

Arkansas Mathematics Standards 4th Grade 2022

## Introduction to the Grades K-8 Arkansas Mathematics Standards

When the Division of Elementary and Secondary Education (DESE) began the process of revising math standards, a diverse group of qualified educators from across the state came together to craft Arkansas standards specific to the schools and students in the state. The result of this work, the Arkansas Mathematics Standards, is contained in this document. These standards reflect what educators across the state know to be best for Arkansas students.

Standards Organization: The revision committee maintained the organizational structure and nomenclature of the previous standards. K-8 Arkansas Mathematics Standards are categorized into domains, clusters, and standards.

- Domains represent the big ideas to be studied at each grade level and sometimes across grade bands. These big ideas support educators in determining the proper amount of focus and instructional time to be given to each of these topics.
- Clusters represent collections of standards grouped to help educators understand the building blocks of rich and meaningful instructional units. These units help students make connections within clusters and avoid seeing mathematics as a discrete list of skills they must master.
- Standards represent the foundational building blocks of math instruction. The standards outlined in this document work together to ensure that students are college and career ready and on track for success.


Standards Support: The revision of the Arkansas Mathematics Standards represent the work of the committee to provide greater clarity, strength, and support of the standards. Additionally, the revised mathematics standards are designed to help educators better understand the areas of emphasis and the focus within the standards. Educators should address the bulleted content as more than a checklist of items that they must teach individually. Content is bulleted to provide specificity of learning expectations included within some extensive standards. In some instances, the standard document includes Arkansas examples, teacher notes, specifications, and italicized words to assist educators with planning, teaching, and student learning.

- Examples included in the original standards were either changed for clarity or separated from the body of the actual standard. The examples included in the body of the standards document in no way reflect all of the possible examples. Likewise, these examples do not mandate curriculum or problem types. Local districts are free to select the high-quality curricula and instructional methods that best meet the needs of their students.
- Teacher notes offer clarification of the standards. These notes are intended to clarify, for teachers, what the expectations are for the learner. Likewise, these notes provide instructional guidance and limitations so that educators can better understand the scope of the standard. This will help with determining what is developmentally appropriate for students when working with specific standards.
- Standard specifications are to strengthen standards. The specifications are precise statements highlighting the need for mastery or function-type parameters for specific standards. This will assist educators in pinpointing the best opportunities for students to gain and master the knowledge and skills needed to succeed in a progression.
- Italicized words are defined in the glossary.

Finally, the Arkansas Mathematics Standards will be a living document. As these standards are implemented across schools in the state, DESE welcomes further suggestions related to notes of clarification, examples, professional development needs, and future revisions of the standards.

## K - 12: Standards for Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

## Fourth Grade Standards: Overview

Abbreviations: The following abbreviations are for the domains for the Arkansas Mathematics Standards.

## Operations and Algebraic Thinking - OA

- Use the four operations with whole numbers to solve problems
- Gain familiarity with factors and multiples
- Generate and analyze patterns


## Number and Operations in Base Ten - NBT

- Generalize place value understanding for multi-digit whole numbers
- Use place value understanding and properties of operations to perform multi-digit arithmetic
- Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.


## Number and Operations - Fractions - NF

- Extend understanding of fraction equivalence and ordering
- Build fractions from unit fractions by applying and extending previous understanding of operations of whole numbers
- Understand decimal notation for fractions, and compare decimal fractions
- Grade 4 expectations in this domain are limited to fractions with denominators $2,3,4,5,6,8,10,12$, and 100.


## Measurement and Data - MD

- Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit
- Represent and interpret data
- Geometric measurement: understand concepts of angle and measure angles

Geometry - G

- Draw and identify lines and angles, and classify shapes by properties of their lines and angles


## 3-5 Grade Band Teacher Note:

Multiplication is represented with the • symbol instead of an $x$. This is to eliminate confusion between the multiplication symbol and the variable $x$.

