Advanced Topics and Modeling in Mathematics

Content Standards Revisions

2022

Course Title:	Advanced Topics and Modeling in Mathematics
Course/Unit Credit:	1
Course Number:	439050
Teacher Licensure:	Please refer to the Course Code Management System (https://adedata.arkansas.gov/ccms/) for the most current licensure codes.
Grades:	9-12
Prerequisite:	Algebra I, Geometry, Algebra II

Advanced Topics and Modeling in Mathematics

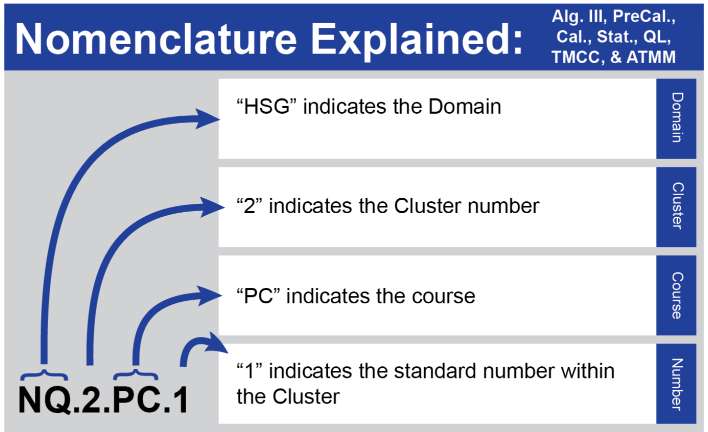
This course builds on Algebra I, Geometry, and Algebra II to explore mathematical topics and relationships beyond Algebra II. Emphasis will be placed on applying modeling as the process of choosing and using appropriate mathematics and statistics to analyze, to better understand, and to improve decisions in analyzing empirical situations. Collection and use of student-generated data should be an aspect of the course. Students will represent and process their reasoning and conclusions numerically, graphically, symbolically, and verbally. Students will be expected to use technology, including graphing calculators, computers, and data gathering equipment throughout the course. Advanced Topics and Modeling in Mathematics does not require Arkansas Department of Education Division of Elementary and Secondary Education approval.

Introduction to Secondary Arkansas Mathematics Standards

When the Division of Elementary and Secondary Education (DESE) began the process of revising math standards, a diverse group of qualified educators from across the state came together to craft Arkansas standards specific to the schools and students in the state. The result of this work, the Arkansas Mathematics Standards, is contained in this document. These standards reflect what educators across the state know to be best for Arkansas students.

Standards Organization: The revision committee maintained the organizational structure and nomenclature of the previous standards. Secondary Arkansas Mathematics Standards are categorized into domains, clusters, and standards.

- **Domains** represent the big ideas to be studied in each course and sometimes across grade bands. These big ideas support educators in determining the proper amount of focus and instructional time to be given to each of these topics.
- **Clusters** represent collections of standards grouped to help educators understand the building blocks of rich and meaningful instructional units. These units help students make connections within clusters and avoid seeing mathematics as a discrete list of skills they must master.
- **Standards** represent the foundational building blocks of math instruction. The standards outlined in this document work together to ensure that students are college and career ready and on track for success.



Standards Support: The revision of the Arkansas Mathematics Standards represent the work of the committee to provide greater clarity, strength, and support of the standards. Additionally, the revised mathematics standards are designed to help educators better understand the areas of emphasis and the focus within the standards. Educators should address the bulleted content as more than a checklist of items that they must teach individually. Content is bulleted to provide specificity of learning expectations included within some extensive standards. In some instances, the standard document includes Arkansas examples, teacher notes, specifications, and italicized words to assist educators with planning, teaching, and student learning.

