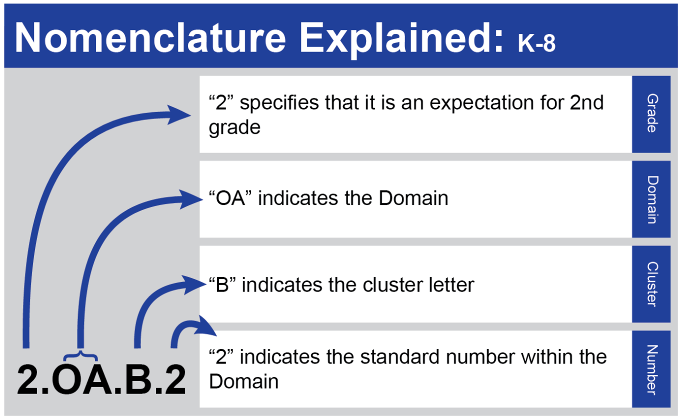
Arkansas Mathematics Standards
2nd Grade
2022

Introduction to the Grades K-8 Arkansas Mathematics Standards

When the Division of Elementary and Secondary Education (DESE) began the process of revising math standards, a diverse group of qualified educators from across the state came together to craft Arkansas standards specific to the schools and students in the state. The result of this work, the Arkansas Mathematics Standards, is contained in this document. These standards reflect what educators across the state know to be best for Arkansas students.

Standards Organization: The revision committee maintained the organizational structure and nomenclature of the previous standards. K-8 Arkansas Mathematics Standards are categorized into domains, clusters, and standards.

- **Domains** represent the big ideas to be studied at each grade level and sometimes across grade bands. These big ideas support educators in determining the proper amount of focus and instructional time to be given to each of these topics.
- **Clusters** represent collections of standards grouped to help educators understand the building blocks of rich and meaningful instructional units. These units help students make connections within clusters and avoid seeing mathematics as a discrete list of skills they must master.
- **Standards** represent the foundational building blocks of math instruction. The standards outlined in this document work together to ensure that students are college and career ready and on track for success.



Standards Support: The revision of the Arkansas Mathematics Standards represent the work of the committee to provide greater clarity, strength, and support of the standards. Additionally, the revised mathematics standards are designed to help educators better understand the areas of emphasis and the focus within the standards. Educators should address the bulleted content as more than a checklist of items that they must teach individually. Content is bulleted to provide specificity of learning expectations included within some extensive standards. In some instances, the standard document includes Arkansas examples, teacher notes, specifications, and italicized words to assist educators with planning, teaching, and student learning.

- **Examples** included in the original standards were either changed for clarity or separated from the body of the actual standard. The examples included in the body of the standards document in no way reflect all of the possible examples. Likewise, these examples do not mandate curriculum or problem types. Local districts are free to select the high-quality curricula and instructional methods that best meet the needs of their students.
- **Teacher notes** offer clarification of the standards. These notes are intended to clarify, for teachers, what the expectations are for the learner. Likewise, these notes provide instructional guidance and limitations so that educators can better understand the scope of the standard. This will help with determining what is developmentally appropriate for students when working with specific standards.
- **Standard specifications** are to strengthen standards. The specifications are precise statements highlighting the need for mastery or function-type parameters for specific standards. This will assist educators in pinpointing the best opportunities for students to gain and master the knowledge and skills needed to succeed in a progression.
- **Italicized words** are defined in the glossary.

Finally, the Arkansas Mathematics Standards will be a living document. As these standards are implemented across schools in the state, DESE welcomes further suggestions related to notes of clarification, examples, professional development needs, and future revisions of the standards.

K-12 Standards for Mathematical Practices	
1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics.	5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Second Grade Standards: Overview

Abbreviations: The following abbreviations are for the domains for the Arkansas Mathematics Standards.

