Common Core State Standards (CCSS) Mathematics Curriculum Materials Analysis Project

Supported by the Council of Chief State School Officers, Brookhill Foundation, and Texas Instruments

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Overview of the Common Core State Standards Mathematics Curriculum Analysis Project

In June 2010, the Council of Chief State School Officers and National Governor's Association released the Common Core State Standards for mathematics and literacy (CCSSO/NGA, 2010). By June 1, 2011, these standards had been adopted by 44 states, the District of Columbia and the US Virgin Islands. This work represents the first significant attempt in our nation's history to systematically align common K-12 mathematics standards across the states in our nation's history, building on previous efforts to create a national vision for mathematics education, including the National Council of Teachers of Mathematics' standards documents (1989, 2000, 2009, 2011). As such, the new *Common Core State Standards for Mathematics* (CCSSM) will stimulate significant and immediate revisions in state mathematics assessments and classroom curriculum materials.

Predictably, some publishers already claim that their existing curriculum materials and textbooks align with CCSSM (Gerertz, 2010); however, as stated by Michael Cohen, president of Achieve, Inc., "Almost no one thinks there are solid processes in place to examine the alignment of instructional materials to state standards." (p. 20) Over the coming years, as textbook companies revise their materials in accordance with the CCSSM, many K-12 teachers and administrators will find themselves in the position of selecting new mathematics curriculum materials. It is critical that educators have quality resources and tools to determine if the revised curriculum materials and textbooks truly align with the scope and intent of the new Standards.

To increase the likelihood that these Standards, including the both the Content Standards and Standards for Mathematical Practice, are fully implemented in mathematics classrooms across the country, school administrators and classroom teachers need immediate guidance to determine the extent to which the revised curriculum materials support implementation of the CCSSM. Given the significant changes represented in the CCSSM, it is unrealistic to expect that educators in school districts and schools have the time, resources, and background to devise independent review processes for these new standards and would require an inefficient use of local resources. To provide this guidance, the CCSS Mathematics Curriculum Analysis Project provides a set of tools to assist textbook selection committees, school administrators, and teachers in the selection of curriculum materials that support implementation of the new CCSSM.

With funding from the Brookhill Foundation and Texas Instruments and support from the Council of Chief State School Officers and National Council of Supervisors of Mathematics, a national team of educators with expertise in mathematics, mathematics education, and school administration developed a set of mathematics curriculum materials analysis tools. The team included the educators listed on the next page:

- William S. Bush (chair), Mathematics Educator, University of Louisville, Kentucky
- Diane J. Briars, Mathematics Education Consultant, Past President, National Council of Supervisors of Mathematics, Pennsylvania
- Jere Confrey, Mathematics Educator, North Carolina State University
- Kathleen Cramer, Mathematics Educator, University of Minnesota
- Carl Lee, Mathematician, University of Kentucky
- W. Gary Martin, Mathematics Educator, Auburn University, Alabama
- Michael Mays, Mathematician, West Virginia University
- Valerie Mills, Supervisor, Mathematics Education, Oakland Schools, Michigan
- Fabio Milner, Mathematician, Arizona State University
- Suzanne Mitchell, Mathematics Educator/Administrator, Executive Director of the Arkansas Science, Technology, Engineering and Mathematics (STEM) Coalition; President, National Council of Supervisors of Mathematics
- Thomas Post, Mathematics Educator, University of Minnesota
- Robert Ronau, Mathematics Educator, University of Louisville, Kentucky
- Donna Simpson Leak, Superintendent, Rich Township High School District 227, Olympia Fields, Illinois
- Marilyn Strutchens, Mathematics Educator, Auburn University; President, Association of Mathematics Teacher Educators, Alabama

Purpose of the Project

The purpose of the CCSS Mathematics Curriculum Analysis Project is to provide a set of tools that will assist K-12 textbook adoption committees, school administrators, and K-12 teachers in selecting mathematics curriculum materials that support implementation of the newly developed CCSSM. The tools are designed to provide educators with objective measures and information to guide their selection of mathematics curriculum materials based on evidence of the materials' alignment with the CCSSM and support for implementation of the CCSSM in classrooms. Ultimately, the choice of which curriculum materials to adopt must be made by committees or individuals charged with that task. The intention of the tools is to provide assistance in collecting useful information focused on salient issues related to the CCSSM, to ensure consistency across reviewers, and promote discussions about newly developed mathematics curriculum materials. Therefore, at the end of the analysis, the decision about which curriculum materials to select is one that must be made based on the collective evidence gathered with the tools and the committee's or reviewer's vision of the need for curriculum materials to support implementation of the CCSSM locally.

CCSSM is substantially different from past national and state standards. They contain standards about content with respect to both mathematical understanding and procedural skill (CCSSM, p. 4) and Mathematical Practices that focus attention on the varieties of mathematical expertise

and thinking that educators at all levels should seek to develop in their students. These Practices provide a detailed description of the way mathematics should be learned and used by students at all grade levels. The following Practices build on the Process Standards from NCTM (2000) and the strands of mathematical proficiency (National Research Council, 2001) that have been widely used in the field. These Practices describe what it means to really "do" mathematics:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Note that the Standards for Mathematical Practice are also standards and thus must be reflected in the assessments and curriculum materials that states and districts will adopt. Moreover, there are significant changes in the approach to the content, in the placement of content by grade level, and in curriculum emphasis. Thus, to ensure reliable results from the reviews of curriculum materials, the tools will be most effective if the teachers and administrators using them are well grounded in the content and in the specific language of the CCSSM. Professional development prior to the review, therefore, will be a critical component of the process. The CCSSM standards are available at http://www.corestandards.org/the-standards/mathematics.

Development of the Tools

The project team met as a group on three occasions (October 2010, January 2011, and May 2011) to develop the resources provided in this package. Three grade-band teams--K-5, 6-8, and 9-12--were formed to develop tools specific for each grade band. Three tools were developed to provide detailed information about the extent to which curriculum materials align with and support the implementation of CCSSM. Tool 1 focuses on mathematics content trajectories; Tool 2 focuses on Mathematical Practices; and Tool 3 focuses on important considerations complimentary to the standards like equity, assessment, and technology that can impact implementation of mathematics curricula. While Tool 1 is specific to a grade band, Tools 2 and 3 are general and apply to all grade bands. All three tools provide different lenses on which to base a comprehensive analysis and ultimately an informed decision.

The three tools went through various layers of development and review before being released more broadly. First, initial versions developed by the team were piloted with elementary, middle and high school mathematics teachers at three locations across the country. The tools were then revised based on these pilots. Second, the tools were sent to educators, including postsecondary mathematicians and mathematics educators and public school administrators, across the country for further

review. Feedback on the tools was also obtained during sessions at the Association of State Supervisors of Mathematics (ASSM) and NCSM Annual Meetings in April 2011. The tools were revised again based on feedback from these reviewers to obtain final versions. The project team then developed a User's Guide and a professional development experience to ensure potential reviewers used the tools as intended.

