Bristol Public Schools

Office of Teaching & Learning

Department	Mathematics
Department Philosophy	*Students learn by doing math, solving problems in mathematical and real-world contexts, and constructing arguments using precise language.* The Bristol mathematics curricula embeds this *learn-by-doing* philosophy by focusing on high expectations for all students and providing students with opportunities that build conceptual understanding, computational and procedural fluency, and problem solving through the use of a variety of strategies, tools, and technologies. The mathematics curriculum is responsive to the individual needs of students, while providing a structure tied to the Common Core State Standards in Connecticut. The *learn-by-doing* philosophy develops mathematically literate and productive students who can effectively and efficiently apply mathematics in their lives to make informed decisions about the world around them by doing math. To be mathematically literate, one must understand major mathematics concepts, possess computational facility, and have the ability to apply these understandings to situations in daily life. Making connections between mathematics and other disciplines is key to the appropriate application of mathematics skills and concepts to solve problems. The ability to read, discuss, and write within the discipline of mathematics is an integral skill that supports mathematical understanding, reasoning and communication. The opportunity to think critically and creatively to solve problems is important to deepen mathematical knowledge and foster innovation. A rich hands-on mathematical experience is essential to provide the foundational knowledge and skills that prepare students to be mathematically literate, productive citizens.
Course	Statistics and Geometry
Grade Level	9, 10
Pre-requisites	Algebra 1

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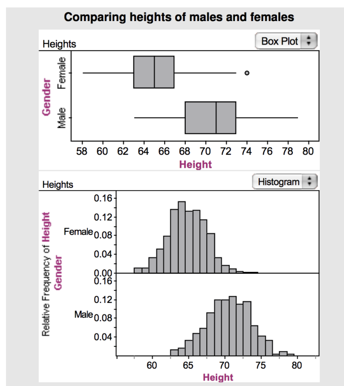
UNIT 1: One-Variable Statistics (iM Algebra 1, Unit 1)

[Illustrative Mathematics Unit Access](#) (Algebra 1, Unit 1)

Essential Questions:

- **How can we use statistics to ask and answer questions?**
- How do various distribution shapes represent data?
- How can spreadsheets help analyze data?
- What happens when data is manipulated?
- How can we analyze data?

UNWRAPPED STANDARDS

Grade Level Standard	Standard Progression	Concepts (Big Ideas/ Understandings)	Academic Vocabulary (Standard Based)
HSS-ID.A.1 Represent data with plots on the real number line (dot plots, histograms, and box plots).	 *Heights of U.S. males and females in the 20–29 age group.* Source: *U.S. Census Bureau, Statistical Abstract of the United States: 2009*, Table 201.	● Produce dot plots, histograms, and box plots ● Explore different data sets and determine their usefulness in prediction and description ● Explain what each display tells about the situation ● Understand how extreme cases are represented in each display	Dot plot, frequency, histogram, interquartile range, median, quartile, bin width
HSS-ID.A.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.	Students should be able to sketch each distribution and answer questions about it just from knowledge of these three facts (shape, center, and spread). For either group, about 68% of the data values will be within one standard deviation of the mean.	● Describe distributions in terms of shape, center, dispersion etc. ● Justify appropriateness of measures based on distributional shape ● Relate the appropriate	Mean, median, interquartile range, standard deviation, distribution, dispersion, shape, symmetry, skew, uniform, bimodal

	They should also observe that the two measures of center, median and mean, tend to be close to each other for symmetric distributions.	measures of dispersion to the best measure of center	
HSS-ID.A.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).	Students should be able to sketch each distribution and answer questions about it just from knowledge of these three facts (shape, center, and spread). For either group, about 68% of the data values will be within one standard deviation of the mean. They should also observe that the two measures of center, median and mean, tend to be close to each other for symmetric distributions.	• Recognize and name different shapes and their characteristics • Recognize what happens to the mean when data is skewed or when there is an outlier • Understand and be able to identify which measure of center and spread are appropriate based on distributional characteristics	Mean, median, interquartile range, shape, distribution, outlier, standard deviation