## Operations and Algebraic Thinking

## Cluster A: Use the four operations with whole numbers to solve problems.


(2) A book at the book fair costs $\$ 15$. That is 3 times more than the cost of a poster. How much does a poster cost?

$$
\begin{aligned}
& 15 \div p=3 \\
& 3 \cdot p=15
\end{aligned}
$$

| 15 |  |  |
| :---: | :---: | :---: |
| $?$ | $?$ | $?$ |

Solve multiplicative comparison word problems:

- Multiply or divide to solve word problems involving multiplicative comparison.
- Use drawings and equations with a variable for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.
Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations (add, subtract, multiply, and divide), including problems in which remainders must be interpreted. Represent these problems using equations with a variable standing for the unknown quantity.
- Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Teacher Note: In multistep word problem situations, students must interpret and use reminders concerning context and the question being asked.

- Quotient needs to be with and without remainders in whole-number form.

Examples:
(1) On a business trip, a salesman travels 314 miles on the first day, 167 miles on the second day, and 26 miles on third day. How many miles did the salesman travel total? How do you know your answer is reasonable?

Possible Responses -
Student A - I first thought about 167 and 26 . I noticed that their sum is about 200 . Then I thought 314 is close to 300 . So my answer is close to 500 .
Student B - I first thought about 167. It is close to 200. There are 3 hundreds in 314 . So that gives me 500 . Then 26 is close to 30 . So my answer is close to 530 .

|  | Student C - I rounded 314 to 300 . I rounded 167 to 200 . I rounded 26 to 30 . I added 300, 200, and 30 so my answer is close to 530. (2) You and your best friend bake cookies to sell for extra money. You bake 187 cookies and want to put them in decorative bags. Each bag will hold 4 cookies. How many bags will you need to hold all the cookies? <br> Possible Response - <br> I know that $4 \bullet 40=160$ and I would have 27 cookies left. I also know that $4 \bullet 6=24$, which would leave me with 3 cookies. The total bags used would be 46 with 3 leftover cookies. So to put all the cookies in bags, I would need 47 bags. |
| :---: | :---: |
| Cluster B: Gain familiarity with factors and multiples. |  |
| AR.Math.Content.4.OA.B.4 | Identify factors and multiples: <br> - Find factor pairs for a whole number in the range 1-100. <br> - Recognize that a whole number is a multiple of each of its factors. <br> - Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number. <br> - Determine whether a given whole number in the range 1-100 is prime or composite. <br> Teacher Note: Informal classroom discussion might include divisibility rules, finding patterns, and other strategies. <br> Examples: <br> (1) <br> - Part A - There are 30 chairs in the PE room. What are the different ways that the chairs can be arranged into equal groups if you want at least 2 groups and want at least 2 chairs in each group? How do you know you've found all the possible arrangements? Write equations to show your answers. <br> - Part B - There are 60 chairs in the cafeteria. What are the different ways the chairs can be arranged into equal groups if you want at least 2 groups and at least 2 chairs in each group? How do you know you've found all the possible arrangements? Write equations to show your answers. <br> - What relationship do you notice about the size of the groups if the chairs were arranged in 5 groups in both Part A and Part B? <br> (2) A school wants to create a garden near the playground. The school can only have between 24 and 30 plants. <br> - Determine how many possible rectangular dimensions you could make if the school purchased 24-30 plants but can only have 10 plants per row. <br> - Which number of plants provides the most flexibility in terms of the possible ways that plants could be arranged in equal groups? Explain your reasoning. |