Curriculum analysis tools that describe alignment among standards and curriculum materials must consider how well both the Mathematics Content Standards and the Standards for Mathematical Practice are embedded in textbooks and curriculum materials. Tool 1 and Tool 2 were designed to analyze the "Core Curriculum" primary source materials, which generally meant the teacher's edition and the student edition. Clearly these primary source materials should consistently align with the Core Content and Mathematical Practices. Tool 3 offers reviewers the opportunity to analyze other materials such as computer software or teaching guides that can be incorporated as integral "must use" components of the curriculum materials.

Tool 1 provides information about the degree to which specific trajectories of mathematics topics are incorporated appropriately across grade-band curriculum materials. To make this analysis manageable and to provide an in-depth review, developers selected key mathematics domains as defined in CCSSM for each grade band. The four criteria for choosing these domains for review were: (1) they represented critical grade level mathematics content as defined by CCSSM; (2) they clearly reflected the standards at each grade band; (3) they formed content trajectories within and across content areas; and (4) they represented a shift from current curricula. Attempting to look at all mathematics standards within a grade band is overly time consuming and not realistic given the number of different curriculum materials currently available to districts and schools. By focusing more deeply on a limited number of standards in key domains, reviewers will be able to conduct in depth reviews with greater reliability in a reasonable time frame.

Tool 1 focuses on key sequences of mathematics content standards across the four grade bands: K-2, 3-5, 6-8, and 9-12 in the CCSSM. These sequences span within and across grade levels in Tool 1. This organization of the standards in Tool 1 is designed to help reviewers determine the extent to which the curriculum materials develop mathematics content across grade levels, as well as within grade levels, according to the Standards. Since CCSSM does not specify course-level standards for high school, Tool 1 for high school content contains a range of domains that would show growth across grades, depending on what curriculum pathway is being considered for high school. Also, because high school mathematics curricula may be organized in two very distinct pathways, that is, traditional course sequence (Algebra I, Geometry, and Algebra II) as well as an integrated course sequence (Mathematics 1, Mathematics 2, Mathematics 3), reviewers should consider how they will assure coherence across courses in their high school curriculum.

The Content Domains (K-8) and Conceptual Categories (9-12) that the development team selected for Tool 1 at

each grade band are listed below. As mentioned above, Tool 1 does not exhaust the standards within each grade or category, but focuses on important domains or standards with within and across grades to provide a representative analysis within a reasonable time.

Domains			Clusters
K-2	**3-5**	**6-8**	**9-12**
• Number/Operations in Base 10 • Operations and Algebraic Thinking • Geometry	• Number/Operations in Base 10 • Operations and Algebraic Thinking • Geometry • Number and Operations-Fractions	• Ratios and Proportional Relationships • Expressions and Equations • Geometry • Statistics and Probability	• Interpreting Functions • Reasoning with Equations & Inequalities • Similarity, Right Triangles and Trigonometry • Geometric Measurement and Dimension • Interpreting Categorical and Quantitative Data

Tool 2 focuses on the extent to which the Standards for Mathematical Practice are embedded and integrated in the curriculum materials. Since the Mathematical Practices describe the essence of "doing mathematics," mathematics curriculum materials that align with the CCSSM must also provide teachers support in incorporating these Mathematical Practices into their lessons, thereby providing students ample opportunities to engage in the Practices.

Tool 3 focuses on the extent to which mathematics curriculum materials address overarching considerations related to equity, assessment, and technology. This tool guides reviewers to find evidence of teacher support with regard to establishing equitable teaching practices, integrating formative assessment into teaching, and using technology to support the learning and teaching of mathematics.

The three tools developed by the team provide school administrators, teachers, and others involved in selecting mathematics curriculum materials with information to carefully analyze the materials based on important criteria and provide evidence on which to base curriculum materials adoption decisions.

The sections that follow include: a User's Guide to assist reviewers in using the tools; grade-level versions of Tool 1, along with Tool 2 and Tool 3; and a Professional Development Guide with PowerPoint slides that can be used to prepare reviewers for using the tools reliably.

References

Common Core State Standards for Mathematics (2010). Washington, D.C.: Council of Chief State School Officers and National Governors Association.

Gewertz, C. (August 25, 2010). "Curriculum Producers Work to Reflect Common Standards." *Education Week*, 30(1), 1, 20-21.

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User's Guide
CCSS Mathematics Curriculum Materials Analysis Project

June 1, 2011

The **User's Guide** offers specific suggestions about how to use the three curriculum analysis tools to analyze mathematics curriculum materials developed for grades K-12 with respect to the Common Core State Standards in Mathematics (CCSSM). Our experiences with curriculum analysis indicate that providing reviewers the tools and asking them to review curriculum materials is unlikely to lead to a successful analysis and selection process. Preparing the reviewers to use the tools reliably involves the following:

1. Providing professional development so that reviewers can familiarize themselves with the CCSSM and the tools;
2. Organizing teams for the work in order to analyze grade level trajectories within and across grades;
3. Using the tools in order from Tool 1 to Tool 2 to Tool 3 because Tool 2 and Tool 3 use information collected during the completion of Tool 1;
4. Providing adequate time for reviewers to conduct a thorough and in-depth reviews; and
5. Gathering teams together at the end to discuss transitions among grade levels and to use their combined evidence to make and justify recommendations regarding selection of materials.

Directions and suggestions for using each of the three tools are provided for reviewers in the User's Guide. The tools are designed to analyze the primary source materials that describe the learning experiences in which the student will be engaged, which generally means the teacher's edition and the student edition materials. All core curriculum materials should be used with all three tools. Other products such as computer software or teaching guides, provided that they are an integral "must use" or "will use" component of the curriculum, can be useful in responding to questions in Tool 3. The final decision should be based on evidence collected from all three tools and reflect the priorities of the school and/or district. **Throughout the process, reviewers should make independent decisions and not rely upon publisher-produced alignment guides.**

Using Tool 1

Tool 1 allows reviewers to analyze mathematics curriculum materials by identifying key content domains at each grade level K-8 and content clusters for high school. For grades K-8, Tool 1 also describes how the content standards can connect within and across grade levels. Tool 1 for grades K-8 was designed differently than Tool 1 for high school. The high school CCSSM addresses conceptual categories (p. 57) rather than grade bands. Furthermore, the high school content standards contain mathematics topics (denoted by +) that students should learn in order to take advanced mathematics courses such as calculus, advanced statistics or discrete mathematics (STEM standards). Reviewers should be aware of the two different content expectations of these two populations (i.e. college and career, advanced mathematics) as they review the curriculum materials.

In Tool 1, reviewers are required first to complete information about themselves and the curriculum materials under review. Below this information section are two sets of rubrics, one focused on the extent to which key mathematics content from the CCSSM is covered in the curriculum materials and one focused on the extent to which the curriculum materials include a balance of understanding and procedural skills. Overall Tool 1 includes four separate sections: (1) personal information about reviewers; (2) information gathered about the mathematics content in the curriculum materials; (3) Notes/Examples noted during the review of the curriculum materials; and (4) responses to 10 specific summary questions about the curriculum materials. The CCSSM specifies that "mathematical understanding and procedural skill are equally important and both are assessable using mathematical tasks of sufficient richness." (p. 4). To help reviewers capture this richness in the curriculum materials, two lenses are used: coverage and balance. **Coverage** refers to the degree to which the curriculum materials attend to the content of a particular standard. The Content Coverage Rubric reports the extent to which reviewers found the designated mathematics content areas listed in Tool 1. Reviewers must decide if (1) the mathematics content area was found, (2) major, some, or a few gaps were found, or (3) the mathematics content area was covered fully. A key consideration is how easily content gaps could be filled by the district, school, or teacher. For example, it might be relatively easy to provide practice on a particular skill that might be under-emphasized. Providing lessons to address development of a concept that is not addressed may be much more difficult.