UNIT 1: One-Variable Statistics (iM Algebra 1, Unit 1)				
CCSS Standards #	**Learning Targets:** I can	**Assessment Strategy** SR-Selected Response CR-Constructed Response P-Performance O-Observation (behavioral)	**Lesson Progression and Connection to Math Practices**	**Common Learning Experiences and Assessments**
[HSS-ID.A.1](#) [HSS-ID.A.2](#)	How can we use statistics to ask and answer questions? ● I can tell statistical questions from non-statistical questions and can explain the difference. (SR) ● I can tell the difference between numerical and categorical data. (SR) ● I can find the five-number summary for data. (P) ● I can use a dot plot, histogram, or box plot to represent data. (P) ● I can graphically represent the data I collected and critique the representations of others. (O)		The mathematical purpose of the first lesson is to understand what makes a question statistical and classify data as numerical or categorical. This unit begins with creating data displays and describing distributions of numerical data.Students learn to recognize statistical questions as questions that anticipate variability in the data. Students classify questions as being statistical or non-statistical, and classify the data that they collect from statistical questions as numerical or categorical. In the second lesson, students represent data using histograms and box plots. They calculate values for the five-number summary and use those values to create dot plots. Students will also create two different histograms that represent the same data set by using different intervals in each of the histograms. Students will also compare a dot plot, box plot, and histogram that represent the same data set. The mathematical purpose of the third lesson is to represent and interpret data using data displays in a less scaffolded way than in the previous lesson. The work of this lesson connects to future work because students will use data displays to more formally describe the shape of distributions, and to determine the appropriate measure of center and measure of variability for a given distribution.	iM Lessons 1, 2, 3
			● MP2 ● MP6 ● MP7	

HSS-ID.A.1 HSS-ID.A.2	How do various distribution shapes represent data? ● I can describe the shape of a distribution using the terms "symmetric, skewed, uniform, bimodal, and bell-shaped." (O) ● I can use a graphical representation of data to suggest a situation that produced the data pictured. (O) ● I can calculate mean absolute deviation, interquartile range, mean, and median for a set of data. (P)	The mathematical purpose of the first lesson in this sequence is to describe distributions using the appropriate terminology. The terminology that is used to describe distributions here include symmetric, skewed, uniform, bimodal and bell shaped. The second lesson reviews how to calculate the mean, median, mean absolute deviation (MAD), and interquartile range (IQR). This lesson connects to upcoming work because students will interpret data using measures of center and measures of variability throughout the unit, so it is important that they understand what these statistics mean. ● MP2	iM Lessons 4, 5
HSS-ID.A.1 HSS-ID.A.2 HSS-ID.A.3	How can spreadsheets help analyze data? ● I can determine basic relationships between cell values in a spreadsheet by changing the values and noticing what happens in another cell. (O) ● I can use a spreadsheet as a calculator to find solutions to word problems. (P) ● I can use shortcuts to fill in cells on a spreadsheet. (P)	The purpose of this sequence of three lessons is to introduce spreadsheet functionality to students who are unfamiliar with it. It is assumed that students know nothing about spreadsheets. ● MP8	Optional iM Lessons 6, 7, 8
HSS-ID.A.1 HSS-ID.A.2 HSS-ID.A.3	What happens when data is manipulated? ● I can create graphic representations of data and calculate statistics using technology. (P) ● I can describe how an extreme value will affect the mean and median. (CR) ● I can use the shape of a distribution to compare the mean and median. (P) ● I can arrange data sets in order of variability given graphic representations. (P) ● I can describe standard deviation as a measure of variability. (CR) ● I can use technology to compute standard deviation. (P) ● I can use standard deviation to say something about a situation.	The mathematical purpose of the first lesson is to create data displays and calculate statistics using technology. Students encounter the term statistics which is a quantity that is calculated from sample data. In this lesson students will enter data into a spreadsheet, find statistics, and create box plots using technology. Next, students will recognize a relationship between the shape of a distribution and the mean and median. Students will use dot plots to investigate this relationship. The purpose of the following lesson is to compare data sets with different measures of variability and to interpret	iM Lessons 9-15