| Cluster C: Generate and analyze patterns. |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| AR.Math.Content.4.OA.C.5 | Generate and analyze patterns: <br> - Generate a number or shape pattern that follows a given <br> - Identify apparent features of the pattern that were not exp <br> Teacher Note: <br> Examples: <br> (1) Keisha and David both babysit on the weekends to earn extra spending money. Keisha charges $\$ 10$ per day and $\$ 2$ per hour. David charges $\$ 4$ per day and $\$ 4$ per hour. Complete the table to show how much money Keisha and David each earn based on the amount of time spent babysitting. |  |  | in the rule it <br> (2) Caleb day he his ba first 5 | a piggy ba is bed, he many quarte | quarters in it. Each 3 more quarters to he bank after the |
|  |  |  |  | Day | Operation | Quarters |
|  |  |  |  | 0 | $3 \cdot 0+4$ | 4 |
|  | \%/hour | $1 / 2 \cdot 2+10=\$ 11$ | $1 / 2 \cdot 4+4=\$ 6$ | 1 | $3 \cdot 1+4$ | 7 |
|  | 1 hour | $1 \cdot 2+10=\$ 12$ | $1 \cdot 4+4=\$ 8$ | 2 | $3 \cdot 2+4$ | 10 |
|  | 1 and $1 /$ hours | $1 \frac{1}{2} \cdot 2+10=513$ | $1 \frac{1}{2} \cdot 4+4=\$ 10$ | 3 | $3 \cdot 3+4$ | 13 |
|  | 2 hours | $2 \cdot 2+10=\$ 14$ | $2 \cdot 4+4=\$ 12$ | 4 | $3 \cdot 4+4$ | 16 |
|  | 2 and $1 /$ hours | $2 \frac{1}{2} \cdot 2+10=\$ 15$ | $2 \frac{1}{2} \cdot 4+4=514$ | 5 | $3 \cdot 5+4$ | 19 |
|  | 3 hours | $3 \cdot 2+10=\$ 16$ | $3 \cdot 4+4=\$ 16$ |  |  |  |


|  | Number and Operations in Base Ten |
| :--- | :--- | :--- |
| Cluster A: Generalize place value understanding for multi-digit whole numbers. |  |
| Grade 4 expectations in this domain are limited to whole numbers less than or equal to $1,000,000$. |  |


| Cluster B: Use place value understanding and properties of operations to perform multi-digit arithmetic. Grade 4 expectations in this domain are limited to whole numbers less than or equal to $1,000,000$. |  |
| :---: | :---: |
| AR.Math.Content.4.NBT.B.4 | Applying computational fluency using a standard algorithm, add and subtract multi-digit whole numbers (up to 1,000,000). <br> Teacher Note: <br> - Computational fluency - refers to having efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate flexibility in the computational methods they choose, understand and can explain these methods, and produce accurate answers efficiently. <br> - A standard algorithm denotes any valid base ten strategy. <br> - A standard algorithm can be viewed as, but should not be limited to, the traditional recording system. |
| AR.Math.Content.4.NBT.B. 5 | Multiply whole numbers: <br> - Using strategies based on place value and the properties of operations, multiply a whole number of up to four digits by a one-digit whole number. <br> - Using strategies based on place value and the properties of operations, multiply two two-digit numbers. <br> Teacher Note: Illustrate and explain the calculation using equations, rectangular arrays, and area models. |
| AR.Math.Content.4.NBT.B. 6 | Divide whole numbers: <br> - Using strategies based on place value, the properties of operations, and the relationship between multiplication and division, find whole-number quotients with up to four-digit dividends and one-digit divisors. <br> - Illustrate and explain the calculation using equations, rectangular arrays, and area models. <br> Teacher Note: <br> - Properties of operations need to be referenced. <br> - Quotient needs to be with and without remainders in whole-number form. |