Balance addresses the degree to which the mathematics content is developed with a balance between mathematical understanding and procedural skill in ways that are consistent with the standard. The rubric is designed to gather specific evidence regarding how the curriculum materials capture understanding and procedural skills as intended in the CCSSM.

Content Coverage Rubric (Cont):	**Balance of Mathematical Understanding and Procedural Skills Rubric (Bal):**
Not Found (N) - The mathematics content was not found.	Not Found (N) - The content was not found.
Low (L) - Major gaps in the mathematics content were found.	Low (L) - The content was not developed or developed superficially.
Marginal (M) - Gaps in the mathematics content, as described in the Standards, were found and these gaps may not be easily filled.	Marginal (M) - The content was found and focused primarily on procedural skills and minimally on mathematical understanding, or ignored procedural skills.
Acceptable (A) - Few gaps in the mathematics content, as described in the Standards, were found and these gaps may be easily filled.	Acceptable (A) -The content was developed with a balance of mathematical understanding and procedural skills consistent with the Standards, but the connections between the two were not developed.
High (H) - The mathematics content was fully formed as described in the Standards.	High (H) - The content was developed with a balance of mathematical understanding and procedural skills consistent with the Standards, and the connections between the two were developed.

The Coverage and Balance rubrics use non-numeric scales, (i.e. Not Found, Low, Marginal, Acceptable, and High) to better capture the qualitative nature of this part of the review. We chose not to use a numerical scale for the rubrics because the categories are more appropriately used as guidance for discussions in order to make reasoned decisions about the curriculum materials rather than to compute an "average" numerical result.

Reviewers are required first to locate evidence of the Standards in the curriculum materials noting the location by page numbers in the first column labeled Chap Pages (chapter pages) beside the Standards. Reviewers are then asked to record their judgments regarding the Content Coverage and Balance Rubrics. **For this analysis, we suggest that the reviewers focus on only the teacher and student books, not any supporting materials.** Throughout the review process, reviewers should make notes of evidence that supports their judgment and write down key examples of what they found during the content analysis so that this information might be shared in subsequent group discussions about the curriculum materials.

At the end of Tool 1, reviewers are asked to respond to a set of questions under the headings *Overall Impressions*, *Content Alignment*, and *Balance between Mathematics Understanding and Procedural Skills*. These questions are designed to provide guidance for within and across grade-band discussions to determine the degree to which the key trajectories and content in the curriculum materials were developed in line with the CCSSM. Recording the final outcomes from these discussions will be useful for subsequent discussions and recommendations.

Using Tool 2

Tool 2 is used to determine the extent to which the curriculum materials were designed to provide students opportunities to engage in the Standards for Mathematical Practice. The CCSSM specify that "the Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important 'processes and proficiencies' with longstanding importance in mathematics education." (p. 6). This tool allows reviewers to determine how well the Mathematical Practices connect to student and teacher activities in the curriculum materials.

To begin the search for Mathematical Practices in curriculum materials, reviewers are pointed to the shaded cells in Tool 1. These content standards were chosen as a basis for reviewing the Mathematical Practices because developers felt that they had the greatest potential to incorporate the Mathematical Practices in the curriculum materials.

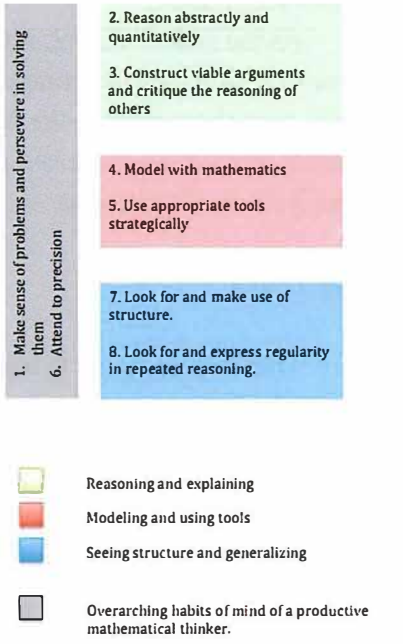

Figure 1

http://commoncoretools.files.wordpress.com/2011/03/practices.pdf

Using the content standards in those cells as a basis, reviewers can use their notes from Tool 1 to locate those content areas in the curriculum materials and analyze specific student tasks, assignments, or projects in the materials to determine, and then to assess, the extent to which the materials reflect the eight Mathematical Practices. Reviewers should record these results in Tool 2. Keep in mind that the identified content standards are only suggestions, not mandates, for where the practices might be addressed. To ensure that reviewers do not miss important aspects of curriculum materials designed to support the Mathematical Practices, reviewers should read the overview in Practices to ascertain the ways in which the materials addresses the Mathematical Practices. Reviewers can then use this information in using Tool 2.

The evidence and notes about the location and nature of the Practices should be recorded in the boxes under each of the eight Practices to facilitate discussions among reviewers later. If no evidence can be found to support a particular Mathematical Practice, a note should be made of this as well. A copy of the Standards for Mathematical Practice, presented as a bulleted list of the ways to engage in doing mathematics for each standard, accompanies Tool 2 to assist in the review. CCSSM places great emphasis on Standards for Mathematical Practice, so reviewers should become very familiar with these Practices and what they mean in order to effectively use this tool. The Mathematical Practices in Tool 2 have been organized in one possible configuration (Figure 1); however, the Practices are not necessarily discrete and other structures

may be possible. One example or task may fit under multiple Mathematical Practices and should be recorded in each.

At the end of Tool 2, reviewers are asked to respond to a set of questions to determine the degree to which the mathematics content reviewed in the curriculum materials support teachers as they engage students in the Mathematical Practices. These questions are designed to provide guidance for within and across grade-band discussions. Recording the final outcomes from these discussions would likely be useful for subsequent discussions and recommendations.

