g., in waveforms) alongside exponential growth or decay. • Trigonometric identities (such as the Pythagorean identity $\sin^2(x) + \cos^2(x) = 1$) allow trigonometric expressions to be simplified into polynomial forms. This is useful for solving equations that may involve both trigonometric and polynomial terms. 	<ul style="list-style-type: none"> • Analytic trigonometry involves the use of identities (like the Pythagorean identity, sum and difference formulas, etc.) to simplify trigonometric expressions and solve equations. For example, solving trigonometric equations often requires applying identities to reduce complex expressions into simpler forms, making the equations easier to handle analytically. • Systems of equations that involve trigonometric functions can often be solved using algebraic techniques, such as substitution or elimination. For example, if you have a system that involves both sine and cosine, you might use the Pythagorean identity $\sin^2(x) + \cos^2(x) = 1$ to convert one equation into a simpler form. • In analytic trigonometry, inverse functions (like arcsin, arccos, and arctan) are used to solve for angles in trigonometric equations. These functions allow us to find specific angle measures when given a trigonometric ratio, which is essential when solving systems of equations involving trigonometric relationships. • In systems of equations, graphing trigonometric functions can visually show the solutions where different trigonometric functions intersect. For instance, solving a system of two sine and cosine equations can be done graphically by plotting the functions and identifying where their graphs intersect, providing a visual understanding of the solution.
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Differentiation through *Universal Design for Learning*

UDL Indicator	Teacher Actions:

Supporting Multilingual/English Learners

Related <i>CELP standards</i> :	Learning Targets:
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<p>CELP Standard 1: Construct meaning from oral presentations and literary and informational text through grade-appropriate listening, reading, and viewing.</p> <ul style="list-style-type: none"> • I can determine the main idea and key details of a text or presentation to show my understanding. • I can use context clues to determine the meaning of unfamiliar words in spoken and written texts. • I can make connections between what I hear, read, and view to build my understanding of a topic. 	
<p>CELP Standard 3: Speak and write about grade-appropriate complex literary and informational texts and topics.</p> <ul style="list-style-type: none"> • I can use details and evidence from texts to support my ideas in discussions and writing. • I can organize my thoughts clearly when speaking or writing about complex topics. • I can revise and improve my writing to make my ideas clearer and stronger. 	
<p>CELP Standard 6: Analyze and critique the arguments of others orally and in writing.</p> <ul style="list-style-type: none"> • I can identify the claim and supporting evidence in an argument. • I can evaluate whether an argument is strong or weak based on the evidence provided. • I can respectfully respond to and challenge the arguments of others using logical reasoning and evidence. 	

Lesson Sequence	Learning Target	Success Criteria/ Assessment	Resources
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4.1 Angles and Radian Measure	I can recognize and use the vocabulary of angles	<ul style="list-style-type: none"> • I can use degree measure. • I can use radian measure. • I can convert between degrees and radians. • I can draw angles in standard position. • I can find coterminal angles. • I can find the length of a circular arc. • I can use linear and angular speed to describe motion on a circular arc.
4.2 Trigonometric Functions: The unit Circle	I can use a unit circle to define trigonometric functions of real numbers	<ul style="list-style-type: none"> • I can recognize the domain and range of sine and cosine functions. • I can find exact values of trigonometric functions • I can evaluate trigonometric functions.
4.3 Right Triangle Trigonometry	I can use right triangles to evaluate trigonometric functions.	<ul style="list-style-type: none"> • I can find function values for 30 degrees and 60 degrees. • I can use right triangle trig to solve applied problems.
6.1 The Law of Sines	I can use the Law of Sines to solve triangles.	<ul style="list-style-type: none"> • I can use the law of sines to solve oblique triangles. • I can use the law of sines to solve, if possible, the triangle or triangles in the ambiguous case. • I can find the area of an oblique triangle. • I can solve applied problems using the law of sines.
6.2 Law of Cosines	I can use the Law of Cosines to solve triangles	<ul style="list-style-type: none"> • I can use the law of cosines to solve oblique triangles. • I can solve applied problems using the law of cosines. • I can use heron's formula to find the area of a triangle
4.4 Trigonometric Functions of Any Angle	I can use the definitions of trigonometric functions of any angle.	<ul style="list-style-type: none"> • I can use the signs of the trigonometric functions. • I can find reference angles. • I can use reference angles to evaluate trigonometric functions.
4.5 Graphs of Sine and Cosine Functions	I can understand the graphs of Sin and Cos Functions	<ul style="list-style-type: none"> • I can graph variations of a sine function. • I can graph variations of a cosine function. • I can use vertical shifts of sine and cosine curves. • I can model periodic behavior.
4.7 Inverse Trigonometric Functions	I can understand and use the inverse sin and cos functions.	<ul style="list-style-type: none"> • I can find exact values of inverse sin and cos functions.
4.8 Applications of Trigonometric Functions	I can use Trigonometry to solve applied problems.	<ul style="list-style-type: none"> • I can solve a right triangle. • I can solve problems involving bearing.