| Number and Operations - Fractions |  |
| :---: | :---: |
| Cluster A: Extend understanding of fraction equivalence and ordering. <br> Grade 4 expectations in this domain are limited to fractions with denominators $2,3,4,5,6,8,10,12$, and 100. |  |
| AR.Math.Content.4.NF.A. 1 | Understand fraction equivalence: <br> - By using visual fraction models, explain why a fraction $\mathbf{a} / \mathbf{b}$ is equivalent to a fraction $(\mathbf{n} \cdot \mathbf{a}) /(\mathbf{n} \cdot \mathbf{b})$ with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. <br> - By using visual fraction models, explain why a fraction $\mathbf{a} / \mathbf{b}$ is equivalent to a fraction $(\mathbf{a} \div \mathbf{n}) /(\mathbf{b} \div \mathbf{n})$ with the denominator of the dividend limited to $4,6,8,10,12$, and 100. <br> - Use this principle to recognize and generate equivalent fractions. <br> Teacher Note: <br> Examples: <br> - $1 / 5$ is equivalent to $(3 \cdot 1) /(3 \cdot 5)$. <br> - $9 / 12$ is equivalent to $(9 \div 3) /(12 \div 3)$. <br> Specification: Students should demonstrate mastery of this standard by the end of 4th grade. |
| AR.Math.Content.4.NF.A. 2 | Compare fractions: <br> - Compare two fractions with different numerators and different denominators (e.g., by creating common denominators or numerators, or by comparing to a benchmark of $0,1 / 2,1$ ). <br> - Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols (>, =, <), and justify the conclusions (e.g., by using a visual fraction model). <br> Teacher Note: Teachers and students should use the correct terminology of symbols to record the results of comparisons. |


| Cluster B: Build fractions from unit fractions by applying and extending previous understanding of operations of whole numbers. |
| :--- | :--- | :--- |
| ${ }^{* *}$ Grade 4 expectations in this domain are limited to fractions with denominators $2,3,4,5,6,8,10,12$, and 100 . |


| AR.Math.Content.4.NF.C. 6 | Use decimal notation for fractions with denominators 10 or 100. <br> Teacher Note: <br> Example: Write 0.62 as $62 / 100$; describe a length as 0.62 meters; locate 0.62 on a number line diagram. |
| :---: | :---: |
| AR.Math.Content.4.NF.C. 7 | Compare decimals: <br> - Compare two decimals to hundredths by reasoning about their size. <br> - Recognize that comparisons are valid only when the two decimals refer to the same whole. <br> - Record the results of comparisons using symbols (>, =, <) and justify the conclusions (e.g., by using a visual model). <br> Teacher Note: Teachers and students should use the correct terminology of symbols to record the results of comparisons. |



| AR.Math.Content.4.MD.A. 2 | Use the four operations (add, subtract, multiply, divide) to solve word problems including: <br> - Distances <br> - Intervals of time that cross the hour <br> - Liquid volumes and masses of objects <br> - Money including the ability to make change (including decimals to the tenths place) <br> - Fractions with like denominators <br> - Expressing measurements given in a larger unit in terms of a smaller unit <br> - Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale. <br> Teacher Note: This standard may be addressed throughout the year focusing on different contexts. |
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| AR.Math.Content.4.MD.A. 3 | Apply the area and perimeter formulas for rectangles in real-world and mathematical problems. <br> Teacher Note: <br> Example: Find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor. |
| Cluster B: Represent and interpret data. |  |
| AR.Math.Content.4.MD.B. 4 | Create and interpret line plots: <br> - Make a line plot to display a data set of measurements in fractions of a unit (e.g., 1/2, 1/4, 1/8). <br> - Solve problems involving addition and subtraction of fractions with like denominators by using information presented in line plots. <br> Teacher Note: <br> Example: From a line plot, find and interpret the difference in length between the longest and shortest specimens in an insect collection (Refer to 4.NF.B.3). |


| Cluster C: Geometric measurement: understand concepts of angle and measure angles. <br> AR.Math.Content.4.MD.C.5 <br> Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of <br> angle measurement: <br> AR.Math.Content.4.MD.C.6 <br> - An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering <br> the fraction of the circular arc between the points where the two rays intersect the circle. |
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| AR.Math.Content.4.MD.C. 7 |
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## Additive Angle Measurement:

- Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts.
- Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems.


