



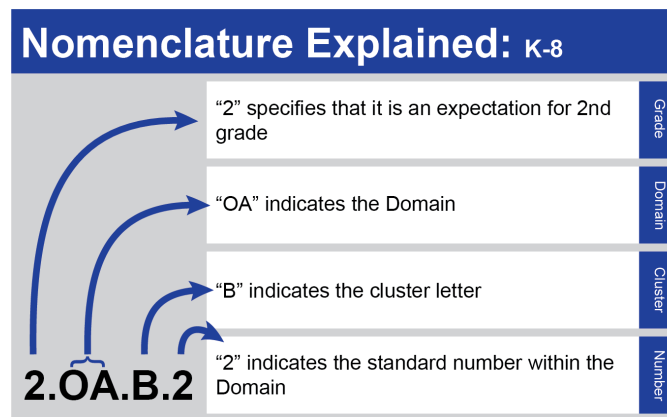
**Arkansas Mathematics Standards  
Kindergarten  
2022**

## Introduction to the Grades K-8 Arkansas Mathematics Standards

When the Division of Elementary and Secondary Education (DESE) began the process of revising math standards, a diverse group of qualified educators from across the state came together to craft Arkansas standards specific to the schools and students in the state. The result of this work, the Arkansas Mathematics Standards, is contained in this document. These standards reflect what educators across the state know to be best for Arkansas students.

**Standards Organization:** The revision committee maintained the organizational structure and nomenclature of the previous standards. K-8 Arkansas Mathematics Standards are categorized into domains, clusters, and standards.

- **Domains** represent the big ideas to be studied at each grade level and sometimes across grade bands. These big ideas support educators in determining the proper amount of focus and instructional time to be given to each of these topics.
- **Clusters** represent collections of standards grouped to help educators understand the building blocks of rich and meaningful instructional units. These units help students make connections within clusters and avoid seeing mathematics as a discrete list of skills they must master.
- **Standards** represent the foundational building blocks of math instruction. The standards outlined in this document work together to ensure that students are college and career ready and on track for success.



**Standards Support:** The revision of the Arkansas Mathematics Standards represent the work of the committee to provide greater clarity, strength, and support of the standards. Additionally, the revised mathematics standards are designed to help educators better understand the areas of emphasis and the focus within the standards. Educators should address the bulleted content as more than a checklist of items that they must teach individually. Content is bulleted to provide specificity of learning expectations included within some extensive standards. In some instances, the standard document includes Arkansas examples, teacher notes, specifications, and italicized words to assist educators with planning, teaching, and student learning.

- **Examples** included in the original standards were either changed for clarity or separated from the body of the actual standard. The examples included in the body of the standards document in no way reflect all of the possible examples. Likewise, these examples do not mandate curriculum or problem types. Local districts are free to select the high-quality curricula and instructional methods that best meet the needs of their students.
- **Teacher notes** offer clarification of the standards. These notes are intended to clarify, for teachers, what the expectations are for the learner. Likewise, these notes provide instructional guidance and limitations so that educators can better understand the scope of the standard. This will help with determining what is developmentally appropriate for students when working with specific standards.
- **Standard specifications** are to strengthen standards. The specifications are precise statements highlighting the need for mastery or function-type parameters for specific standards. This will assist educators in pinpointing the best opportunities for students to gain and master the knowledge and skills needed to succeed in a progression.
- **Italicized words** are defined in the glossary.

Finally, the Arkansas Mathematics Standards will be a living document. As these standards are implemented across schools in the state, DESE welcomes further suggestions related to notes of clarification, examples, professional development needs, and future revisions of the standards.