Unit Title	
Unit 5: Analytic Trigonometry	
Relevant Standards: Bold indicates priority	
<p>F.T.F.C.8 Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, $\tan(\theta)$ and the quadrant of the angle.</p> <p>F.T.F.C.9 Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems</p> <p>A.SSE.A.2 Use the structure of an expression to identify ways to rewrite it.</p> <p>G.SRT.D.9 Apply trigonometry to general triangles. Derive the formula $A = (1/2)ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.</p> <p>G.SRT.D.10 Apply trigonometry to general triangles. Prove the Laws of Sines and Cosines and use them to solve problems.</p> <p>G.SRT.D.11 Apply trigonometry to general triangles. Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).</p>	
Essential Question(s)	Enduring Understanding(s)
<ul style="list-style-type: none"> Why do we structure trig functions in different ways? What is the difference between solving an equation and verifying a trigonometric identity? How do you decide which strategies to use when verifying a trigonometric identity? How are trigonometric identities used in the process of solving trig equations? How are trigonometric ratios used in solving real-world problems? How do you determine whether the law of sines or law of cosines is most appropriate for solving a particular problem? In ambiguous cases, how many possible triangles can the Law of Sines produce? Is it possible to solve for the area of a triangle if the altitude is not known? What kind of real-life situations can be modeled by sinusoidal equations? How can sinusoidal equations be used to predict future values? How can the area of a triangle be found using the law of sines and cosines? 	<ul style="list-style-type: none"> Trigonometric functions are useful for modeling periodic phenomena. Expressions can be written in multiple ways using the rules of Trigonometry and Algebra and used to solve trigonometric functions. The law of sines and cosines are fundamental trigonometry laws that can be used to solve problems involving the sides and angles of non-right triangles. In a non-right triangle the given information determines which law is most appropriate to solve a problem. When you know two sides and a non-included angle (SSA), there can be zero, one, or two possible triangles, known as the "ambiguous case." If the altitude of a non-right triangle is not given, it can be expressed using trigonometry and used to calculate its area.
Demonstration of Learning:	Pacing for Unit
Homework Class Practice Readiness Quizzes Skills Check Quizzes Mid-Unit Checkpoint End of Unit Assessment	5 blocks
Family Overview (link below)	Integration of Technology
Precalculus ACA - Family Overview (2024-25) Precalculus ACA - Family Overview (2024-25) SPANISH	TI-84 graphing calculator Desmos calculator MyMathLab interactive homework, quizzes and tests MyMathLab video explanations
Unit-specific Vocabulary	Aligned Unit Materials, Resources, and Technology (beyond core resources)