Operations and Algebraic Thinking – OA

- Represent and solve problems involving addition and subtraction
- Add and subtract within 20
- Work with equal groups of objects to gain foundations for multiplication

Number and Operations in Base Ten – NBT

- Understand place value
- Use place value understanding and properties of operations to add and subtract

Measurement and Data – MD

- Measure and estimate lengths in standard units
- Relate addition and subtraction to length
- Work with time and money
- Represent and interpret data

Geometry – G

- Reason with shapes and their attributes

Operations and Algebraic Thinking	
Cluster A: Represent and solve problems involving addition and subtraction.	
AR.Math.Content.2.OA.A.1	Solve one and two-step word problems: ● Use addition and subtraction within 100 to solve one and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing with unknowns in all positions. ● Represent a strategy with a related *equation* including a symbol or *variable* for the unknown number.
Cluster B: Add and subtract within 20.	
AR.Math.Content.2.OA.B.2	Demonstrate *mastery* of *computational fluency* with addition and subtraction within 20, using mental strategies such as: ● Making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14). ● Decomposing a number leading to a ten (e.g., 13 - 4 = 13 - 3 - 1 = 10 - 1 = 9). ● Using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4). ● Creating equivalent but easier or known *sums* (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13). Teacher Note: ● *<u>Computational fluency</u>* - refers to having efficient and accurate methods for computing. Students exhibit computational *fluency* when they demonstrate flexibility in the computational methods they choose, understand and can explain these methods, and produce accurate answers efficiently. ● *<u>Mastery</u>* - refers to teaching in a way that students develop a deep understanding of mathematical concepts rather than memorizing key procedures or resorting to rote learning or simple memorization of facts. ● Specification: Students should demonstrate *mastery* of this standard by the end of 2nd grade.
Cluster C: Work with equal groups of objects to gain foundations for multiplication.	
AR.Math.Content.2.OA.C.3	Determine whether a group of objects (up to 20) has an odd or even number of members (e.g., by pairing objects or counting them by 2's). ● Write an *equation* to express an even number (up to 20) as a *sum* of two equal *addends.* Teacher Note: ● An even number is an amount that can be made of two equal parts with a remainder of zero. An odd number is an amount that is not even and cannot be made of two equal parts. ● Students should understand that if a number can be decomposed (broken apart) into two equal *addends* or doubles addition facts (e.g., 10 = 5 + 5), then that number is an even number.
AR.Math.Content.2.OA.C.4	Use addition to find the total number of objects arranged in *rectangular arrays* with up to 5 rows and up to 5 columns. ● Write an *equation* to express the total as a *sum* of equal *addends.* Teacher Note: This standard relates to array work in standard 2.G.A.2 which prepares students for multiplication. It also connects to the 3rd grade standard 3.MD.C.5 and 6.

Number and Operations in Base Ten	
Cluster A: Understand place value.	
AR.Math.Content.2.NBT.A.1	Extend place value understanding: ● Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones (e.g., 726 equals 7 hundreds, 2 tens, 6 ones). ● Understand that 100 can be thought of as a group of ten tens - called a "hundred". ● Understand that the numbers 100, 200, 300, 400, 500, 600, 700, 800, and 900 refer to one, two, three, four, five, six, seven, eight, or nine groups of 100.
AR.Math.Content.2.NBT.A.2	Count within 1000: ● Count forward and backward by ones from any given number. (Students can continue a count from the given number without having to start their counting at one). ● Skip-count forward by 5s, 10s, and 100s. ● Skip-count by 10s and 100s forward and backward beginning at any number between zero and 1000. Teacher Note: This standard supports students' understanding of AR.Math.Content.2.NBT.B.5 and 7. Specification: Students should demonstrate *mastery* of this standard by the end of 2nd grade.
AR.Math.Content.2.NBT.A.3	Read, write, and model numbers: ● Read and write numbers up to 1000 using base ten numerals, number names, and a variety of *expanded forms.* ● Model and describe numbers within 1000 as groups of 10 in a variety of ways. Teacher Note: When reading and writing *whole numbers*, the word "and" should not be used (235 is stated and written as "two hundred thirty-five").
AR.Math.Content.2.NBT.A.4	Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits. Use >, =, and < symbols with correct terminology for the symbols to record the results of comparisons.
Cluster B: Use place value understanding and properties of operations to add and subtract.	
AR.Math.Content.2.NBT.B.5	Add and subtract within 100 with *computational fluency* using strategies based on *place value*, *properties of operations*, and the relationship between addition and subtraction. Teacher Note: ● *Computational fluency* - refers to having efficient and accurate methods for computing. Students exhibit *computational fluency* when they demonstrate flexibility in the computational methods they choose, understand and can explain these methods, and produce accurate answers efficiently. ● The standard *algorithm* of carrying or borrowing is neither an expectation nor a focus in Second Grade. Students develop strategies for addition and subtraction in Grades K-3.
AR.Math.Content.2.NBT.B.6	Add up to four two-digit numbers using strategies based on *place value* and *properties of operations.*