## K - 12: Standards for Mathematical Practices

- |  |  |
|--|--|
| <ol style="list-style-type: none"><li>1. Make sense of problems and persevere in solving them.</li><li>2. Reason abstractly and quantitatively.</li><li>3. Construct viable arguments and critique the reasoning of others.</li><li>4. Model with mathematics.</li></ol> | <ol style="list-style-type: none"><li>5. Use appropriate tools strategically.</li><li>6. Attend to precision.</li><li>7. Look for and make use of structure.</li><li>8. Look for and express regularity in repeated reasoning.</li></ol> |
|--|--|

## Kindergarten Standards: Overview

**Abbreviations:** The following abbreviations are for the domains for the Arkansas Mathematics Standards.

### Counting and Cardinality – CC

- Know Number Names and The Count Sequence
- Count to Tell the Number of Objects
- Compare Numbers

### Operations and Algebraic Thinking – OA

- Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from

### Number and Operations in Base Ten – NBT

- Work with numbers 11-20 to gain foundations for place value

### Measurement and Data – MD

- Describe and compare measurable attributes
- Classify objects and count the number of objects in each category
- Work with time and money

### Geometry – G

- Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres)
- Analyze, compare, create, and compose shapes

## Counting and Cardinality

### Cluster A: Know number names and the count sequence.

|                          |  |
|--------------------------|--|
| AR.Math.Content.K.CC.A.1 | Count to 100 by ones, fives, and tens.   |
| AR.Math.Content.K.CC.A.2 | Count forward, by ones, from any given number up to 100.   |
| AR.Math.Content.K.CC.A.3 | <p>Read, write, and represent numerals from 0 to 20.</p> <p>Teacher Note: K.CC.A.3 addresses the writing of numbers and uses the written numerals 0-20 to describe the amount of a set of objects. Due to the varied progression of fine motor and visual development, a reversal of numerals is anticipated for the majority of students. While reversals should be pointed out to students, the emphasis is on using numerals to represent quantities rather than the correct handwriting of the actual number itself.</p> |

### Cluster B: Count to tell the number of objects.

|                          |  |
|--------------------------|--|
| AR.Math.Content.K.CC.B.4 | <p>Understand the relationship between numbers and quantities; connect counting to <i>cardinality</i>.</p> <p>When counting objects:</p> <ul style="list-style-type: none"> <li>• Say the numbers in order, pairing each object with only one number, and each number with only one object (one-to-one correspondence).</li> <li>• Understand that the last number said tells the number of objects counted.</li> <li>• Understand that each successive number refers to a quantity that is one larger.</li> </ul> <p>Teacher Note: Students should understand that the number of objects is the same regardless of their arrangement or the order in which they were counted.</p> |
| AR.Math.Content.K.CC.B.5 | <p>Count to answer 'how many?'</p> <ul style="list-style-type: none"> <li>• Count up to 20 objects in any arrangement.</li> <li>• Count up to 10 objects in a scattered configuration.</li> <li>• Count out a specified number of objects when given a number from 1-20.</li> </ul> <p>Teacher Note: Any of the following would be considered age appropriate.<br/>Example: Move objects while counting, line up objects in order to count, touching scattered items as they count, or visually counting without touching or moving.</p>   |

### Cluster C: Compare numbers.

|                          |   |
|--------------------------|---|
| AR.Math.Content.K.CC.C.6 | <p>Identify whether the number of objects in one group from 0-10 is greater than (more, most), less than (less, fewer, least), or equal to (same as) the number of objects in another group of 0-10.</p> <p>Teacher Note: Use matching and counting strategies to compare <i>values</i>.</p>                      |
| AR.Math.Content.K.CC.C.7 | <p>Compare two numbers between 0 and 20 presented as written numerals.</p> <p>Teacher Note: The usage of the symbols for greater than/less than should not be introduced in this grade level. Appropriate terminology for this standard would be more than/ greater than, less than, or the same as/equal to.</p> |

|                          |   |
|--------------------------|---|
| AR.Math.Content.K.CC.C.8 | <p><i>Subitize</i> (quickly identify a number of items in a set without counting) from 0-10.</p> <p>Teacher Note: Utilize manipulatives or visuals.<br/>Examples: dominoes, dot cubes, tally marks, five frames, ten frames, etc.</p> |
|--------------------------|---|