<p>Sine, Cosine, Tangent, Secant, Cosecant, Cotangent, Sum Formula, Difference Formula, Double Angle Formula, Half Angle Formulas, Pythagorean identity, Pythagorean theorem, Unit circle, Quotient identity, Law of Sines, Law of Cosines, Angle of Elevation, Angle of Depression, Bearings, Heron's Formula, Ambiguous Case, Included Angle, Solving Triangles, Acute Triangle, Right Triangle, Obtuse Triangle, Oblique Triangle</p>	<p>TI-84 graphing calculator Desmos calculator Mini white boards Teacher created activities Vertical surfaces</p>
<p>Opportunities for Interdisciplinary Connections:</p>	<p>Anticipated misconceptions:</p>
<p>Physics - Wave Motion and Oscillations: Trigonometric functions (especially sine and cosine) model periodic phenomena such as sound waves, light waves, and mechanical vibrations. Understanding the relationships between the amplitude, frequency, and phase of wave functions helps students in physics to analyze oscillatory systems.</p> <p>Engineering - Signal Processing and Electrical Engineering: Analytical trigonometry plays a key role in signal processing, particularly in analyzing alternating current (AC) circuits and Fourier transforms. Engineers use sine and cosine functions to model signals and study the frequency and phase shifts in electrical systems.</p> <p>Astronomy - Orbital Mechanics: Trigonometric functions are used to calculate the positions and motions of celestial bodies. The concepts of angle measurements, periodicity, and phase shifts help describe the orbits of planets, satellites, and the apparent motion of stars.</p> <p>Geography and Navigation: Bearings, Angle of Elevation, and Angle of Depression are essential concepts used in geography and navigation. In these fields, trigonometric principles, like the Law of Sines and Law of Cosines, are applied to mapmaking, surveying, and GPS navigation.</p>	<ul style="list-style-type: none"> • Students often confuse the unit circle with a simple circle, not realizing that its radius is always 1 and the coordinates on the circle represent the cosine and sine values of the corresponding angles. • Incorrectly assuming that trigonometric functions, like sine or cosine, have a range of all real numbers. • Students may incorrectly apply trigonometric identities, such as assuming $\sin(\theta) = \cos(\theta)$ for all angles or misapplying the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$. • Overlooking the periodic nature of trigonometric functions and assuming that functions like sine, cosine, and tangent behave as if they are linear and continue to increase or decrease indefinitely. • Students may mix up angle measurements in degrees and radians, especially when using trigonometric functions. • Confusing angle of elevation and angle of depression. • Incorrect application of the Law of Sines and Law of Cosines.
<p>Connections to Prior Units:</p>	<p>Connections to Future Units:</p>
<ul style="list-style-type: none"> • Trigonometric functions (sine, cosine, tangent, etc.) can be analyzed as functions with specific domains and ranges. These functions can be graphed on the coordinate plane, providing insights into the relationship between angles and side lengths in triangles, similar to how algebraic functions are graphed and interpreted. This connection reinforces the understanding of functions, their graphs, and the properties of periodicity, amplitude, and symmetry. • Just as algebraic functions undergo transformations (shifts, stretches, reflections), trigonometric functions can also be transformed. For example, the graph of $y = \sin(x)$ can be shifted, reflected, and stretched. These transformations are central to understanding how the periodic nature of trigonometric functions is affected by changes in the input. • The concept of inverse functions extends from algebra to trigonometry. Just as logarithmic functions are inverses of exponential functions, 	<ul style="list-style-type: none"> • Many trigonometric equations, such as $\sin(x) = 1/2$ or $\tan(x) = 3$, can be solved as part of a system of equations. By applying inverse trigonometric functions, students can solve for angles and use the solutions to form systems of equations with multiple variables. • Trigonometry plays a significant role in polar coordinates, where equations involving trigonometric functions can describe curves or points. Students may need to solve systems of equations involving sine and cosine functions to find points of intersection or curves represented in polar form. • Trigonometric identities can be used to simplify and solve systems of trigonometric equations. For example, by using the sum and difference identities, students can simplify equations in terms of sine and cosine and solve systems of equations to find values of angles or unknowns. • Sometimes, trigonometric functions are involved in linear systems, such as solving systems of