## Teacher Note

Example: In the rectangle below, angle CAD can be found by solving $73^{\circ}+x=90^{\circ}$ or $90^{\circ}-73^{\circ}=x$


## Geometry

Cluster A: Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

| AR.Math.Content.4.G.A. 1 | Draw and identify lines and angles: <br> - Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. <br> - Identify these in two-dimensional figures. <br> - Shapes to include square, rectangle, trapezoid, parallelogram, rhombus, and triangle. |
| :---: | :---: |
| AR.Math.Content.4.G.A. 2 | Classify shapes: <br> - Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. <br> - Shapes to include: Quadrilaterals - trapezoid, parallelogram, rectangle, square, rhombus; Triangles - right, acute, obtuse <br> Teacher Note: Trapezoids should be defined to be a quadrilateral with at least one pair of opposite sides parallel; therefore, all parallelograms are trapezoids. |
| AR.Math.Content.4.G.A. 3 | Identify symmetry: <br> - Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. <br> - Identify line-symmetric figures and draw lines of symmetry. |


| K-5 Glossary |  |
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| Addend | Any of the numbers added to find a sum |
| Additive Comparison | Compare two amounts by asking how much more or less is one amount than the other. |
| Additive inverses | Two numbers whose sum is 0 are additive inverses of one another; example: $3 / 4$ and $(-3 / 4)$ are additive inverses of one <br> another because $3 / 4+(-3 / 4)=0$ |
| Algorithm | An explicit step-by-step procedure for performing a mathematical computation or for solving a mathematical problem. |
| Associative Property of <br> addition | A property of real numbers that states that the sum of a set of numbers is the same, regardless of how the numbers are <br> grouped. <br> Example: $(4+8)+3=4+(8+3)$ |
| Associative Property of <br> multiplication | A property of real numbers that states that the product of a set of numbers is the same, regardless of how the numbers are <br> grouped. <br> Example: $(4 \bullet 8) \bullet 3=4 \bullet(8 \bullet 3)$ |
| Attributes | Characteristics or properties of an object |
| Axis | A vertical or horizontal number line, both of which are used to define a coordinate grid. The horizontal axis is the $x$-axis, and the <br> vertical axis is the y-axis. The plural of axis is axes. |
| Benchmark Fraction | A common fraction used when comparing other fractions (e.g., $1 / 2,1 / 4)$ |
| Cardinality | The understanding that when you count items, the number word applied to the last object counted represents the total amount. |
| Commutative Property of <br> addition | A property of real numbers that states that the sum of two terms is unaffected by the order in which the terms are added; i.e., <br> the sum remains the same. <br> Example: $5+9=9+5$ |
| Commutative Property of <br> multiplication | A property of real numbers that states that the product of two factors is unaffected by the order in which the factors are <br> multiplied, i.e., the product remains the same. <br> Example: $5 \cdot 9=9 \bullet 5$ |
| Composite | A number with more than two factors. |
| Composite Shape | Shapes composed of two or more shapes. |
| Congruent | Identical in form |
| Coordinate | An ordered pair of numbers in the form $(x, y)$ that describes the location of a point on a coordinate plane. |
| Coordinate Plane | A plane divided by perpendicular number lines creating four quadrants. The perpendicular number lines represent the axes and <br> where they intersect represents the origin $(0,0) . ~ P o i n t s ~ c a n ~ b e ~ i d e n t i f i e d ~ u s i n g ~ c o o r d i n a t e s ~(x, y) ~ f o u n d ~ w i t h i n ~ t h e ~ q u a d r a n t s ~$ <br> (example below). |