## Operations and Algebraic Thinking

### Cluster A: Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

|                          |  |
|--------------------------|--|
| AR.Math.Content.K.OA.A.1 | <p>Represent addition and subtraction using objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, <i>expressions</i> (e.g., <math>2+3</math>), or <i>equations</i> (e.g., <math>2+3 = 5</math>).</p> <p>Teacher Note: Students should see <i>equations</i> and be encouraged to recognize that the two parts make the whole. However, writing <i>equations</i> is not required.</p>   |
| AR.Math.Content.K.OA.A.2 | <p>Solve real-world problems that involve addition and subtraction within 10 (e.g., by using objects or drawings to represent the problem).</p>  |
| AR.Math.Content.K.OA.A.3 | <p>Use objects or drawings to decompose (break apart) numbers less than or equal to 10 into pairs in more than one way and record each decomposition (part) by a drawing or an <i>equation</i> (e.g., <math>5 = 2 + 3</math> and <math>5 = 4 + 1</math>).</p> <p>Teacher Note: Students should see <i>equations</i> and be encouraged to recognize that the two parts make the whole. However, writing <i>equations</i> is not required.</p>   |
| AR.Math.Content.K.OA.A.4 | <p>Find the number that makes 10 when added to the given number (e.g., by using objects or drawings) and record the answer with a drawing or <i>equation</i>.</p> <p>Teacher Note:</p> <ul style="list-style-type: none"> <li>● Use of different manipulatives (e.g., ten-frames, cubes, two-color counters) assists students in visualizing these number pairs.</li> <li>● Students should see <i>equations</i> and be encouraged to recognize that the two parts make the whole. However, writing <i>equations</i> is not required.</li> </ul> |
| AR.Math.Content.K.OA.A.5 | <p>Fluently add and subtract within 10 by using various strategies and manipulatives.</p> <p>Teacher Note: <i>Fluency</i> is developed by working with many different kinds of objects over an extended period of time. This standard does not require that the students automatically know the answer. <i>Fluency</i> in this standard means accuracy (correct answer), efficiency (a reasonable amount of time), and flexibility (using various strategies).</p>   |

## Number and Operations in Base Ten

### Cluster A: Work with numbers 11-20 to gain foundations for place value.

AR.Math.Content.K.NBT.A.1

Develop an initial understanding of *place value* and the base-ten number system by showing equivalent forms of *whole numbers* up to 20 as groups of tens and ones using objects and drawings.

Teacher Note: Equivalency is shown as 12 ones or 1 ten 2 ones (e.g., use concrete representations such as snap cubes or manipulatives that can be broken apart).

## Measurement and Data

### Cluster A: Describe and compare measurable attributes.

AR.Math.Content.K.MD.A.1

Describe several measurable attributes of a single object, including but not limited to length, weight, height, and temperature.

Teacher Note: Vocabulary may include short, long, heavy, light, tall, hot, cold, warm, or cool.

AR.Math.Content.K.MD.A.2

Describe the *difference* when comparing two objects (side-by-side) with a measurable attribute in common to see which object has more of or less of the common *attribute*.

Teacher Note: Vocabulary may include, but is not limited to, shorter, longer, taller, lighter, heavier, warmer, cooler, or holds more.

### Cluster B: Classify objects and count the number of objects in each category.

AR.Math.Content.K.MD.B.3

Classify, sort, and count objects using both measurable and non-measurable *attributes* such as size, number, color, or shape.

Teacher Note: Limit category count to be less than or equal to 10. Students should be able to explain their reasoning for the way the objects were sorted.

### Cluster C: Work with time and money.

AR.Math.Content.K.MD.C.4

Understand concepts of time:

- Including morning, afternoon, evening, today, yesterday, tomorrow, day, week, month, and year
- Understand that clocks, both analog and digital, and calendars are tools that measure time.

AR.Math.Content.K.MD.C.5

Read time to the hour on digital and analog clocks.