- **Examples** included in the original standards were either changed for clarity or separated from the body of the actual standard. The examples included in the body of the standards document in no way reflect all of the possible examples. Likewise, these examples do not mandate curriculum or problem types. Local districts are free to select the high-quality curricula and instructional methods that best meet the needs of their students.
- **Teacher notes** offer clarification of the standards. These notes are intended to clarify, for teachers, what the expectations are for the learner. Likewise, these notes provide instructional guidance and limitations so that educators can better understand the scope of the standard. This will help with determining what is developmentally appropriate for students when working with specific standards.
- **Standard specifications** are to strengthen standards. The specifications are precise statements highlighting the need for mastery or function-type parameters for specific standards. This will assist educators in pinpointing the best opportunities for students to gain and master the knowledge and skills needed to succeed in a progression.
- **Asterisks (*)** are denoted to represent the modeling component of the standards. These standards should be presented in a modeling context which allows students to engage in the modeling process that is outlined in the Standards for Mathematical Process. (See Appendix A)
- **Italicized words** are defined in the glossary.

Finally, the Arkansas Mathematics Standards will be a living document. As these standards are implemented across schools in the state, DESE welcomes further suggestions related to notes of clarification, examples, professional development needs, and future revisions of the standards.

K - 12 Standards for Mathematical Practices	
1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics.*	5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Advanced Topics and Modeling in Mathematics Standards: Overview	
Abbreviations: The following abbreviations are for the domains for the Arkansas Mathematics Standards.	
Functions - F	
	1. Students will analyze and interpret functions using different representations in terms of an authentic contextual application.
	2. Students will construct and compare various types of functions and build models to represent and solve problems.
Vectors - V	
	3. Students will represent and model vector quantities and perform operations on vectors.
Matrix Operations - MO	
	4. Students will perform operations on matrices and use matrices in applications.
Probability and Statistics- PS	
	5. Students will interpret linear models, calculate expected values to solve problems, and use probability to evaluate outcomes of decisions.

Functions	
Cluster 1: Students will analyze and interpret functions using different representations in terms of an authentic contextual application.	
F.1.ATMM.1*	Interpret key features of graphs and tables in terms of two quantities, which extends to function families beyond linear and quadratic, that model a relationship between the quantities in a contextual application and/or student-generated data.
F.1.ATMM.2*	Graph functions expressed symbolically and show key features of the graph using technology. Teacher Note: Function types include but are not limited to the standards listed in F.1.ATTMM.3-6.
F.1.ATMM.3*	Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions, with or without the appropriate technology.

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F.1.ATMM.2*	Graph functions expressed symbolically and show key features of the graph using technology. Teacher Note: Function types include but are not limited to the standards listed in F.1.ATTMM.3-6.
	Teache Note: Examples: The contextual situations are not limited to the following list: ● Cube Root (Minimizing packaging on cubic boxes, Geostationary satellites) ● Piecewise (Postage Stamp Function, Teacher Salary, using GPS for distance) ● Square Root (Distance via Pythagorean Theorem)
F.1.ATMM.4*	Graph polynomial functions, identifying *zeros* when suitable factorizations are available and showing end behavior, with or without the appropriate technology.
F.1.ATMM.5*	Graph rational functions, identifying *zeros* and asymptotes (vertical, horizontal, and/or oblique) when suitable factorizations are available and showing end behavior, with or without the appropriate technology. Teacher Note: Example: The contextual situations for rational functions are not limited to the following list: ● Average cost function ● Drug concentration in bloodstream
F.1.ATMM.6*	Graph with or without the appropriate technology: ● Exponential and logarithmic functions, showing intercepts and end behavior. ● Trigonometric functions, showing period, midline, and amplitude. Teacher Note: Example: The contextual situations are not limited to the following list: ● Exponential (half-life, investments, population growth or decay) ● Logarithmic (Richter scale, Decibels, pH scale) ● Trigonometric (phases of the moon, daylight hours, light waves)
F.1.ATMM.7*	Interpret the *parameters* of functions beyond the level of linear and quadratic in terms of a context.

Cluster 2: Students will construct and compare various types of functions and build models to represent and solve problems.	
F.2.ATMM.1*	Model equations using *regression equations* in two or more variables to represent relationships between quantities, which extends to function families beyond linear and quadratic, with or without the appropriate technology. Teacher Note: See Appendix A.
F.2.ATMM.2*	Represent constraints or inequalities using systems of equations and/or inequalities; interpret solutions as viable or non-viable options in a modeling context, excluding linear and quadratic functions.