## K-5 Glossary

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| Counting Back | A strategy for finding the difference using backward counting. For example, if a stack of books has 12 books and someone borrows 4 books to read, how many books are left? A student may start at 12 and count back for spaces or numbers saying 12...11, 10, 9, 8 ; there are 8 books left in the stack. |
| Counting On | A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books has 8 books and 3 more books are added to the top, it is unnecessary to count the stack all over again. One can find the total by counting on pointing to the top book and saying "eight", following this with "nine, ten, eleven." There are eleven books now. |
| Data Set | A collection of numbers related to a topic. |
| Decompose | Breaking a quantity into smaller quantities/units in order to assist computation. |
| Denominator | The term of a fraction, usually written under the line, that indicates the number of equal parts into which the unit is divided; divisor |
| Difference | The distance between two values; result of a subtraction problem. |
| Distributive Property | When a single-term expression is being multiplied by a sum or difference, the single-term expression can be multiplied by each term before finding the sum or difference. <br> Examples: $3(7+5)=3 \cdot 7+3 \cdot 5$ $w(5-2)=5 w-2 w$ |
| Dividend | A number that is being divided by another number (divisor) |
| Divisor | The number by which another number is being divided |
| Equation | A statement that has one number or expression equal to another number or expression, such as $8+3=11$ or $2 x-3=7$. |
| Evaluate | Calculate or solve |
| Expanded form | A multi-digit number is expressed in expanded form when it is written as a sum of the single-digit multiples of powers of ten. For example, $643=600+40+3$ |
| Exponent | A symbol that is written above and to the right of a number to show how many times the number is to be multiplied by itself |

## K-5 Glossary

| Expression | A mathematical phrase consisting of numbers, variables, and operations |
| :---: | :---: |
| Fluency | There are different types of fluency. All of them require students to be accurate, efficient, and flexible. The types are defined as follows: <br> Basic fact fluency- fluency with operations involving single digit numbers. <br> Computational fluency - having efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate flexibility in the computational methods they choose, understand, and can explain these methods as well as produce accurate answers efficiently. <br> Procedural fluency - Procedural fluency is the ability to apply procedures accurately, efficiently, and flexibly; to transfer procedures to different problems and contexts; to build or modify procedures from other procedures, and to recognize when one strategy or procedure is more appropriate to apply than another. (NCTM) |
| Factor | One or more numbers (or variables) that are multiplied together to get a product ( 5 and 2 are both factors because $5 \cdot 2=10$ ) |
| Fraction | A number expressible in the form $a / b$ where $a$ is a whole number and $b$ is a whole number. (The word fraction in these standards, K-5, always refers to a non-negative number.) This includes all forms of fractions - fractions less than one, fractions greater than one (improper fractions), and mixed numbers. See also: rational number |
| Identity property of 0 | The property that asserts the sum of an original addend plus zero is equal to the original addend. Example: $58+0=58$ |
| Identity property of 1 | The property that asserts the product of an original factor times one is equal to the original factor. Example: 58 - $1=58$ |
| Inequality Symbols | Symbols used to show a comparison between quantities. Also known as the greater than and less than symbols (<,>). |
| Interval | Includes all the numbers that come between two particular numbers. |
| Inverse (Operation) | An operation that is the opposite of, or undoes, another operation. Addition and subtraction are inverse operations as are multiplication and division. |
| Iterating | Repeating; repetition of a process in order to generate a sequence of outcomes. |
| Line plot | A method of visually displaying a distribution of data values where each data value is shown as an X or mark above a number line. Also known as a dot plot. |
| Mass | The amount of matter in an object. Often measured by the amount of material it contains which causes it to have weight. However, mass is not to be confused with weight. Weight is determined by the force of gravity on an object while mass is not. For example, an watermelon on Jupiter would have a greater weight than one on Earth because Jupiter's gravity is stronger than Earth's. The mass of the watermelon would be the same on both planets. |
| Mastery | Refers to teaching in a way that students learn to develop a deep understanding of mathematical concepts rather than memorizing key procedures or resorting to rote learning of steps or facts. |
| Multiplicative Comparison | Compare two amounts by asking how many times larger or smaller is one amount than the other. |