AR.Math.Content.2.NBT.B.7	Addition and subtraction within 1000: ● Add and subtract within 1000, using strategies such as concrete models, drawings, or numerical representations (i.e., *equation* or *expression*) based on *place value*, *properties of operations*, and the relationship between addition and subtraction; relate the strategy to a written *expression* or *equation*. ● Explain why addition and subtraction strategies work using *place value* and their understanding of the *properties of operations*. (e.g., A student may say, "I started with the larger number when adding because I know when adding I can start with any number." This shows they understand the *commutative property of addition*. Drawings or objects could support explanations). Teacher Note: ● Students need to be aware of the formal terms of these properties but do not have to name them specifically. However, students should use precise mathematical language. Therefore, they should use formal terms rather than informal ones (e.g., "flip-flop" or "turn around"). ● Teachers should utilize the formal terminology for the *properties of operations* such as the *commutative property of addition* and *associative property of addition*.
AR.Math.Content.2.NBT.B.8	Mentally add 10 or 100 to a given number 100-900, and mentally subtract 10 or 100 from a given number 100- 900.

Measurement and Data

Cluster A: Measure and estimate lengths in standard units.	
AR.Math.Content.2.MD.A.1	Select and use appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes to measure the length of an object to the nearest whole unit (This standard requires students to estimate in order to choose the appropriate tool to measure given objects and to determine the nearest whole unit). Teacher Note: ● Customary Units - inches, feet, yards ● Metric - centimeters, meters
AR.Math.Content.2.MD.A.2	Measure the length of an object twice with two different length units within the same measuring system (Customary with Customary or Metric with Metric). ● Describe how the two measurements relate to the size of the unit chosen. Teacher Note: Example: A bulletin board is measured in inches and feet. A student compares the size of the unit of measure and the number of those units to determine the reasonableness of the tool used.
AR.Math.Content.2.MD.A.4	Measure to determine how much longer or shorter one object is than another, expressing the length *difference* in terms of a standard length whole unit. Teacher Note: The word "standard" refers to a unit of measurement. Both the Customary and Metric systems can be used.
Cluster B: Relate addition and subtraction to length.	
AR.Math.Content.2.MD.B.5	Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units and write *equations* with a symbol or *variable* for the unknown number to represent the problem.

<table>
<tr><td>AR.Math.Content.2.MD.B.6</td><td>Solve addition and subtraction problems on a number line diagram by using whole numbers as lengths with appropriately spaced points corresponding to the numbers to solve problems within 100.

Teacher Note: Appropriately spaced allows students to represent adding or subtracting by their increment of choice (i.e, a change of 100 would be a larger jump compared to ten, ones, or another increment).</td></tr>
<tr><td colspan="2">Cluster C: Work with time and money.</td></tr>
<tr><td>AR.Math.Content.2.MD.C.7</td><td>Tell and write time from analog and digital clocks to the nearest five minutes using a.m. and p.m.