Teacher Note: This is an introductory skill that will be addressed more formally in the upcoming grade levels.

| Progression of Time Skills |   |
|----------------------------|---|
| Grade Level                | Time Learning Expectation   |
| Kindergarten               | Tell time to the nearest hour   |
| First Grade                | Tell time to the nearest half-hour  |
| Second Grade               | Tell time to the nearest 5 minutes  |
| Third Grade                | Tell time to the nearest quarter<br>Tell time to the nearest minute<br>Solve word problems involving time |

AR.Math.Content.K.MD.C.6

Identify pennies, nickels, and dimes, and know the *value* of each.

Teacher Note: This is an introductory skill that will be addressed more formally in the upcoming grade levels. In addition, this standard supports K.CC.A.1 and K.MD.B.3.

| Progression of Money Skills |  |  |
|-----------------------------|--|--|
| Grade Level                 | Coins                                      | Money Learning Expectation   |
| Kindergarten                | penny, nickel, dime                        | Know name and value  |
| First Grade                 | penny, nickel, dime, quarter               | Know name and value<br>Count collections of like coins   |
| Second Grade                | penny, nickel, dime, quarter, dollar bills | Know name and value<br>Use dollar and cent symbol<br>Solve word problems: <ul style="list-style-type: none"><li>• within 99 cents</li><li>• using whole dollar amounts</li></ul> |
| Fourth Grade                | all coins and bills                        | Solve word problems including making change and decimals   |

## Geometry

### Cluster A: Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).

|                         |  |
|-------------------------|--|
| AR.Math.Content.K.G.A.1 | <p>Describe the positions of objects in the environment and geometric shapes in space using names of shapes, and describe the relative positions of these objects.</p> <ul style="list-style-type: none"> <li>Shapes to include: squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres</li> </ul> <p>Teacher Note: Positions could include inside, outside, between, above, below, near, far, under, over, up, down, behind, in front of, next to, to the left of, to the right of, or beside.</p> |
| AR.Math.Content.K.G.A.2 | <p>Correctly name shapes regardless of their orientations or overall size.</p> <ul style="list-style-type: none"> <li>Shapes to include: squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres</li> </ul> <p>Teacher Note: Orientation refers to the way the shape is turned (upside down, diagonally).</p>  |
| AR.Math.Content.K.G.A.3 | <p>Identify shapes as 2D (flat) or 3D (solid).</p>   |

### Cluster B: Analyze, compare, create, and compose shapes.

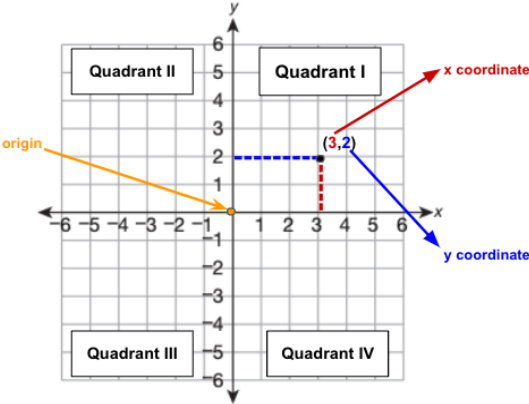
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|-------------------------|--|
| AR.Math.Content.K.G.B.4 | <p>Analyze and compare 2D and 3D shapes in different sizes and orientations using informal language to describe their similarities, <i>differences</i>, parts (e.g., number of sides, <i>vertices</i>), and other <i>attributes</i> (e.g., length of sides, straight or curved lines).</p> <p>Teacher Note:<br/>This is an introductory skill and supports K.MD.B.3.<br/>The following two and three-dimensional shapes are grade-level appropriate:</p> <ul style="list-style-type: none"> <li>2D shapes: squares, circles, triangles, rectangles, and hexagons</li> <li>3D shapes: cube, cone, cylinder, and sphere</li> </ul> |
| AR.Math.Content.K.G.B.5 | <p>Model shapes in the world by building and drawing shapes from components (e.g., sticks and clay balls).</p> <p>Teacher Note: This is an introductory skill and supports geometric understanding.</p>  |
| AR.Math.Content.K.G.B.6 | <p>Use two or more 2D shapes to compose different 2D shapes.</p> <p>Teacher Note: This is an introductory skill and supports geometric understanding.<br/>Example: Join two squares to make a rectangle or join six equilateral triangles to form a hexagon.</p>   |