F.2.ATMM.3*	Compose functions. Teacher Note: Example: If *T(y)* is the temperature in the atmosphere as a function of height and *h(t)* is the height of a weather balloon as a function of time, then *T(h(t))* is the temperature at the location of the weather balloon as a function of time.
F.2.ATMM.4*	Write *arithmetic* and *geometric sequences* both *recursively* and with an *explicit formula*; use the sequences to model situations and translate between the two forms.
F.2.ATMM.5*	Understand that restricting a trigonometric function to a *domain* on which it is always increasing or always decreasing allows its inverse to be constructed.
F.2.ATMM.6*	Use *inverse functions* to solve trigonometric equations that arise in modeling context; evaluate the solutions using technology and interpret them in terms of the context.

Vectors	
Cluster 3: Students will represent and model vector quantities and perform operations on vectors.	
V.3.ATMM.1	Recognize *vector* quantities as having both *magnitude* and direction; represent *vector* quantities by directed line segments and use appropriate symbols for *vector* and their *magnitudes (*e.g. vector, ***v***; magnitude ||***v***|| or |***v***| and *scalar* multiple, *c).*
V.3.ATMM.2	Find the *components of a vector* by subtracting the coordinates of an initial point from the coordinates of a terminal point.
V.3.ATMM.3	Solve problems involving velocity and other quantities that can be represented by *vectors.*
V.3.ATMM.4	Add *vectors* end-to-end, component-wise, and by the parallelogram rule; understand that the *magnitude* of a sum of two *vectors* is typically not the sum of *magnitudes.*
V.3.ATMM.5	Determine the *magnitude* and direction of the sum of two given *vectors* in *magnitude* and direction form.
V.3.ATMM.6	Understand *vector* subtraction; ***v*** – ***w*** as ***v*** + *(-**w**)*, where -***w*** is the additive inverse of ***w**,* with the same *magnitude* as ***w*** pointing in the opposite direction; represent *vector* subtraction graphically by connecting the tips in the appropriate order and perform *vector* subtraction component-wise.
V.3.ATMM.7	Represent scalar multiplication graphically by scaling *vectors* and possibly reversing their direction; perform scalar multiplication component-wise (e.g., as *c(vx, vy) = (c vx, c vy)*).
V.3.ATMM.8	Compute the *magnitude* of a scalar multiple *c**v*** using ||*c**v***|| = |*c*|***v***; compute the direction of *c**v*** knowing that when the |*c*|***v*** $\neq 0$, the direction *c**v*** is either along ***v****(for c > 0)* or against ***v****(c < 0).*

Matrix Operations	
Cluster 4: Students will perform operations on matrices and use matrices in applications.	
MO.4.ATMM.1*	Use matrices to represent and manipulate data (e.g., to represent payoffs or incidence relationships in a network).
MO.4.ATMM.2*	Multiply matrices by scalars to produce new matrices using technology (e.g., all the payoffs in a game are doubled).
MO.4.ATMM.3*	Add, subtract, and multiply matrices of appropriate dimensions using technology.
MO.4.ATMM.4*	Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation but still satisfies the associative and distributive properties.

MO.4.ATMM.5*	Understand the following: ● *Zero* and *identity matrices* play a role in matrix addition and multiplication similar to 0 and 1 in real numbers. ● The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
MO.4.ATMM.6*	Represent a system of linear equations as a single matrix equation in a *vector* variable. Teacher Note: Example: $\begin{bmatrix} 2 & 6 \\ 5 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2x + 6y \\ 5x + 2y \end{bmatrix} = \begin{bmatrix} 8 \\ 10 \end{bmatrix}$
MO.4.ATMM.7*	Find the *inverse of a matrix* if it exists; use it to solve systems of linear equations; utilize technology to find the *inverse of matrices* with dimensions of 3 x 3 or greater.