Using Tool 3

Tool 3 is designed to address three overarching considerations that will impact the materials' effectiveness in supporting the CCSSM. It should be used after reviewing mathematics curriculum materials using Tool 1 (Content Analysis) and Tool 2 (Mathematics Practices Analysis). Based on what reviewers have noted in reviewing the materials, as well as in additional software or materials that have been identified by the committee as an integral "must use" or "will use" component of the curriculum, reviewers should answer the questions reflecting how well the curriculum materials support teachers with regard to the three important overarching issues of **Equity/Diversity/Access, Formative Assessment,** and **Technology** that support teaching the Mathematics Core Content and Mathematical Practices. With regard to **Equity/Diversity/Access**, the National Council of Teachers of Mathematics (NCTM,1991) asked teachers to: (1) build on how students' linguistic, ethnic, racial, gender, and socioeconomic backgrounds influence their learning; (2) help students become aware of the role of mathematics in society and culture; (3) expose students to the contributions of various cultures to the advancement of mathematics; (4) show students how mathematics relates to other subjects; and (5) provide students with opportunities to apply mathematics to authentic contexts. CCSSM also notes that, "The Standards should be read as allowing for the widest possible range of students to participate fully from the outset, along with appropriate accommodations to ensure maximum participation of students with special education needs." **Formative Assessment** is an instructional process that, if implemented appropriately, can improve student learning. Curriculum materials can provide a variety of levels of support for formative assessment, including extra homework exercises, classroom tests, and ongoing tasks including innovative projects and other student products. Finally, the increasing availability of **Technology** offers opportunities to use technology mindfully in ways that assist teachers in teaching mathematics and enable students to explore and deepen their understanding of mathematical concepts and procedures, as well as improving problem-solving and reasoning skills.

Tool 3 requires reviewers to focus their analysis on answering individual questions related to the extent that the curriculum materials reflect equitable practices, embed high quality and high cognitive formative assessments, and encourage the use of technology in rich and appropriate ways. Reviewers might wish to revisit the curriculum materials as they address the questions in Tool 3. After answering the questions using the rubric, reviewers should write

comments regarding their rating in spaces provided on the left hand side of the Tool. The rubric is listed below:

Rubric for answering questions about **Overarching Considerations**:

Not Found (NF)	The curriculum materials do not support this element.
Low (L)	The curriculum materials contain limited support for this element, but the support is not embedded or consistently present within and across grades.
Medium (M)	The curriculum materials contain support for this element, but it is not always embedded or consistently present within and across grades.
High (H)	The curriculum materials contain embedded support for this element so that it is consistently present within and across grades.

The rubric describes the extent to which the materials provide teachers support in these three critical overarching considerations. **In contrast to the previous tools, we suggest here that reviewers consider supporting materials in addition to the teacher and student materials.**

At the end of Tool 3, reviewers are asked to summarize their responses through questions about the three overarching considerations. These questions were designed to provide guidance and stimulate discussion to determine the degree to which these issues were addressed in the curriculum materials. Recording the final outcomes from these discussions will be useful for subsequent discussions and recommendations.

Reaching a Conclusion

As mentioned earlier, these tools were designed to assist reviewers of mathematics curriculum materials in gathering information that can be used to determine the extent to which the materials provide teachers and students with the best opportunities to meet the CCSSM. The next step is to bring reviewers together and examine collectively the evidence gathered with the tools. In order to address the trajectories in the CCSSM, reviewers should collect the evidence across teams and grade bands--i.e., grades K to grade 2, grade 3 to grade 5, grade 6 to grade 8, and content areas in grades 9 to 12. Groups of reviewers are encouraged to work together to determine the strengths and weaknesses of each set of curriculum materials under review. We encourage them to identify those features that will provide teachers and students opportunities to meet the requirements of the CCSSM and prepare students for the upcoming assessments based on the CCSSM.

Tool 1
Mathematics Content
Grades 6-8

CCSSM Curriculum Analysis Tool 1— Ratios and Proportional Relationships for Grades 6-8

Name of Reviewer _____ **School/District** _____ **Date** _____

Name of Curriculum Materials _____ **Publication Date** _____ **Grade Level(s)** _____

Content Coverage Rubric (Cont):
Not Found (N) -The mathematics content was not found.
Low (L) - Major gaps in the mathematics content were found.
Marginal (M) - Gaps in the content, as described in the Standards, were found and these gaps may not be easily filled.
Acceptable (A) - Few gaps in the content, as described in the Standards, were found and these gaps may be easily filled.
High (H) - The content was fully formed as described in the Standards.

Balance of Mathematical Understanding and Procedural Skills Rubric (Bal):
Not Found (N) -The content was not found.
Low (L)- The content was not developed or developed superficially.
Marginal (M) - The content was found and focused primarily on procedural skills and minimally on mathematical understanding, or ignored procedural skills.
Acceptable (A)-The content was developed with a balance of mathematical understanding and procedural skills consistent with the Standards, but the connections between the two were not developed.
High (H)-The content was developed with a balance of mathematical understanding and procedural skills consistent with the Standards, and the connections between the two were developed.

CCSSM Grade 6				CCSSM Grade 7				CCSSM Grade 8			
6.RP Ratios and Proportional Relationships	**Chap. Pages**	**Cont N-L-M-A-H**	**Bal N-L-M-A-H**	**7.RP Ratios and Proportional Relationships**	**Chap. Pages**	**Cont N-L-M-A-H**	**Bal N-L-M-A-H**	**8.EE Expressions and Equations**	**Chap. Pages**	**Cont N-L-M-A-H**	**Bal N-L-M-A-H**
Understand ratio concepts and use ratio reasoning to solve problems.				**Analyze proportional relationships and use them to solve real-world and mathematical problems.**				**Understand connections between proportional relationships, lines, and linear equations.**			
1. Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak."				1. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction 1/2/1/4 miles per hour, equivalently 2 miles per hour.				5. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.			

CCSSM Curriculum Analysis Tool 1—Ratios and Proportional Relationships for Grades 6-8												
CCSSM Grade 6				**CCSSM Grade 7**				**CCSSM Grade 8**				
Understand ratio concepts and use ratio reasoning to solve problems.				**Analyze proportional relationships and use them to solve real-world and mathematical problems.**				**Understand connections between proportional relationships, lines, and linear equations.**				
2. Understand the concept of a unit rate a/b associated with a ratio a:b with $b \neq 0$, and use rate language in the context of a ratio relationship. For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."				2. Recognize and represent proportional relationships between quantities. 2a. Decide whether two quantities are in a proportional relationship by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. 2d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation.				6. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.			
3. Use ratio and rate reasoning to solve real-world and mathematical problems by reasoning. 3c. Find a percent of a quantity as a rate per 100; solve problems involving finding the whole, given a part and the percent.				2b. Identify the constant of proportionality in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. 2c. Represent proportional relationships by equations.								
3a. Make tables of equivalent ratios relating quantities with whole umber measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.				3. Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease.*								
3b. Find a percent of a quantity as a rate per 100; solve problems involving finding the whole, given a part and the percent.												
3d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.												

CCSSM Curriculum Analysis Tool 1—Ratios and Proportional Relationships for Grades 6-8	
Notes and Examples:	
Overall Impressions: 1. What are your overall impressions of the curriculum materials examined? 2. What are the strengths and weaknesses of the materials you examined? **Standards Alignment:** 3. Have you identified gaps within this domain? What are they? If so, can these gaps be realistically addressed through supplementation? 4. Within grade levels, do the curriculum materials provide sufficient experiences to support student learning within this standard? 5. Within this domain, is the treatment of the content across grade levels consistent with the progression within the Standards?	**Balance between Mathematical Understanding and Procedural Skills** 6. Do the curriculum materials support the development of students' mathematical understanding? 7. Do the curriculum materials support the development of students' proficiency with procedural skills? 8. Do the curriculum materials assist students in building connections between mathematical understanding and procedural skills? 9. To what extent do the curriculum materials provide a balanced focus on mathematical understanding and procedural skills? 10. Do student activities build on each other within and across grades in a logical way that supports mathematical understanding and procedural skills?