Teacher Note: This standard is a continuation of previous instruction at lower grades with the expectation of mastery in telling time to the nearest minute by the end of third grade.
<table>
<tr><th colspan="2">Progression of Time Skills</th></tr>
<tr><th>Grade Level</th><th>Time Learning Expectation</th></tr>
<tr><td>Kindergarten</td><td>Tell time to the nearest hour</td></tr>
<tr><td>First Grade</td><td>Tell time to the nearest half-hour</td></tr>
<tr><td>Second Grade</td><td>Tell time to the nearest 5 minutes</td></tr>
<tr><td>Third Grade</td><td>Tell time to the nearest quarter hour
Tell time to the nearest minute
Solve word problems involving time</td></tr>
</table></td></tr>
<tr><td>AR. Math.Content.2.MD.C.8</td><td>Count and solve problems with money:
• Count collections of mixed coins.
• Solve word problems involving:
    ○ Quarters, dimes, nickels, and pennies within 99¢, using ¢ symbols appropriately.
    ○ Whole dollar amounts, using the $ symbol appropriately.
Teacher Note:
<table>
<tr><th colspan="3">Progression of Money Skills</th></tr>
<tr><th>Grade Level</th><th>Coins</th><th>Money Learning Expectation</th></tr>
<tr><td>Kindergarten</td><td>penny, nickel, dime</td><td>Know name and value</td></tr>
<tr><td>First Grade</td><td>penny, nickel, dime, quarter</td><td>Know name and value
Count collections of like coins</td></tr>
<tr><td>Second Grade</td><td>penny, nickel, dime, quarter, dollar bills</td><td>Know name and value
Use dollar and cent symbol
Solve word problems:
• within 99 cents
• using whole dollar amounts</td></tr>
<tr><td>Fourth Grade</td><td>all coins and bills</td><td>Solve word problems including making change and decimals</td></tr>
</table>
Specification: Students should demonstrate mastery of this standard by the end of 2nd grade.</td></tr>
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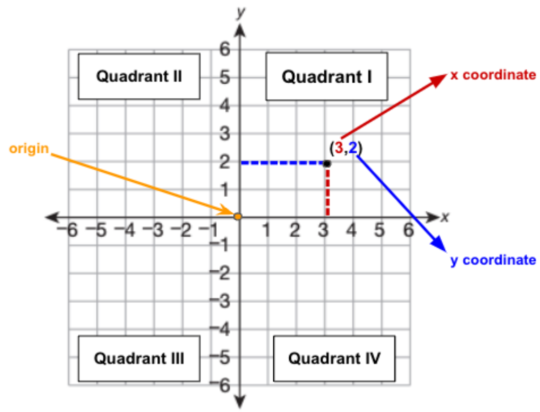
Cluster D: Represent and interpret data.	
AR.Math.Content.2.MD.D.9	Create and interpret line plots: ● Generate data by measuring the same *attribute* of similar objects to the nearest whole unit (Example *attributes:* - length or width; similar objects - pencils, books, shoes, etc.). ● Make a *line plot* where the horizontal scale is marked off in whole-number units. ● Display the measurement data to compare *attributes* of objects. Teacher Note: This is a measurement standard. Students should have opportunities to measure to generate data. After several experiences with generating data to use, the students can be given data already generated to create the *line plot.*
AR.Math.Content.2.MD.D.10	Create and solve problems with bar graphs: ● Draw a picture graph and a bar graph, with a single-unit scale, to represent a *data set* with up to four categories. ● Solve simple put-together, take-apart, and compare problems using information presented in a bar graph.

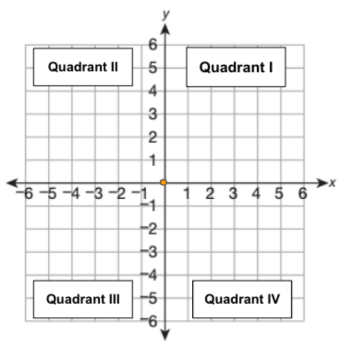
Geometry	
Cluster A: Reason with shapes and their attributes.	
AR.Math.Content.2.G.A.1	Identify and describe shapes based on attributes: ● Identity, describe, and draw the following 2D shapes: triangles, pentagons, hexagons, and quadrilaterals (specifically the following quadrilaterals: square, rectangle, *trapezoid*, parallelogram, and rhombus). Describe the shapes based on defining *attributes* (e.g., number of *vertices*, number of sides, etc.). ● Identify and describe defining *attributes* in 3D shapes: *rectangular prisms*, cubes, and square-based pyramids. Describe the shapes based on the shapes of faces, number of faces, number of edges, and number of *vertices* (*Students are not expected to draw three-dimensional objects).* 2D is defined as length and width. 3D is defined as length, width, and height. Teacher Note: ● Teachers should utilize the vocabulary term *vertices* rather than corners. ● Teachers should utilize the proper names for various quadrilaterals, but students should not be assessed on the names in this grade. ● Specified *attributes* should be limited to the number and length of sides as well as the number of *vertices*. Other terms referring to the relationship between the shape's sides or the measure of the angles are **not** introduced or addressed at this grade level.
AR.Math.Content.2.G.A.2	Partition a rectangle into rows and columns of same-size squares and count to find the total number of squares. Teacher Note: This standard relates to array work in standard 2.OA.C.4 which prepares students for multiplication. It also connects to the 3rd-grade standards 3.MD.C.5. and 6.