## Glossary

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| Addend                                 | Any of the numbers added to find a sum   |
| Additive Comparison                    | Compare two amounts by asking how much more or less is one amount than the other.  |
| Additive inverses                      | Two numbers whose sum is 0 are additive inverses of one another; example: $\frac{3}{4}$ and $(-\frac{3}{4})$ are additive inverses of one another because $\frac{3}{4} + (-\frac{3}{4}) = 0$                   |
| Algorithm                              | An explicit step-by-step procedure for performing a mathematical computation or for solving a mathematical problem.  |
| Associative Property of addition       | A property of real numbers that states that the sum of a set of numbers is the same, regardless of how the numbers are grouped.<br>Example: $(4 + 8) + 3 = 4 + (8 + 3)$  |
| Associative Property of multiplication | A property of real numbers that states that the product of a set of numbers is the same, regardless of how the numbers are grouped.<br>Example: $(4 \cdot 8) \cdot 3 = 4 \cdot (8 \cdot 3)$                    |
| Attributes                             | Characteristics or properties of an object   |
| Axis                                   | A vertical or horizontal number line, both of which are used to define a coordinate grid. The horizontal axis is the x-axis, and the vertical axis is the y-axis. The plural of axis is axes.                  |
| Benchmark Fraction                     | A common fraction used when comparing other fractions (e.g., $\frac{1}{2}$ , $\frac{1}{4}$ )   |
| Cardinality                            | The understanding that when you count items, the number word applied to the last object counted represents the total amount.   |
| Commutative Property of addition       | A property of real numbers that states that the sum of two terms is unaffected by the order in which the terms are added; i.e., the sum remains the same.<br>Example: $5 + 9 = 9 + 5$                          |
| Commutative Property of multiplication | A property of real numbers that states that the product of two factors is unaffected by the order in which the factors are multiplied, i.e., the product remains the same.<br>Example: $5 \cdot 9 = 9 \cdot 5$ |
| Composite                              | A number with more than two factors.   |
| Composite Shape                        | Shapes composed of two or more shapes.   |
| Congruent                              | Identical in form  |
| Coordinate                             | An ordered pair of numbers in the form $(x, y)$ that describes the location of a point on a coordinate plane.  |

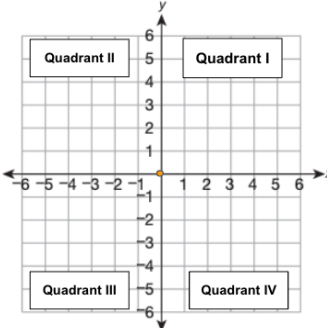
## Glossary

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|-----------------------|--|
| Coordinate Plane      | <p>A plane divided by perpendicular number lines creating four quadrants. The perpendicular number lines represent the axes and where they intersect represents the origin (0,0). Points can be identified using coordinates (x,y) found within the quadrants (example below).</p>    |
| Counting Back         | A strategy for finding the difference using backward counting. For example, if a stack of books has 12 books and someone borrows 4 books to read, how many books are left? A student may start at 12 and count back for spaces or numbers saying 12...11, 10, 9, 8; there are 8 books left in the stack.   |
| Counting On           | A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books has 8 books and 3 more books are added to the top, it is unnecessary to count the stack all over again. One can find the total by <i>counting on</i> pointing to the top book and saying “eight”, following this with “nine, ten, eleven.” There are eleven books now. |
| Data Set              | A collection of numbers related to a topic.  |
| Decompose             | Breaking a quantity into smaller quantities/units in order to assist computation.  |
| Denominator           | The term of a fraction, usually written under the line, that indicates the number of equal parts into which the unit is divided; divisor   |
| Difference            | The distance between two values; result of a subtraction problem.  |
| Distributive Property | When a single-term expression is being multiplied by a sum or difference, the single-term expression can be multiplied by each term before finding the sum or difference.<br>Examples: $3(7 + 5) = 3 \cdot 7 + 3 \cdot 5$<br>$w(5 - 2) = 5w - 2w$  |
| Dividend              | A number that is being divided by another number (divisor)   |
| Divisor               | The number by which another number is being divided  |
| Equation              | A statement that has one number or expression equal to another number or expression, such as $8 + 3 = 11$ or $2x - 3 = 7$ .  |
| Evaluate              | Calculate or solve   |
| Expanded form         | A multi-digit number is expressed in expanded form when it is written as a sum of the single-digit multiples of powers of ten. For example, $643 = 600 + 40 + 3$   |