CCSSM Curriculum Analysis Tool 1—Geometry for Grades 6-8

Name of Reviewer _____ School/District _____ Date _____

Name of Curriculum Materials _____ Publication Date _____ Grade Level(s) _____

Content Coverage Rubric (Cont):
Not Found (N) -The mathematics content was not found.
Low (L) - Major gaps in the mathematics content were found.
Marginal (M) - Gaps in the content, as described in the Standards, were found and these gaps may not be easily filled.
Acceptable (A) - Few gaps in the content, as described in the Standards, were found and these gaps may be easily filled.
High (H) - The content was fully formed as described in the Standards.

Balance of Mathematical Understanding and Procedural Skills Rubric (Bal):
Not Found (N) -The content was not found.
Low (L) - The content was not developed or developed superficially.
Marginal (M)-The content was found and focused primarily on procedural skills and minimally on mathematical understanding, or ignored procedural skills.
Acceptable (A)-The content was developed with a balance of mathematical understanding and procedural skills consistent with the Standards, but the connections between the two were not developed.
High (H)-The content was developed with a balance of mathematical understanding and procedural skills consistent with the Standards, and the connections between the two were developed.

CCSSM Grade 6				CCSSM Grade 7				CCSSM Grade 8			
6.G Geometry	**Chap Pages**	**Cont N-L-M-A-H**	**Bal N-L-M-A-H**	**7.G Geometry**	**Chap Pages**	**Cont N-L-M-A-H**	**Bal N-L-M-A-H**	**8.G Geometry**	**Chap Pages**	**Cont N-L-M-A-H**	**Bal N-L-M-A-H**
Solve real-world and mathematical problems involving area, surface area, and volume.				**Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.**				**Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.***			
1. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.				4. Know the formulas for area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.							
2. Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply formulas $V=l\,w\,h$ and $V = bh$ to find volumes to solve				6. Solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.							

real-world and mathematical problems.											
CCSSM Curriculum Analysis Tool 1—Geometry for Grades 6-8											
CCSSM Grade 6				**CCSSM Grade 7**				**CCSSM Grade 8**			
6.G Geometry	**Chap Pages**	**Cont N-L-M-A-H**	**Bal N-L-M-A-H**	**7.G Geometry**	**Chap Pages**	**Cont N-L-M-A-H**	**Bal N-L-M-A-H**	**8.G Geometry**	**Chap Pages**	**Cont N-L-M-A-H**	**Bal N-L-M-A-H**
Solve real-world/math problems involving area, surface area, and volume.				**Solve real-world/math problems involving angle measure, area, surface area, and volume.**				**Solve real-world/ mathematical problems involving volume of cylinders, cones, and spheres.**			
4. Represent 3-dimensional figures using nets of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.				3. Describe the two-dimensional figures that result from slicing three dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.				9. Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.			
				Draw, construct, and describe geometrical figures and describe the relationships between them.				**Understand congruence and similarity using physical models, transparencies, or geometry software.**			
3. Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate.				5. Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.				5. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.			
								1. Verify the properties of rotations, reflections, and translations: a. lines are taken to lines and the line segments to line segments of the same length; b. angles are taken to angles; c. parallel lines are taken to parallel lines.			
								3. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.			
				1. Solve problems involving scale drawings of geometric figures, including computing actual lengths				4. Understand that a 2-dimensional figure is similar to another if the second can be obtained from the			

				and areas from a scale drawing and reproducing a scale drawing at a different scale.				first by rotations, reflections, translations, and dilations; given two similar figures, describe sequences that make them similar.				
CCSSM Curriculum Analysis Tool 1—Geometry for Grades 6-8												
CCSSM Grade 6				**CCSSM Grade 7**				**CCSSM Grade 8**				
6.G Geometry	**Chap Pages**	**Cont N-L-M-A-H**	**Bal N-L-M-A-H**	**7.G Geometry**	**Chap Pages**	**Cont N-L-M-A-H**	**Bal N-L-M-A-H**	**8.G Geometry**	**Chap Pages**	**Cont N-L-M-A-H**	**Bal N-L-M-A-H**	
				Draw, construct, and describe geometrical figures and describe the relationships between them.				**Understand congruence and similarity using physical models, transparencies, or geometry software.**				
				2. Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.				2. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits congruence between them.				
								Understand and apply the Pythagorean Theorem				
								6. Explain a proof of the Pythagorean Theorem and its converse.				
								7. Apply the Pythagorean Theorem to determine the unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.				
								8. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.				

Notes/Examples:

CCSSM Curriculum Analysis Tool 1--Geometry for Grades 6-8	
Overall Impressions: 11. What are your overall impressions of the curriculum materials examined? 12. What are the strengths and weaknesses of the materials you examined? **Standards Alignment:** 13. Have you identified gaps within this domain? What are they? If so, can these gaps be realistically addressed through supplementation? 14. Within grade levels, do the curriculum materials provide sufficient experiences to support student learning within this standard? 15. Within this domain, is the treatment of the content across grade levels consistent with the progression within the Standards?	**Balance between Mathematical Understanding and Procedural Skills:** 16. Do the curriculum materials support the development of students' mathematical understanding? 17. Do the curriculum materials support the development of students' proficiency with procedural skills? 18. Do the curriculum materials assist students in building connections between mathematical understanding and procedural skills? 19. To what extent do the curriculum materials provide a balanced focus on mathematical understanding and procedural skills? 20. Do student activities build on each other within and across grades in a logical way that supports mathematical understanding and procedural skills?

CCSSM Curriculum Analysis Tool 1—Expressions and Equations for Grades 6-8

Name of Reviewer _____ **School/District** _____ **Date** _____

Name of Curriculum Materials _____ **Publication Date** _____ **Grade Level(s)** _____

Content Coverage Rubric (Cont):
Not Found (N) -The mathematics content was not found.
Low (L) - Major gaps in the mathematics content were found.
Marginal (M) - Gaps in the content, as described in the Standards, were found and these gaps may not be easily filled.
Acceptable (A) - Few gaps in the content, as described in the Standards, were found and these gaps may be easily filled.
High (H) - The content was fully formed as described in the Standards.

Balance of Mathematical Understanding and Procedural Skills Rubric (Bal):
Not Found (N) -The content was not found.
Low (L)- The content was not developed or developed superficially.
Marginal (M) - The content was found and focused primarily on procedural skills and minimally on mathematical understanding, or ignored procedural skills.
Acceptable (A)-The content was developed with a balance of mathematical understanding and procedural skills consistent with the Standards, but the connections between the two were not developed.
High (H)-The content was developed with a balance of mathematical understanding and procedural skills consistent with the Standards, and the connections between the two were developed.