AR.Math.Content.2.G.A.3	Partition circles and rectangles into two, three, or four equal shares; describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Teacher Note: Students are expected to know the terminology but not required to recognize or write fraction notation (numerator over denominator, a/b).
AR.Math.Content.2.G.A.4	Recognize that equal shares of identical wholes need not have the same shape. Teacher Note: Example: A square can be divided into halves, thirds, or fourths vertically or horizontally. Although the shape of the pieces is different, they still represent the same fractional amount.

K-5 Glossary	
Addend	Any of the numbers added to find a sum
Additive Comparison	Compare two amounts by asking how much more or less is one amount than the other.
Additive inverses	Two numbers whose sum is 0 are additive inverses of one another; example: 3/4 and (-3/4) are additive inverses of one another because 3/4 + (- 3/4) = 0
Algorithm	An explicit step-by-step procedure for performing a mathematical computation or for solving a mathematical problem.
Associative Property of addition	A property of real numbers that states that the sum of a set of numbers is the same, regardless of how the numbers are grouped. Example: (4 + 8) + 3 = 4 + (8 + 3)
Associative Property of multiplication	A property of real numbers that states that the product of a set of numbers is the same, regardless of how the numbers are grouped. Example: $(4 \bullet 8) \bullet 3 = 4 \bullet (8 \bullet 3)$
Attributes	Characteristics or properties of an object
Axis	A vertical or horizontal number line, both of which are used to define a coordinate grid. The horizontal axis is the x-axis, and the vertical axis is the y-axis. The plural of axis is axes.
Benchmark Fraction	A common fraction used when comparing other fractions (e.g., 1/2 , 1/4)
Cardinality	The understanding that when you count items, the number word applied to the last object counted represents the total amount.
Commutative Property of addition	A property of real numbers that states that the sum of two terms is unaffected by the order in which the terms are added; i.e., the sum remains the same. Example: $5 + 9 = 9 + 5$
Commutative Property of multiplication	A property of real numbers that states that the product of two factors is unaffected by the order in which the factors are multiplied, i.e., the product remains the same. Example: $5 \bullet 9 = 9 \bullet 5$
Composite	A number with more than two factors.
Composite Shape	Shapes composed of two or more shapes.
Congruent	Identical in form
Coordinate	An ordered pair of numbers in the form (*x, y*) that describes the location of a point on a coordinate plane.
Coordinate Plane	A plane divided by perpendicular number lines creating four quadrants. The perpendicular number lines represent the axes and where they intersect represents the origin (0,0). Points can be identified using coordinates (x,y) found within the quadrants (example below).

	
Counting Back	A strategy for finding the difference using backward counting. For example, if a stack of books has 12 books and someone borrows 4 books to read, how many books are left? A student may start at 12 and count back for spaces or numbers saying 12…11, 10, 9, 8; there are 8 books left in the stack.
Counting On	A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books has 8 books and 3 more books are added to the top, it is unnecessary to count the stack all over again. One can find the total by *counting on* pointing to the top book and saying "eight", following this with "nine, ten, eleven." There are eleven books now.
Data Set	A collection of numbers related to a topic.
Decompose	Breaking a quantity into smaller quantities/units in order to assist computation.
Denominator	The term of a fraction, usually written under the line, that indicates the number of equal parts into which the unit is divided; divisor
Difference	The distance between two values; result of a subtraction problem.
Distributive Property	When a single-term expression is being multiplied by a sum or difference, the single-term expression can be multiplied by each term before finding the sum or difference. Examples: $3(7+5) = 3 \bullet 7 + 3 \bullet 5$ $w(5-2) = 5w - 2w$
Dividend	A number that is being divided by another number (divisor)
Divisor	The number by which another number is being divided
Equation	A statement that has one number or expression equal to another number or expression, such as 8 + 3 = 11 or 2x- 3 = 7.
Evaluate	Calculate or solve
Expanded form	A multi-digit number is expressed in expanded form when it is written as a sum of the single-digit multiples of powers of ten. For example, 643 = 600 + 40 + 3
Exponent	A symbol that is written above and to the right of a number to show how many times the number is to be multiplied by itself
Expression	A mathematical phrase consisting of numbers, variables, and operations