## Glossary

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|---------------------------|---|
| Exponent                  | A symbol that is written above and to the right of a number to show how many times the number is to be multiplied by itself   |
| Expression                | A mathematical phrase consisting of numbers, variables, and operations  |
| Fluency                   | <p>There are different types of <b>fluency</b>. All of them require students to be accurate, efficient, and flexible. The types are defined as follows:</p> <p><u>Basic fact fluency</u> - fluency with operations involving single digit numbers.</p> <p><u>Computational fluency</u> - having efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate flexibility in the computational methods they choose, understand, and can explain these methods as well as produce accurate answers efficiently.</p> <p><u>Procedural fluency</u> - Procedural fluency is the ability to apply procedures accurately, efficiently, and flexibly; to transfer procedures to different problems and contexts; to build or modify procedures from other procedures, and to recognize when one strategy or procedure is more appropriate to apply than another. (NCTM)</p> |
| Factor                    | One or more numbers (or variables) that are multiplied together to get a product (5 and 2 are both factors because $5 \cdot 2 = 10$ )   |
| Fraction                  | A number expressible in the form $a/b$ where $a$ is a whole number and $b$ is a whole number. (The word fraction in these standards, K-5, always refers to a non-negative number.) This includes all forms of fractions - fractions less than one, fractions greater than one (improper fractions), and mixed numbers. <i>See also:</i> rational number   |
| Identity property of 0    | The property that asserts the sum of an original addend plus zero is equal to the original addend.<br>Example: $58 + 0 = 58$  |
| Identity property of 1    | The property that asserts the product of an original factor times one is equal to the original factor.<br>Example: $58 \cdot 1 = 58$  |
| Inequality Symbols        | Symbols used to show a comparison between quantities. Also known as the greater than and less than symbols (<,>).   |
| Interval                  | Includes all the numbers that come between two particular numbers.  |
| Inverse (Operation)       | An operation that is the opposite of, or undoes, another operation. Addition and subtraction are inverse operations as are multiplication and division.   |
| Iterating                 | Repeating; repetition of a process in order to generate a sequence of outcomes.   |
| Line plot                 | A method of visually displaying a distribution of data values where each data value is shown as an X or mark above a number line. Also known as a dot plot.   |
| Mass                      | The amount of matter in an object. Often measured by the amount of material it contains which causes it to have weight. However, mass is not to be confused with weight. Weight is determined by the force of gravity on an object while mass is not. For example, an watermelon on Jupiter would have a greater weight than one on Earth because Jupiter's gravity is stronger than Earth's. The mass of the watermelon would be the same on both planets.   |
| Mastery                   | Refers to teaching in a way that students learn to develop a deep understanding of mathematical concepts rather than memorizing key procedures or resorting to rote learning of steps or facts.   |
| Multiplicative Comparison | Compare two amounts by asking how many times larger or smaller is one amount than the other.  |