Probability and Statistics	
Cluster 5: Students will interpret linear models, calculate expected values to solve problems, and use probability to evaluate outcomes of decisions.	
PS.5.ATMM.1	Define a *random variable* for a quantity of interest by assigning a numerical value to each event in a *sample space*; graph the corresponding *probability distribution* using the same graphical displays as for *data distributions.* Teacher Note: Students should make connections between probability distributions & *data distribution*s.
PS.5.ATMM.2	Calculate the *expected value* of a *random variable*; interpret it as the *mean* of the *probability distribution.* Teacher Note: Students should be able to interpret the expected value in the context of a problem to demonstrate understanding.
PS.5.ATMM.3	Develop a *probability distribution* for a *random variable* defined for a *sample space* in which *theoretical probabilities* can be calculated; find the *expected value.* Teacher Note: Example: Find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices; find the expected grade under various grading schemes.
PS.5.ATMM.4	Develop a *probability distribution* for a *random variable* defined for a *sample space* in which probabilities are assigned empirically; find the *expected value*. Teacher Note: Example: Find a current data distribution on the number of TV sets per household in the United States and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?
PS.5.ATMM.5	Find the expected payoff for a game of chance.

	Teacher Note: Example: Find the expected winnings from a state lottery or a game at a fast-food restaurant.
PS.5.ATMM.6	Evaluate and compare strategies on the basis of *expected values*. Teacher Note: Example: Compare a high-deductible versus a low-deductible automobile insurance policy using various but reasonable chances of having a minor or major accident.

Glossary

Arithmetic Sequence	A sequence in which each term after the first is equal to the previous term added to a constant value Note: constant value in an arithmetic sequence is called the common difference
Components of a Vector	Each part of a two-dimensional vector which depicts the influence of that vector in a given direction; the combined influence of the two components is equivalent to the influence of the single two-dimensional vector; the single two-dimensional vector could be replaced by the two components
Domain	The set of values of the independent variable(s) for which a function or relation is defined
Expected Value	A quantity equal to the average result of an experiment after a large number of trials
Explicit Formula	An equation in which the dependent variable can be written explicitly in terms of the independent variable
Geometric Sequence	A sequence in which each term after the first is found by multiplying the previous term by a constant, called the common ratio, *r*
Identity Matrices	A square matrix which has a 1 for each element on the main diagonal and 0 for all other elements
Inverse Functions	Two functions f and g are inverse functions, if and only if, both of their compositions yield the identity function; p for example, $[f \circ g](x) = x$ and $[g \circ f](x) = x$
Inverse of a Matrix (Inverse of Matrices)	For a square matrix A, the inverse is written A^{-1}; when A is multiplied by A^{-1}, the result is the identity matrix; non-square matrices do not have inverses Note: not all square matrices have inverses; a square matrix which has an inverse is called invertible or nonsingular, and a square matrix without an inverse is called noninvertible or singular
Magnitude	The length of a vector
Mean	A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list; (e.g., for the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the mean absolute deviation is 20)
Parameter	A quantity that influences the output or behavior of a function but is viewed as being held constant.
Probability Distribution	The set of possible values of a random variable with a probability assigned to each
Random Variable	An assignment of a numerical value to each outcome in a sample space
Recursive Rule	Defines the *n*th term of a sequence in relation to the previous term
Regression Equation	A relationship, if any exists, between sets of data.
Sample Space	A list of the individual outcomes that are to be considered
Theoretical Probability	Probability is a likelihood that an event will happen $P(event) = \frac{NumberofFavorableOutcomes}{TotalNumberofPossibleOutcomes}$
Vector	A quantity with magnitude and direction in the plane or in space, defined by an ordered pair or triple of real numbers
Zero Matrix	A matrix for which all elements are equal to 0
Zero	A value of x which makes a function $f(x)$ equal 0; a zero may be real or complex

Appendix A.

Mathematical Modeling Cycle

The basic modeling cycle is summarized in this diagram. It involves: (1) identifying variables in the situation and selecting those that represent essential features; (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables; (3) analyzing and performing operations on these relationships to draw conclusions; (4) interpreting the results of the mathematics in terms of the original situation; (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable; (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.