K-5 Glossary	
Fluency	There are different types of ***fluency***. All of them require students to be accurate, efficient, and flexible. The types are defined as follows: Basic fact fluency - fluency with operations involving single digit numbers. Computational fluency - having efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate flexibility in the computational methods they choose, understand, and can explain these methods as well as produce accurate answers efficiently. Procedural fluency - Procedural fluency is the ability to apply procedures accurately, efficiently, and flexibly; to transfer procedures to different problems and contexts; to build or modify procedures from other procedures, and to recognize when one strategy or procedure is more appropriate to apply than another. (NCTM)
Factor	One or more numbers (or variables) that are multiplied together to get a product (5 and 2 are both factors because 5 • 2 =10)
Fraction	A number expressible in the form *a/b* where *a* is a whole number and *b* is a whole number. (The word fraction in these standards, K-5, always refers to a non-negative number.) This includes all forms of fractions - fractions less than one, fractions greater than one (improper fractions), and mixed numbers. *See also*: rational number
Identity property of 0	The property that asserts the sum of an original addend plus zero is equal to the original addend. Example: $58 + 0 = 58$
Identity property of 1	The property that asserts the product of an original factor times one is equal to the original factor. Example: $58 \bullet 1 = 58$
Inequality Symbols	Symbols used to show a comparison between quantities. Also known as the greater than and less than symbols (<,>).
Interval	Includes all the numbers that come between two particular numbers.
Inverse (Operation)	An operation that is the opposite of, or undoes, another operation. Addition and subtraction are inverse operations as are multiplication and division.
Iterating	Repeating; repetition of a process in order to generate a sequence of outcomes.
Line plot	A method of visually displaying a distribution of data values where each data value is shown as an X or mark above a number line. Also known as a dot plot.
Mass	The amount of matter in an object. Often measured by the amount of material it contains which causes it to have weight. However, mass is not to be confused with weight. Weight is determined by the force of gravity on an object while mass is not. For example, an watermelon on Jupiter would have a greater weight than one on Earth because Jupiter's gravity is stronger than Earth's. The mass of the watermelon would be the same on both planets.
Mastery	Refers to teaching in a way that students learn to develop a deep understanding of mathematical concepts rather than memorizing key procedures or resorting to rote learning of steps or facts.
Multiplicative Comparison	Compare two amounts by asking how many times larger or smaller is one amount than the other.
Multiplicative inverses	Two numbers whose product is 1 are multiplicative inverses of one another. Examples: 3/4 and 4/3 are multiplicative inverses of one another because 3/4 • 4/3 = 1 6 and $\frac{1}{6}$ are also multiplicative inverses because $6 \bullet \frac{1}{6}$ = 1

K-5 Glossary	
Natural Numbers	Counting numbers 1, 2, 3, 4, 5, 6…
Number line diagram	A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity
Numerator	The number in a fraction that is above the fraction line and that is divided by the number below the fraction line
Order of Operations	A specific sequence in which operations are to be performed when an expression requires more than one operation.
Origin	The point in a Cartesian coordinate system where axes intersect
Place value	The value of the place of a digit in a numeral; the relative worth of each number that is determined by its position
Polygons	A closed two-dimensional figure made up of straight sides.
Prime	A number with only two factors, 1 and itself.
Prism	A three-dimensional (solid) figure that has two congruent and parallel faces that are polygons called bases. The remaining faces, called lateral faces, are parallelograms (often rectangles). Prisms are named by the shape of their bases.
Product	The number or expression resulting from the multiplication together of two or more numbers or expressions (factor • factor = product)
Properties of operations	Rules that apply to the operations with real numbers. (See Table 1 below)
Quadrant	One of the four sections of a coordinate plane separated by horizontal and vertical axes. 
Quotient	The number that results when one number is divided by another
Rational Numbers	A real number which can be written in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$. The set of rational numbers include the set of integers.
Rectangular array	A set of quantities arranged in rows and columns
Rectangular Prism	A three-dimensional figure whose six faces are rectangles.
Rectilinear Figures	A polygon with all right angles.
Subitize	Instantly see how many objects are in a group without counting.
Sum	The result of adding two or more numbers
Trapezoid	A quadrilateral with *at least* one pair of parallel sides
Unit fraction	A fraction where the numerator is 1 and the denominator is the positive integer
Value	Numerical worth or amount
Variable	A symbol used to represent an unknown value, usually a letter such as x

K-5 Glossary	
Vertices	A point where two or more line segments meet. (vertex is singular, plural is vertices)
Visual fraction model	A tape diagram, number line diagram, or area model
Volume	Amount of space occupied by a 3D object, measured in cubic units
Whole numbers	The numbers 0, 1, 2, 3…..