CCSSM Grade 6				CCSSM Grade 7				CCSSM Grade 8			
6.EE Expressions and Equations	**Chap Pages**	**Cont N-L-M-A-H**	**Bal N-L-M-A-H**	**7.EE Expressions and Equations**	**Chap Pages**	**Cont N-L-M-A-H**	**Bal N-L-M-A-H**	**8.EE Expressions and Equations**	**Chap Pages**	**Cont N-L-M-A-H**	**Bal N-L-M-A-H**
Apply and extend previous understandings of arithmetic to algebraic expressions				**Use properties of operations to generate equivalent expressions**				**Work with radicals and integer exponents**			
1. Write and evaluate numerical expressions involving whole number exponents.								1. Know and apply the properties of integer exponents to generate equivalent numerical expressions.			
								4. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities. Interpret scientific notation that has been generated by technology.			

CCSSM Curriculum Analysis Tool 1—Expressions and Equations for Grades 6-8											
CCSSM Grade 6				**CCSSM Grade 7**				**CCSSM Grade 8**			
6.EE Expressions and Equations	**Chap Pages**	**Cont N-L-M-A-H**	**Bal N-L-M-A-H**	**7.EE Expressions and Equations**	**Chap Pages**	**Cont N-L-M-A-H**	**Bal N-L-M-A-H**	**8.EE Expressions and Equations**	**Chap Pages**	**Cont N-L-M-A-H**	**Bal N-L-M-A-H**
Apply and extend previous understandings of arithmetic to algebraic expressions				**Use properties of operations to generate equivalent expressions**				**Work with radicals and integer exponents**			
2. Write, read, and evaluate expressions in which letters stand for numbers. a. Write expressions that record operations with numbers and with letters standing for numbers. b. Identify parts of an expression using mathematical terms (sum, term, product, quotient, coefficient); view one or more parts of an expression as a single entity.											
c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations that include whole-number exponents, in the order when there are no parentheses to specify order.				1. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.							
3. Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property or properties of operations.*				2. Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities are related.							
4. Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them).											
6. Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or any number in a specified set.											

CCSSM Curriculum Analysis Tool 1—Expressions and Equations for Grades 6-8										
CCSSM Grade 6				**CCSSM Grade 7**				**CCSSM Grade 8**			
6.EE Expressions and Equations	**Chap Pages**	**Cont N-L-M-A-H**	**Bal N-L-M-A-H**	**7.EE Expressions and Equations**	**Chap Pages**	**Cont N-L-M-A-H**	**Bal N-L-M-A-H**	**8.EE Expressions and Equations**	**Chap Pages**	**Cont N-L-M-A-H**	**Bal N-L-M-A-H**
Reason about and solve one-variable equations and inequalities				**Solve real life and mathematical problems using numerical and algebraic expressions and equations**				**Analyze and solve linear equations and pairs of simultaneous linear equations**			
5. Understand solving an equation or inequality as a process of answering a question: Which values form a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.								7. Solve linear equations in one variable. a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.			
7. Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.				4. Use variables to represent quantities in a real-world and mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, r are specific rational numbers. Solve equations like these fluently.							
8. Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of inequalities on number lines.				b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.							

CCSSM Curriculum Analysis Tool 1—Expressions and Equations for Grades 6-8										
CCSSM Grade 6				**CCSSM Grade 7**				**CCSSM Grade 8**			
6.EE Expressions and Equations	**Chap Pages**	**Cont N-L-M-A-H**	**Bal N-L-M-A-H**	**7.EE Expressions and Equations**	**Chap Pages**	**Cont N-L-M-A-H**	**Bal N-L-M-A-H**	**8.EE Expressions and Equations**	**Chap Pages**	**Cont N-L-M-A-H**	**Bal N-L-M-A-H**
								8. Analyze and solve pairs of linear equations. a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations. b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. c. Solve real-world and math problems leading to two linear equations in two variables.			

Notes/Examples:

Overall Impressions:

1. What are your overall impressions of the curriculum materials examined?
2. What are the strengths and weaknesses of the materials you examined?

Standards Alignment:

3. Have you identified gaps within this domain? What are they? If so, can these gaps be realistically addressed through supplementation?
4. Within grade levels, do the curriculum materials provide sufficient experiences to support student learning within this standard?
5. Within this domain, is the treatment of the content across grade levels consistent with the progression within the Standards?

Balance between Mathematical Understanding and Procedural Skills

6. Do the curriculum materials support the development of students' mathematical understanding?
7. Do the curriculum materials support the development of students' proficiency with procedural skills?
8. Do the curriculum materials assist students in building connections between mathematical understanding and procedural skills?
9. To what extent do the curriculum materials provide a balanced focus on mathematical understanding and procedural skills?
10. Do student activities build on each other within and across grades in a logical way that supports mathematical understanding and procedural skills?

CCSSO Curriculum Analysis Tool 1—Statistics and Probability for Grades 6-8

Name of Reviewer _____ School/District _____ Date _____

Name of Curriculum Materials _____ Publication Date _____ Grade Level(s) _____

Content Coverage Rubric (Cont):
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CCSSM Grade 6				CCSSM Grade 7				CCSSM Grade 8			
6.SP Statistics and Probability	Chap Pages	**Cont N-L-M-A-H**	**Bal N-L-M-A-H**	**7.SP Statistics and Probability**	Chap Pages	**Cont N-L-M-A-H**	**Bal N-L-M-A-H**	**8.SP Statistics and Probability**	Chap Pages	**Cont N-L-M-A-H**	**Bal N-L-M-A-H**
Develop understanding of statistical variability.				**Use random sampling to draw inferences about a population.**				**Investigate patterns of association in bivariate data.**			
1. Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.*				1. Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Know that random sampling produces samples and supports valid inferences.				1. Construct and interpret scatterplots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.			
				2. Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions.				2. Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.			

CCSSM Curriculum Analysis Tool 1—Statistics and Probability for Grades 6-8											
CCSSM Grade 6				**CCSSM Grade 7**				**CCSSM Grade 8**			
6.SP Statistics and Probability	Chap Pages	**Cont N-L-M-A-H**	**Bal N-L-M-A-H**	**7.SP Statistics and Probability**	Chap Pages	**Content N-L-M-A-H**	**Bal N-L-M-A-H**	**8.SP Statistics and Probability**	Chap Pages	**Cont N-L-M-A-H**	**Bal N-L-M-A-H**
Develop understanding of statistical variability				**Draw informal comparative inferences about two populations**							
2. Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.				3. Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable*							
3. Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.				4. Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*				4. Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables			
Notes/Examples											

CCSSM Curriculum Analysis Tool 1—Statistics and Probability for Grades 6-8										
CCSSM Grade 6				**CCSSM Grade 7**				**CCSSM Grade 8**			
6.SP Statistics and Probability	Chap Pages	**Cont N-L-M-A-H**	**Bal N-L-M-A-H**	**7.SP Statistics and Probability**	Chap Pages	**Content N-L-M-A-H**	**Bal N-L-M-A-H**	**8.SP Statistics and Probability**	Chap Page	**Cont N-L-M-A-H**	**Bal N-L-M-A-H**
Summarize and describe distributions.											
4. Display numerical data in plots on a number line, including dot plots, histograms, and box plots.											
5. Summarize numerical data sets in relation to their context, such as by: a. Reporting the number of observations; b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement; c. giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered; and d. relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.											
				Investigate chance processes and develop, use, and evaluate probability models							
				5. Understand that the probability of a chance event is a between 0 and 1 and expresses the likelihood of the event. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is not unlikely or likely, and a probability near 1 indicates a likely event.							