## K-5 Glossary

| Multiplicative inverses | Two numbers whose product is 1 are multiplicative inverses of one another. Examples: $3 / 4$ and $4 / 3$ are multiplicative inverses of one another because $3 / 4 \bullet 4 / 3=1$ 6 and $\frac{1}{6}$ are also multiplicative inverses because $6 \cdot \frac{1}{6}=1$ |
| :---: | :---: |
| Natural Numbers | Counting numbers 1, 2, 3, 4, 5, 6... |
| Number line diagram | A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity |
| Numerator | The number in a fraction that is above the fraction line and that is divided by the number below the fraction line |
| Order of Operations | A specific sequence in which operations are to be performed when an expression requires more than one operation. |
| Origin | The point in a Cartesian coordinate system where axes intersect |
| Place value | The value of the place of a digit in a numeral; the relative worth of each number that is determined by its position |
| Polygons | A closed two-dimensional figure made up of straight sides. |
| Prime | A number with only two factors, 1 and itself. |
| Prism | A three-dimensional (solid) figure that has two congruent and parallel faces that are polygons called bases. The remaining faces, called lateral faces, are parallelograms (often rectangles). Prisms are named by the shape of their bases. |
| Product | The number or expression resulting from the multiplication together of two or more numbers or expressions (factor $\bullet$ factor $=$ product) |
| Properties of operations | Rules that apply to the operations with real numbers. (See Table 1 below) |
| Quadrant | One of the four sections of a coordinate plane separated by horizontal and vertical axes. |
| Quotient | The number that results when one number is divided by another |
| Rational Numbers | A real number which can be written in the form $\frac{a}{b}$, where $a$ and $b$ are integers and $b \neq 0$. The set of rational numbers include the set of integers. |
| Rectangular array | A set of quantities arranged in rows and columns |
| Rectangular Prism | A three-dimensional figure whose six faces are rectangles. |
| Rectilinear Figures | A polygon with all right angles. |

## K-5 Glossary

| Subitize | Instantly see how many objects are in a group without counting. |
| :--- | :--- |
| Sum | The result of adding two or more numbers |
| Trapezoid | A quadrilateral with at least one pair of parallel sides |
| Unit fraction | A fraction where the numerator is 1 and the denominator is the positive integer |
| Value | Numerical worth or amount |
| Variable | A symbol used to represent an unknown value, usually a letter such as $x$ |
| Vertices | A point where two or more line segments meet. (vertex is singular, plural is vertices) |
| Visual fraction model | A tape diagram, number line diagram, or area model |
| Volume | Amount of space occupied by a 3D object, measured in cubic units |
| Whole numbers | The numbers $0,1,2,3 \ldots$. |

## Appendix

## Table 1: Properties of Operations

| Associative property of addition | $(a+b)+c=a+(b+c)$ |
| :---: | :---: |
| Commutative property of addition | $a+b=b+a$ |
| Additive identity property of 0 | $a+0=0+a=a$ |
| Existence of additive inverses | For every a there exists -a so that $a+(-a)=(-a)+a=0$ |
| Associative property of multiplication | $(\mathrm{a} \cdot \mathrm{b}) \cdot \mathrm{c}=\mathrm{a} \cdot(\mathrm{b} \cdot \mathrm{c})$ |
| Commutative property of multiplication | $a \cdot b=b \cdot a$ |
| Multiplicative identity property 1 | $\mathrm{a} \cdot 1=1 \mathrm{a}=\mathrm{a}$ |
| Existence of multiplication inverses | For every $a \neq 0$ there exists $1 / \mathrm{a}$ so that $\mathrm{a} \cdot 1 / \mathrm{a}=1 / \mathrm{a} \cdot \mathrm{a}=1$ |
| Distributive property of multiplication over addition | $a \cdot(b+c)=a \cdot b+a \cdot c$ |

Table 2: Properties of Equality

| Reflexive property of equality | $a=a$ |
| :--- | :--- |
| Symmetric property of equality | If $a=b$, then $b=a$. |
| Transitive property of equality | If $a=b$ and $b=c$, then $a=c$. |
| Addition property of equality | If $a=b$, then $a+c=b+c$. |
| Subtraction property of equality | If $a=b$, then $a-c=b-c$. |
| Multiplication property of equality | If $a=b$, then $a \bullet c=b \bullet c$. |
| Division property of equality | If $a=b$ and $c \neq 0$, then $a \div c=b \div c$. |
| Substitution property of equality | If $a=b$, then $b$ may $b e$ substituted for $a$ in any expression containing $a$. |