## Glossary

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|--------------------------|--|
| Multiplicative inverses  | Two numbers whose product is 1 are multiplicative inverses of one another.<br>Examples: $\frac{3}{4}$ and $\frac{4}{3}$ are multiplicative inverses of one another because $\frac{3}{4} \cdot \frac{4}{3} = 1$<br>$6$ and $\frac{1}{6}$ are also multiplicative inverses because $6 \cdot \frac{1}{6} = 1$ |
| Natural Numbers          | Counting numbers 1, 2, 3, 4, 5, 6...   |
| Number line diagram      | A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity  |
| Numerator                | The number in a fraction that is above the fraction line and that is divided by the number below the fraction line   |
| Order of Operations      | A specific sequence in which operations are to be performed when an expression requires more than one operation.   |
| Origin                   | The point in a Cartesian coordinate system where axes intersect  |
| Place value              | The value of the place of a digit in a numeral; the relative worth of each number that is determined by its position   |
| Polygons                 | A closed two-dimensional figure made up of straight sides.   |
| Prime                    | A number with only two factors, 1 and itself.  |
| Prism                    | A three-dimensional (solid) figure that has two congruent and parallel faces that are polygons called bases. The remaining faces, called lateral faces, are parallelograms (often rectangles). Prisms are named by the shape of their bases.   |
| Product                  | The number or expression resulting from the multiplication together of two or more numbers or expressions (factor $\cdot$ factor = product)  |
| Properties of operations | Rules that apply to the operations with real numbers. (See Table 1 below)  |
| Quadrant                 | One of the four sections of a coordinate plane separated by horizontal and vertical axes.<br>  |
| Quotient                 | The number that results when one number is divided by another  |
| Rational Numbers         | A real number which can be written in the form $\frac{a}{b}$ , where $a$ and $b$ are integers and $b \neq 0$ . The set of rational numbers include the set of integers.  |
| Rectangular array        | A set of quantities arranged in rows and columns   |
| Rectangular Prism        | A three-dimensional figure whose six faces are rectangles.   |
| Rectilinear Figures      | A polygon with all right angles.   |
| Subitize                 | Instantly see how many objects are in a group without counting.  |
| Sum                      | The result of adding two or more numbers   |

## Glossary

|                       |  |
|-----------------------|--|
| Trapezoid             | A quadrilateral with <i>at least</i> one pair of parallel sides                        |
| Unit fraction         | A fraction where the numerator is 1 and the denominator is the positive integer        |
| Value                 | Numerical worth or amount  |
| Variable              | A symbol used to represent an unknown value, usually a letter such as $x$              |
| Vertices              | A point where two or more line segments meet. (vertex is singular, plural is vertices) |
| Visual fraction model | A tape diagram, number line diagram, or area model                                     |
| Volume                | Amount of space occupied by a 3D object, measured in cubic units                       |
| Whole numbers         | The numbers 0, 1, 2, 3.....  |

## Appendix

**Table 1: Properties of Operations**

|   |   |
|---|---|
| Associative property of addition                      | $(a + b) + c = a + (b + c)$   |
| Commutative property of addition                      | $a + b = b + a$   |
| Additive identity property of 0                       | $a + 0 = 0 + a = a$   |
| Existence of additive inverses                        | For every $a$ there exists $-a$ so that $a + (-a) = (-a) + a = 0$               |
| Associative property of multiplication                | $(a \cdot b) \cdot c = a \cdot (b \cdot c)$                                     |
| Commutative property of multiplication                | $a \cdot b = b \cdot a$   |
| Multiplicative identity property 1                    | $a \cdot 1 = 1a = a$  |
| Existence of multiplication inverses                  | For every $a \neq 0$ there exists $1/a$ so that $a \cdot 1/a = 1/a \cdot a = 1$ |
| Distributive property of multiplication over addition | $a \cdot (b + c) = a \cdot b + a \cdot c$                                       |

**Table 2: Properties of Equality**

|                                     |   |
|-------------------------------------|---|
| Reflexive property of equality      | $a = a$   |
| Symmetric property of equality      | If $a = b$ , then $b = a$ .   |
| Transitive property of equality     | If $a = b$ and $b = c$ , then $a = c$ .   |
| Addition property of equality       | If $a = b$ , then $a + c = b + c$ .   |
| Subtraction property of equality    | If $a = b$ , then $a - c = b - c$ .   |
| Multiplication property of equality | If $a = b$ , then $a \cdot c = b \cdot c$ .   |
| Division property of equality       | If $a = b$ and $c \neq 0$ , then $a \div c = b \div c$ .                            |
| Substitution property of equality   | If $a = b$ , then $b$ may be substituted for $a$ in any expression containing $a$ . |