CCSSM Curriculum Analysis Tool 1—Statistics and Probability for Grades 6-8											
CCSSM Grade 6				**CCSSM Grade 7**				**CCSSM Grade 8**			
6.SP Statistics and Probability	Chap Pages	**Cont N-L-M-A-H**	**Bal N-L-M-A-H**	**7.SP Statistics and Probability**	Chap Pages	**Cont N-L-M-A-H**	**Bal N-L-M-A-H**	**8.SP Statistics and Probability**	Chap Page	**Cont N-L-M-A-H**	**Bal N-L-M-A-H**
				Investigate chance processes and develop, use, and evaluate probability models							
				6. Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.							
				7. Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.							
				7a. Develop a probability model by assigning equal probability to all outcomes, and use the model to find probabilities of events.							
				7b. Develop a probability model by observing frequencies in data generated from a chance process (which may not be uniform) by observing frequencies in data generated from a chance process. 7c. Design and use a stimulation to generate frequencies for compound events.							
				8. Find probabilities of compound events using lists, tables, tree diagrams, and simulation. a. Understand that the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. 8c. Design and use a simulation to generate frequencies for compound events.							

CCSSM Curriculum Analysis Tool 1—Statistics and Probability for Grades 6-8	
Notes/Examples	
Overall Impressions: 1. What are your overall impressions of the curriculum materials examined? 2. What are the strengths and weaknesses of the materials you examined? **Standards Alignment:** 3. Have you identified gaps within this domain? What are they? If so, can these gaps be realistically addressed through supplementation? 4. Within grade levels, do the curriculum materials provide sufficient experiences to support student learning within this standard? 5. Within this domain, is the treatment of the content across grade levels consistent with the progression within the Standards?	**Balance between Mathematical Understanding and Procedural Skills** 6. Do the curriculum materials support the development of students' mathematical understanding? 7. Do the curriculum materials support the development of students' proficiency with procedural skills? 8. Do the curriculum materials assist students in building connections between mathematical understanding and procedural skills? 9. To what extent do the curriculum materials provide a balanced focus on mathematical understanding and procedural skills? 10. Do student activities build on each other within and across grades in a logical way that supports mathematical understanding and procedural skills?

Tool 2
Mathematical Practices

Name of Reviewer _____ School/District _____ Date _____

Name of Curriculum Materials _____ Publication Date _____ Grade Level(s) _____

Tool 1 Domain Considered _____

Opportunities to Engage in the Standards for Mathematical Practices Found Across the Content Standards

Overarching Habits of Mind	1. Make sense of problems and persevere in solving them.	6. Attend to precision.
Evidence of how the Standards for Mathematics Practice were addressed (with page numbers)		
Reasoning and Explaining	**2. Reason abstractly and quantitatively.**	**3. Construct viable arguments and critique the reasoning of others.**
Evidence of how the Standards for Mathematics Practice were addressed (with page numbers)		

CCSSM Mathematical Practices Analysis Tool 2		Page 2
Modeling and Using Tools	**4. Model with mathematics.**	**5. Use appropriate tools strategically.**
Evidence of how the Standards for Mathematics Practice were addressed (with page numbers)		
Seeing Structure and Generalizing	**7. Look for and make use of structure.**	**8. Look for and express regularity in repeated reasoning.**
Evidence of how the Standards for Mathematics Practice were addressed (with page numbers)		

Synthesis of Standards for Mathematical Practice	Page 3
(Mathematical Practices → Content) To what extent do the materials demand that students engage in the Standards for Mathematical Practice as the primary vehicle for learning the Content Standards?	
(Content → Mathematical Practices) To what extent do the materials provide opportunities for students to develop the Standards for Mathematical Practice as "habits of mind" (ways of thinking about mathematics that are rich, challenging, and useful) throughout the development of the Content Standards?	
To what extent do accompanying assessments of student learning (such as homework, observation checklists, portfolio recommendations, extended tasks, tests, and quizzes) provide evidence regarding students' proficiency with respect to the Standards for Mathematical Practice?	
What is the quality of the instructional support for students' development of the Standards for Mathematical Practice as habits of mind?	
Summative Assessment **(Low) – The Standards for Mathematical Practice are not addressed or are addressed superficially.** **(Marginal) The Standards for Mathematical Practice are addressed, but not consistently in a way that is embedded in the development of the Content Standards.** **(Acceptable) – Attention to the Standards for Mathematical Practice is embedded throughout the curriculum materials in ways that may help students to develop them as habits of mind.**	**Explanation for score**

COMMON CORE STATE STANDARDS FOR MATHEMATICS

Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning , strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately) and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1 Make sense of problems and persevere in solving them.

Mathematically proficient students:

- explain to themselves the meaning of a problem and looking for entry points to its solution.
- analyze givens, constraints, relationships, and goals.
- make conjectures about the form and meaning of the solution attempt.
- plan a solution pathway rather than simply jumping into a solution.
- consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution.
- monitor and evaluate their progress and change course if necessary.
- transform algebraic expressions or change the viewing window on their graphing calculator to get information.
- explain correspondences between equations, verbal descriptions, tables, and graphs.
- draw diagrams of important features and relationships, graph data, and search for regularity or trends.
- use concrete objects or pictures to help conceptualize and solve a problem.
- check their answers to problems using a different method.
- ask themselves, "Does this make sense?"
- understand the approaches of others to solving complex problems and identify correspondences between approaches.

2. Reason abstractly and quantitatively.

Mathematically proficient students:

- make sense of quantities and their relationships in problem situations.
- Bring two complementary abilities to bear on problems involving quantitative relationships:
  - ✓ *decontextualize* (abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents and
  - ✓ *contextualize* (pause as needed during the manipulation process in order to probe into the referents for the symbols involved).
- use quantitative reasoning that entails creating a coherent representation of the problem at hand, considering the units involved, and attending to the meaning of quantities, not just how to compute them
- know and flexibly use different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students:

- understand and use stated assumptions, definitions, and previously established results in constructing arguments.
- make conjectures and build a logical progression of statements to explore the truth of their conjectures.
- analyze situations by breaking them into cases
- recognize and use counterexamples.
- justify their conclusions, communicate them to others, and respond to the arguments of others.