Table 3: Properties of Inequality

| Exactly one of the following is true: $a<b, a=b, a>b$. |
| :--- |
| If $a>b$ and $b>c$, then $a>c$. |
| If $a>b, b<a$. |
| If $a>b$, then $a \pm c>b \pm c$. |
| If $a>b$ and $c>0$, then $a \bullet c>b \bullet c$. |
| If $a>b$ and $c<0$, then $a \bullet c<b \bullet c$. |
| If $a>b$ and $c>0$, then $a \div c>b \div c$. |
| If $a>b$ and $c<0$, then $a \div c<b \div c$. |

Table 4: Common Problem Types for Addition and Subtraction

|  | RESULT UNKNOWN | CHANGE UNKNOWN | START UNKNOWN |
| :---: | :---: | :---: | :---: |
| ADD TO | Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2+3=$ ? | Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5 | Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? $+3=5$ |
| TAKE FROM | Five apples were on the table. I ate two apples. How many apples are on the table now?5-2 = ? | Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5-?=3$ | Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before?? -2 = 3 |
|  | TOTAL UNKNOWN | ADDEND UNKNOWN | BOTH ADDENDS UNKNOWN |
| PUT TOGETHER / TAKE APART | Three red apples and two green apples are on the table. How many apples are on the table? $3+2=$ ? | Five apples are on the table. Three are red and the rest are green. How many apples are green? $3+$ ? $=5,5-3=$ ? | Grandma has five flowers. How many can she put in the red vase and how many in her blue vase? $5=0+5,5+05=1$ $+4,5=4+15=2+3,5=3+2$ |
| COMPARE | DIFFERENCE UNKNOWN | BIGGER UNKNOWN | SMALLER UNKNOWN |
|  | ("How many more?" version):Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have then Julie? 2 + ? = $5,5-2=$ ? | (Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2+3=$ ? $3+2=$ ? | (Version with "more"):Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have?(Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? $5-3=?, ?+3=5$ |

Table 5: Common Problem Types for Multiplication and Division

|  | UNKNOWN PRODUCT | GROUP SIZE UNKNOWN ("HOW MANY IN EACH GROUP?" DIVISION) | NUMBER OF GROUPS UNKNOWN ("HOW MANY GROUPS?" DIVISION) |
| :---: | :---: | :---: | :---: |
|  | $3 \cdot 6=$ ? | $3 \cdot ?=18$, and $18 \div 3=$ ? | $? \cdot 6=18$, and $18 \div 6=$ ? |
| EQUAL GROUPS | There are 3 bags with 6 plums in each bag. How many plums are there in all? Measurement example. You need 3 lengths of string, each 6 inches long. How much string will you need altogether? | If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? Measurement example. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be? | If 18 plums are to be packed 6 to a bag, then how many bags are needed? Measurement example. You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have? |
| ARRAYS, AREA | There are 3 rows of apples with 6 apples in each row. How many apples are there? Area example. What is the area of a 3 cm by 6 cm rectangle? | If 18 apples are arranged into 3 equal rows, how many apples will be in each row? Area example. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it? | If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? Area example. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it? |
| COMPARE | A blue hat costs $\$ 6$. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? Measurement example. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long? | A red hat costs $\$ 18$ and that is 3 times as much as a blue hat costs. How much does a blue hat cost? Measurement example. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first? | A red hat costs $\$ 18$ and a blue hat costs $\$ 6$. How many times as much does the red hat cost as the blue hat? Measurement example. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first? |
| GENERAL | $\mathrm{a} \cdot \mathrm{b}=$ ? | $\mathrm{a} \cdot \mathrm{?}=\mathrm{p}$ and $\mathrm{p} \div \mathrm{a}=$ ? | $? \cdot \mathrm{~b}=\mathrm{p}$, and $\mathrm{p} \div \mathrm{b}=$ ? |