**Table 3: Properties of Inequality**

|   |
|---|
| Exactly one of the following is true: $a < b$ , $a = b$ , $a > b$ . |
| If $a > b$ and $b > c$ , then $a > c$ .                             |
| If $a > b$ , $b < a$ .  |
| If $a > b$ , then $a \pm c > b \pm c$ .                             |
| If $a > b$ and $c > 0$ , then $a \cdot c > b \cdot c$ .             |
| If $a > b$ and $c < 0$ , then $a \cdot c < b \cdot c$ .             |
| If $a > b$ and $c > 0$ , then $a \div c > b \div c$ .               |
| If $a > b$ and $c < 0$ , then $a \div c < b \div c$ .               |

**Table 4: Common Problem Types for Addition and Subtraction**

|                                  | <b>RESULT UNKNOWN</b>   | <b>CHANGE UNKNOWN</b>   | <b>START UNKNOWN</b>  |
|----------------------------------|---|---|---|
| <b>ADD TO</b>                    | Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$   | Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$   | Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$   |
| <b>TAKE FROM</b>                 | Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$  | Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$  | Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$  |
|                                  | <b>TOTAL UNKNOWN</b>  | <b>ADDEND UNKNOWN</b>   | <b>BOTH ADDENDS UNKNOWN</b>   |
| <b>PUT TOGETHER / TAKE APART</b> | Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$   | Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5$ , $5 - 3 = ?$  | Grandma has five flowers. How many can she put in the red vase and how many in her blue vase? $5 = 0 + 5$ , $5 = 0 + 5$ , $5 = 1 + 4$ , $5 = 4 + 1$ , $5 = 2 + 3$ , $5 = 3 + 2$   |
| <b>COMPARE</b>                   | <b>DIFFERENCE UNKNOWN</b>   | <b>BIGGER UNKNOWN</b>   | <b>SMALLER UNKNOWN</b>  |
|                                  | (“How many more?” version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? (“How many fewer?” version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have then Julie? $2 + ? = 5$ , $5 - 2 = ?$ | (Version with “more”): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with “fewer”): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?$ , $3 + 2 = ?$ | (Version with “more”): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with “fewer”): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? $5 - 3 = ?$ , $? + 3 = 5$ |

**Table 5: Common Problem Types for Multiplication and Division**

|                     | <b>UNKNOWN PRODUCT</b>  | <b>GROUP SIZE UNKNOWN (“HOW MANY IN EACH GROUP?” DIVISION)</b>  | <b>NUMBER OF GROUPS UNKNOWN (“HOW MANY GROUPS?” DIVISION)</b>  |
|---------------------|---|---|--|
|                     | $3 \cdot 6 = ?$   | $3 \cdot ? = 18$ , and $18 \div 3 = ?$  | $? \cdot 6 = 18$ , and $18 \div 6 = ?$   |
| <b>EQUAL GROUPS</b> | There are 3 bags with 6 plums in each bag. How many plums are there in all? <i>Measurement example.</i> You need 3 lengths of string, each 6 inches long. How much string will you need altogether?   | If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? <i>Measurement example.</i> You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?                                       | If 18 plums are to be packed 6 to a bag, then how many bags are needed? <i>Measurement example.</i> You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?  |
| <b>ARRAYS, AREA</b> | There are 3 rows of apples with 6 apples in each row. How many apples are there? <i>Area example.</i> What is the area of a 3 cm by 6 cm rectangle?   | If 18 apples are arranged into 3 equal rows, how many apples will be in each row? <i>Area example.</i> A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?   | If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? <i>Area example.</i> A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?  |
| <b>COMPARE</b>      | A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? <i>Measurement example.</i> A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long? | A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? <i>Measurement example.</i> A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first? | A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat? <i>Measurement example.</i> A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first? |
| <b>GENERAL</b>      | $a \cdot b = ?$   | $a \cdot ? = p$ and $p \div a = ?$  | $? \cdot b = p$ , and $p \div b = ?$   |