- reason inductively about data, making plausible arguments that take into account the context from which the data arose
- compare the effectiveness of plausible arguments
- distinguish correct logic or reasoning from that which is flawed and, if there is a flaw, explain what it is
  - ✓ elementary students construct arguments using concrete referents such as objects, drawings, diagrams, and actions..
  - ✓ later students learn to determine domains to which an argument applies.
- listen or read the arguments of others, decide whether they make sense, and ask useful question to clarify or improve arguments

4 Model with mathematics.

Mathematically proficient students:

- apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.
  - ✓ In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community.
  - ✓ By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another.
- make assumptions and approximations to simplify a complicated situation, realizing that these may need revision later.
- identify important quantities in a practical situation
- map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas.
- analyze those relationships mathematically to draw conclusions.
- interpret their mathematical results in the context of the situation.
- reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students

- consider available tools when solving a mathematical problem. (These tools might include pencil and paper, concrete models, a ruler, protractor, calculator, spreadsheet, computer algebra system, a statistical package, or dynamic geometry software.
- are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations.
  - ✓ High school students analyze graphs of functions and solutions generated using a graphing calculator
- detect possible errors by using estimations and other mathematical knowledge.
- know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data.
- identify relevant mathematical resources and use them to pose or solve problems.
- use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students:

- try to communicate precisely to others.
- try to use clear definitions in discussion with others and in their own reasoning.
- state the meaning of the symbols they choose, including using the equal sign consistently and appropriately.
- specify units of measure and label axes to clarify the correspondence with quantities in a problem.
- calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context.
  - ✓ In the elementary grades, students give carefully formulated explanations to each other.
  - ✓ In high school, students have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.

Mathematically proficient students:

- look closely to discern a pattern or structure.
  - ✓ Young students might notice that three and seven more is the same amount as seven and three more or they may sort a collection of shapes according to how many sides the shapes have.

- ✓ Later, students will see 7 x 8 equals the well remembered 7 x 5 + 7 x 3, in preparation for the distributive property.
- ✓ In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems.

- step back for an overview and can shift perspective.
- see complicated things, such as some algebraic expressions, as single objects or composed of several objects.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students:

- notice if calculations are repeated
- look both for general methods and for shortcuts.
  - ✓ Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeated decimal.
  - ✓ Middle school students might abstract the equation (y-2)/((x-1)=3 by paying attention to the calculation of slope as they repeatedly check whether the points are on the line through (1,2) with a slope 3.
  - ✓ Noticing the regularity in the way terms cancel when expanding $(x-1)(x+1)(x^2+1)$ and $(x-1)(x^3+x^2+x+1)$ might lead high school students to the general formula for the sum of a geometric series.
- maintain oversight of the process of solving a problem, while attending to the details.
- continually evaluate the reasonableness of intermediate results.

Tool 3
Overarching Considerations

Equity
Formative Assessment
Technology

CCSSM Curriculum Analysis Tool 3 (Overarching Considerations)

This tool should be used after reviewing mathematics curriculum materials using Tool 1 (Content Analysis) and Tool 2 (Mathematical Practices Analysis). After reviewing the curriculum materials carefully, answer the questions below reflecting important overarching considerations with regard to the materials. Overarching considerations are those that support the teaching of Mathematics Core Content and Practices. **Equity:** NCTM (1991) calls for teachers to build on how students' linguistic, ethnic, racial, gender, and socioeconomic backgrounds influence their learning; to help students to become aware of the role of mathematics in society and culture; to expose students to the contributions of various cultures to the advancement of mathematics; and to show students how mathematics relates to other subjects; and to provide students with opportunities to apply mathematics to authentic contexts. CCSSM also notes that, "The Standards should be read as allowing for the widest possible range of students to participate fully from the outset, along with appropriate accommodations to ensure maximum participation of students with special education needs." **Formative Assessment** is a critical part of classroom instruction, and curriculum materials can provide a variety of levels of support with regard to information to teachers about student learning. Finally, the increasing availability of **technology** offers opportunities to use technology mindfully in ways that enable students to explore and deepen their understanding of mathematical concepts.

Name of Reviewer _____ School/District _____ Date _____

Name of Curriculum Materials _____ Publication Date _____ Grade Level(s) _____

Rubric for answering questions about Overarching Considerations:

Not Found (N) - The curriculum materials do not support this element.
Low (L) - The curriculum materials contain limited support for this element, but the support is not embedded or consistently present within or across grades.
Medium (M) - The curriculum materials contain support for this element, but it is not always embedded or consistently present within or across grades.
High (H) - The curriculum materials contain embedded support for this element so that it is consistently present within and across grades.

Questions about Overarching Considerations (Page 1)	See Rubric	Comments/Examples
Equity	**N-L-M-H**	
To what extent do the materials:		
1. Provide teachers with strategies for meeting the needs of a range of learners?		
2. Provide instructional support to help teachers sequence or scaffold lessons so that students move from what they know to what they do not know?		
3. Provide opportunities for teachers to use a variety of grouping strategies?		
4. Embed tasks with multiple entry-points that can be solved using a variety of solution strategies or representations?		
5. Suggest accommodations and modifications for English language learners that will support their regular and active participation in learning mathematics?		

CCSSM Instructional Materials Analysis Project--Overarching Considerations (Tool 3)		Page 2
Questions about Overarching Considerations (Page 2)	**See Rubric**	**Comments/Examples**
To what extent do the materials:	**N-L-M-H**	
6. Provide opportunities to use reading, writing, and speaking in mathematics lessons.		
7. Encourage teachers to draw upon home language and culture to facilitate learning?		
8. Encourage teachers to draw on multiple resources such as objects, drawings, and graphs to facilitate learning?		
9. Draw upon students' personal experiences to facilitate learning?		
10. Provide opportunities for teacher and students to connect mathematics to other subject areas?		
11. Provide both individual and collective opportunities for students to learn using mathematical tasks with a range of challenge?		
12. Provide opportunities for advanced students to investigate mathematics content at greater depth?		
13. Provide a balanced portrayal of various demographic and personal characteristics?		
Assessment		
14. Provide strategies for gathering information about students' prior knowledge and background?		
15. Provide strategies for teachers to identify common student errors and misconceptions?		
16. Assess students at a variety of knowledge levels (e.g., memorization, understanding, reasoning, problem solving)?		
17. Encourage students to monitor their own progress?		
18. Provide opportunities for ongoing review and practice with feedback related to learning concepts, and skills.		
19. Provide support for a varied system of on-going formative and summative assessment (formal or informal observations, interviews, surveys, performance assessments, target problems)?		

Questions about Overarching Considerations (Page 2)	See Rubric	Comments/Examples
Technology	**N-L-M-H**	
To what extent do the materials:		
20. Integrate technology such as interactive tools, virtual manipulatives/objects, and dynamic mathematics software in ways that engage students in the Mathematical Practices?		
21. Include or reference technology that provides opportunities for teachers and/or students to communicate with each other (e.g. websites, discussion groups, webinars)?		
22. Include opportunities to assess student mathematical understandings and knowledge of procedural skills using technology?		
23. Include or reference technology that provides teachers additional tasks for students?		
24. Include teacher guidance for the mindful use of embedded technology to support and enhance student learning?		

Notes/Examples:

Summary Discussion Questions

1. Equity: To what extent do the materials contain embedded support for elements of equity consistently within and across grades?
2. Assessment: To what extent do the materials contain embedded support for elements of assessment consistently within and across grades?
3. Technology: To what extent do the materials contain embedded support for elements of technology consistently within and across grades?
4. Overall: To what extent do the materials incorporate the Overarching Consideration elements to advance students' learning of mathematical content and engagement in the mathematical practices?