<p>inverse trigonometric functions (such as arcsine, arccosine, and arctangent) allow for the solving of equations involving trigonometric functions. Understanding their domains and ranges is similar to understanding how logarithmic functions are used to solve exponential equations.</p> <ul style="list-style-type: none"> Trigonometric identities, such as the Pythagorean identities, sum and difference identities, and double-angle formulas, often require algebraic manipulation for simplification and solving equations. For example, simplifying expressions involving trigonometric functions can be approached in a manner similar to simplifying polynomial or rational expressions in algebra. 	<p>equations where one equation is trigonometric (e.g., $\sin(x) + 2 = 3$) and the other is linear or polynomial. These systems can be solved using algebraic techniques, substitution, or numerical methods.</p> <ul style="list-style-type: none"> Analytical trigonometry connects to real-world systems, such as modeling wave motion, oscillations, or sound waves. In these cases, students may solve systems of equations involving trigonometric functions to model physical phenomena like harmonic motion or electrical circuits, where multiple trigonometric relationships interact.
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Differentiation through *Universal Design for Learning*

UDL Indicator	Teacher Actions

Supporting Multilingual/English Learners

Related <u>CELP standards:</u>	Learning Targets
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CELP Standard 1:
Construct meaning from oral presentations and literary and informational text through grade-appropriate listening, reading, and viewing.

- I can determine the main idea and key details of a text or presentation to show my understanding.
- I can use context clues to determine the meaning of unfamiliar words in spoken and written texts.
- I can make connections between what I hear, read, and view to build my understanding of a topic.

CELP Standard 3:
Speak and write about grade-appropriate complex literary and informational texts and topics.

- I can use details and evidence from texts to support my ideas in discussions and writing.
- I can organize my thoughts clearly when speaking or writing about complex topics.
- I can revise and improve my writing to make my ideas clearer and stronger.

CELP Standard 6:
Analyze and critique the arguments of others orally and in writing.

- I can identify the claim and supporting evidence in an argument.
- I can evaluate whether an argument is strong or weak based on the evidence provided.
- I can respectfully respond to and challenge the arguments of others using logical reasoning and evidence.

Lesson Sequence	Learning Target	Success Criteria/ Assessment	Resources
5.1 Verifying Trigonometric Identities	I can use the fundamental trigonometric identities to verify identities.	<ul style="list-style-type: none"> I can show equivalence of expressions using trigonometric identities 	
5.5 Trigonometric Equations	I can solve trigonometric equations	<ul style="list-style-type: none"> I can find all solutions of a trigonometric equation. I can solve equations with multiple angles. I can solve trigonometric equations in linear form. I can solve trigonometric equations in quadratic form. I can use factoring to separate different functions in trigonometric equations. I can use identities to solve equations. I can use a calculator to solve trigonometric equations. 	

Unit Title	
Unit 6: Systems of Equations and Inequalities	
Relevant Standards: Bold indicates priority	
<p>HSA.CED.A.3: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.</p> <p>HSA.REI.C.6: Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.</p> <p>HSA.REI.C.7: Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.</p> <p>HSA.REI.D.11: Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations.</p> <p>HSA.REI.D.12: Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.</p>	
Essential Question(s)	Enduring Understanding(s)
<ul style="list-style-type: none"> • How can we model real-world situations using systems of equations or inequalities? • How do we interpret solutions to systems of equations and inequalities in the context of a problem? • How do constraints affect the solutions to systems of equations or inequalities? • What methods can we use to solve systems of linear equations exactly (algebraically and graphically)? • How do the solutions to systems of linear equations relate to their graphical representations? • How do the solutions to a system consisting of a linear and a quadratic equation differ from those in a purely linear system? • How do we solve systems involving both linear and quadratic equations algebraically and graphically? • Why do the points of intersection of two graphs represent the solutions to the system of equations? • How do the solutions to systems of equations change based on the nature of the functions? 	<ul style="list-style-type: none"> • Systems of equations and inequalities model real-world scenarios, and their solutions represent viable and nonviable options depending on the constraints of the problem. • Representing constraints through equations and inequalities provides insight into relationships between different variables in a given context. • Systems of equations can be used to predict outcomes, make decisions, and optimize processes in various fields. • A system of linear equations can be solved using various methods, including substitution, elimination, and graphing. • Solutions to systems of linear equations can be interpreted graphically as the points of intersection of the corresponding lines. • The process of solving systems of linear equations builds a foundation for more complex systems, including those involving nonlinear equations. • Solutions to linear inequalities are represented as half-planes on the coordinate plane, and the solution set of a system of inequalities is the intersection of these half-planes. • The boundary of the solution set in a system of inequalities is defined by the equation of the inequality, and whether the boundary is included or excluded depends on whether the inequality is strict or non-strict. • Graphing systems of inequalities provides a visual representation of the range of possible solutions, helping to understand the feasibility of different options.
Demonstration of Learning	Pacing for Unit
Homework Class Practice Readiness Quizzes Skills Check Quizzes	12 blocks