Appendix

Table 1: Properties of Operations

Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of addition	$a + b = b + a$
Additive identity property of 0	$a + 0 = 0 + a = a$
Existence of additive inverses	For every a there exists $–a$ so that $a + (-a) = (-a) + a = 0$
Associative property of multiplication	$(a \bullet b) \bullet c = a \bullet (b \bullet c)$
Commutative property of multiplication	$a \bullet b = b \bullet a$
Multiplicative identity property 1	$a \bullet 1 = 1a = a$
Existence of multiplication inverses	For every $a \neq 0$ there exists $1/a$ so that $a \bullet 1/a = 1/a \bullet a = 1$
Distributive property of multiplication over addition	$a \bullet (b + c) = a \bullet b + a \bullet c$

Table 2: Properties of Equality

Reflexive property of equality	$a = a$
Symmetric property of equality	If $a = b$, then $b = a$.
Transitive property of equality	If $a = b$ and $b = c$, then $a = c$.
Addition property of equality	If $a = b$, then $a + c = b + c$.
Subtraction property of equality	If $a = b$, then $a – c = b – c$.
Multiplication property of equality	If $a = b$, then $a \bullet c = b \bullet c$.
Division property of equality	If $a = b$ and $c \neq 0$, then $a \div c = b \div c$.
Substitution property of equality	If $a = b$, then b may be substituted for a in any expression containing a.

Table 3: Properties of Inequality

Exactly one of the following is true: $a < b$, $a = b$, $a > b$.
If $a > b$ and $b > c$, then $a > c$.
If $a > b$, $b < a$.
If $a > b$, then $a \pm c > b \pm c$.
If $a > b$ and $c > 0$, then $a \bullet c > b \bullet c$.
If $a > b$ and $c < 0$, then $a \bullet c < b \bullet c$.
If $a > b$ and $c > 0$, then $a \div c > b \div c$.
If $a > b$ and $c < 0$, then $a \div c < b \div c$.

Table 4: Common Problem Types for Addition and Subtraction

	RESULT UNKNOWN	**CHANGE UNKNOWN**	**START UNKNOWN**
ADD TO	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 =5
TAKE FROM	Five apples were on the table. I ate two apples. How many apples are on the table now?5-2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before?? -2 = 3
	TOTAL UNKNOWN	**ADDEND UNKNOWN**	**BOTH ADDENDS UNKNOWN**
PUT TOGETHER / TAKE APART	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5-3 = ?	Grandma has five flowers. How many can she put in the red vase and how many in her blue vase? 5 = 0 + 5, 5 + 0 5 = 1 +4, 5 = 4 +1 5 = 2 + 3, 5 = 3 + 2
COMPARE	**DIFFERENCE UNKNOWN**	**BIGGER UNKNOWN**	**SMALLER UNKNOWN**
	("How many more?" version):Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy?("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have then Julie? 2 + ? = 5, 5 – 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "more"):Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have?(Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

Table 5: Common Problem Types for Multiplication and Division

	UNKNOWN PRODUCT	GROUP SIZE UNKNOWN ("HOW MANY IN EACH GROUP?" DIVISION)	NUMBER OF GROUPS UNKNOWN ("HOW MANY GROUPS?" DIVISION)
	3 • 6 = ?	3 • ? = 18, and 18 ÷ 3 = ?	? • 6 = 18, and 18 ÷ 6 = ?
EQUAL GROUPS	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example.* You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example.* You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example.* You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
ARRAYS, AREA	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example.* What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example.* A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example.* A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
COMPARE	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example.* A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example.* A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example.* A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
GENERAL	a • b = ?	a • ? = p and p ÷ a = ?	? • b = p, and p ÷ b = ?