Mid-Unit Checkpoint End of Unit Assessment	
Family Overview (link below)	Integration of Technology
Precalculus ACA - Family Overview (2024-25) Precalculus ACA - Family Overview (2024-25) SPANISH	TI-84 graphing calculator Desmos calculator MyMathLab interactive homework, quizzes and tests MyMathLab video explanations
Unit-specific Vocabulary	Aligned Unit Materials, Resources, and Technology (beyond core resources)
System of Equations, Solution to a System, Linear System, Linear Inequality, Substitution Method, Elimination Method, Graphical Method, Intersection, Ordered Pair, Ordered Triple, Feasible Region, Constraint, Dependent System, Independent System, Inconsistent System, Partial Solution, Graphing Linear Inequalities, Bounded Region, Unbounded Region, Dual System, Substituting into Inequalities, Critical Points, Convex Set, Basic Feasible Solution, Linear Programming, Objective Function, Slopes of Lines, Parallel Lines, Coincident Lines, Linear Transformation	TI-84 graphing calculator Desmos calculator Mini white boards Teacher created activities Vertical surfaces
Opportunities for Interdisciplinary Connections	Anticipated misconceptions
<p>Economics (Optimization and Resource Allocation): Systems of equations and inequalities are commonly used in economics to model scenarios such as resource allocation, budget constraints, and supply and demand.</p> <p>Engineering (Structural Analysis and Design): Systems of equations in three variables and inequalities are often used in engineering to analyze and design structures, such as bridges, buildings, and mechanical systems. For example, equations can represent forces, moments, and equilibrium conditions, while inequalities can describe safety factors or material strength constraints.</p> <p>Environmental Science (Modeling Population Growth and Resource Management): In environmental science, systems of equations and inequalities are used to model population dynamics, resource consumption, and environmental factors.</p>	<ul style="list-style-type: none"> • Misunderstanding the meaning of solutions- Students often think that the solution to a system of equations represents a single point on a graph, rather than the set of all points that satisfy the system. • Confusing solutions with intersections- Some students believe that the solution is only where the graphs of the equations physically intersect on the graph, ignoring other possible solution sets such as parallel lines (no solutions) or coincident lines (infinitely many solutions). • Students may forget to consider the domain and range of the inequalities, assuming all values of x and y are solutions within a system of inequalities. • Students may graph systems of inequalities by shading regions incorrectly, confusing the boundary lines (whether the inequality is strict or non-strict). • Students may confuse systems of linear equations with nonlinear equations, leading to incorrect graphing or solution methods. • Students may assume that solving systems of three variables is always more complex and that it is unnecessary to check for special cases like infinitely many solutions or no solution. • Students sometimes mix up the approach to solving systems of equations with systems of inequalities, believing that the solutions are always specific points rather than regions of possible solutions.
Connections to Prior Units	Connections to Future Units
<ul style="list-style-type: none"> • Systems of equations in two or more variables involve algebraic manipulation to isolate variables, solve for unknowns, and simplify expressions. 	<ul style="list-style-type: none"> • In introductory college math, students often learn about matrices and determinants as tools for solving systems of linear equations. The concepts

<p>These processes connect directly to basic algebraic concepts such as substitution, elimination, and solving for unknowns.</p> <ul style="list-style-type: none"> • A system of equations often represents two or more functions (e.g., linear, quadratic) and their intersections. Understanding how functions relate to each other graphically is crucial for solving these systems, whether through graphical or algebraic methods. • Systems of equations in two variables are often solved graphically by plotting the equations on the coordinate plane and identifying the intersection points. This graphical representation connects to visualizing functions and understanding the relationships between them. • Systems may involve polynomials or rational functions, especially when dealing with non-linear systems (e.g., quadratic systems or rational equations). Solving these systems can involve factoring polynomials, solving quadratic equations, or working with rational expressions. 	<p>of row reduction, matrix multiplication, and matrix inverses are introduced as methods for solving systems efficiently, especially for larger systems.</p> <ul style="list-style-type: none"> • Systems of inequalities are foundational to linear programming and optimization problems, often introduced in introductory college courses. These systems model constraints and objective functions in maximizing or minimizing quantities, such as cost or profit. • In introductory calculus, systems of equations often appear in optimization problems where students analyze functions to find maximum and minimum values. Systems may also be used in solving problems that involve rates of change, like related rates or optimization under constraints.
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Differentiation through Universal Design for Learning	
UDL Indicator	Teacher Actions

Supporting Multilingual/English Learners	
Related CELP standards:	Learning Targets

<p>CELP Standard 2: Interactive Support for Communication</p> <ul style="list-style-type: none"> • Provide graphs, charts, and step-by-step diagrams to illustrate systems of equations and inequalities. • Use color coding to differentiate equations in a system when solving by substitution or elimination. • Show real-world examples of optimization problems using visuals (e.g., maximizing profit in a business). • Use labeled coordinate planes to demonstrate graphing linear and nonlinear inequalities. <p>CELP Standard 3: Language for Content-Specific Learning</p> <ul style="list-style-type: none"> • Provide sentence frames for explaining reasoning: The ordered pair (x, y) is/ is not a solution because...I substituted x into the equation and found... • Offer structured writing prompts to explain how they verified a solution. • Use a word bank with mathematical vocabulary and definitions (e.g., solution, substitution, elimination, system, inequality). <p>CELP Standard 1: Engagement in Complex Tasks</p> <ul style="list-style-type: none"> • Pair students to work through problems using think-pair-share strategies. • Encourage group work with structured roles (e.g., one student explains while the other solves). • Use real-world scenarios and project-based learning where students solve applied problems collaboratively. <p>CELP Standard 4: Meaningful Engagement with Complex Texts</p> <ul style="list-style-type: none"> • Use videos or animations to model solving systems of equations. • Implement interactive online graphing tools (e.g., Desmos) for hands-on practice. • Provide audio support for word problems and key concepts to reinforce comprehension. <p>CELP Standard 5: Constructing and Expanding Knowledge</p> <ul style="list-style-type: none"> • Connect mathematical problems to students' real-life experiences (e.g., budgeting, business, physics applications). • Allow students to explain solutions in their home language before translating into English. • Support comprehension with bilingual glossaries or peer support when needed. 	
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Lesson Sequence	Learning Target	Success Criteria/ Assessment	Resources
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6.1 System of equations in two variable.	I can solve a system of linear equations with 2 variables.	<ul style="list-style-type: none"> I can determine whether an ordered pair is a solution of a linear system I can solve linear systems by substitution I can solve linear systems by elimination I can identify systems that do not have exactly one ordered pair solutions Solve applied problems using a system of linear equations.
6.2 Systems of linear equations in 3 variables	I can solve a system of equations with 3 variables.	<ul style="list-style-type: none"> I can verify the solution of a system of linear equations in 3 variables. I can solve a system of linear equations in 3 variables. I can solve apple problems using a system in 3 variables.
6.4 systems of nonlinear equations in 2 variables	I can solve a nonlinear system of equations	<ul style="list-style-type: none"> I can recognize systems of nonlinear equations in 2 variables. I can solve nonlinear systems by substitution. I can solve nonlinear systems by addition. I can solve applied problems using a system of linear equations.
6.5 System of inequalities	I can graph a system of linear inequalities	<ul style="list-style-type: none"> I can graph a system of linear inequalities in 2 variables. I can graph a nonlinear inequality I can graph a system of inequalities
6.6 Linear programming	I can use linear programming to solve problems.	<ul style="list-style-type: none"> I can write an objective function describing a quantity that must be maximized or minimized. I can use inequalities to describe limitations in a situation.

NOTE:

Pacing adds up to 88 blocks. Adjust as needed to ensure that there is time for the midterm exam and review and final exam and review.