	(CR) ● I can find values that are outliers, investigate their source, and figure out what to do with them. (P) ● I can tell how an outlier will impact mean, median, IQR, or standard deviation. (CR) ● I can compare and contrast situations using measures of center and measures of variability. (CR)	data sets with greater MADs or IQRs as having greater variability. Students make connections between different data displays and measures of center and measures of variability. The next lesson in this sequence will introduce the concept of standard deviation to students. They will then continue to deepen their understanding of standard deviation by interpreting it in various contexts. The mathematical purpose of the next lesson is for students to recognize outliers, to investigate their source, to make decisions about excluding them from the data set, and to understand how the presence of outliers impacts measures of center and measures of variability. The mathematical purpose of the last lesson in this sequence is for students to compare measures of center and the standard deviation and the IQR for different data sets. This lesson provides students another opportunity to compare measures of center and variability, but now using standard deviation and outliers too in the comparisons.	
		● MP8 ● MP4 ● MP7 ● MP5	
HSS-ID.A.1 HSS-ID.A.2 HSS-ID.A.3	How can we analyze data? ● I can collect data from an experiment and compare the results using measures of center and measures of variability. (P)	The mathematical purpose of this final lesson is to pose and answer a statistical question by designing an experiment, collecting data, and analyzing data. Students will determine how best to display data, select appropriate measures of center and variability, and answer a statistical question involving two different treatments. The work of this lesson connects to previous work because students learned about the shape, measures of center, and measures of variability for data distributions. This connects to upcoming work because students will investigate bivariate data using two-way tables, relative frequency tables, scatter plots, and lines	iM Lesson 16

		of best fit.	
		● MP1	

COMMON MISCONCEPTIONS	PRIOR KNOWLEDGE NEEDED TO MASTER STANDARDS FOR THIS UNIT	ADVANCED STANDARDS FOR STUDENTS WHO HAVE DEMONSTRATED PRIOR MASTERY	OPPORTUNITIES FOR STUDENT-DIRECTED LEARNING WITHIN THE UNIT
● Incorrect use of terms ● Unable to see connections between measures of center and measures of variability	● 6.SP.B.4 ● 6.SP.B.5		

UNIT 2: Two-Variable Statistics (iM Algebra 1, Unit 3)

Illustrative Mathematics Unit Access **(Algebra 1, Unit 3)**

Essential Questions:

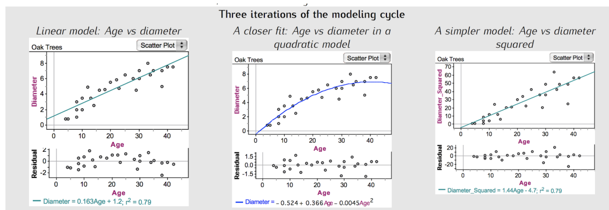
- What can learn from two-way tables?
- How can we use scatter plots to analyze data?
- How do we find and interpret correlation coefficients?
- How can we draw conclusions from data?

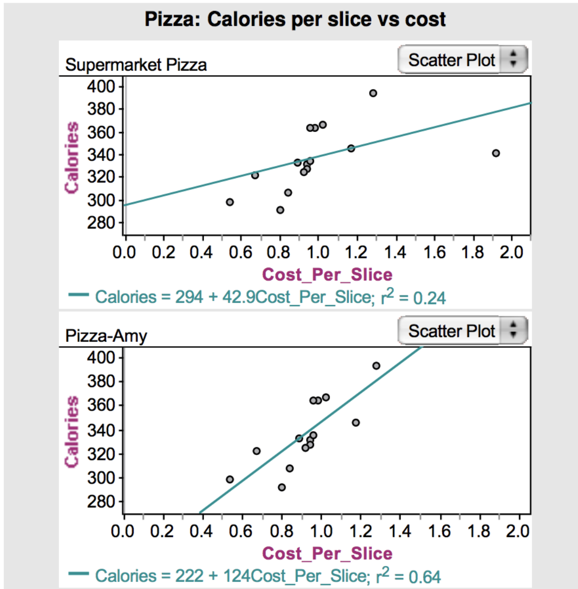
UNWRAPPED STANDARDS

Grade Level Standard	Standard Progression	Concepts Big Ideas/ Understandings)	Academic Vocabulary (Standard Based)
HSS-ID.A Summarize, Represent, And Interpret Data On A Single Count Or Measurement Variable	Students find and interpret information representing a univariate data set, including measures of central tendency and spread. Students develop values of spread, building on the conceptual foundations of mean absolute deviation from middle grades content and calculated using technology such as calculators, computer software designed for student learning, and/or tables. Students may find relationships between mean and median of a data set and may also decide when each is appropriate for use based on characteristics of the distribution. In addition, students justify the placement of means and median based on spread of measurements, distributional shape, and extreme values in the data set. Teachers should emphasize the connection of the mean and standard deviation with outliers. After students have related and developed measures of center and spread, students relate those measures to the number of observations and percentages under a normal curve. Though the empirical rule is not addressed in the standard, the antecedent can serve as a conceptual framework to investigate the relationship between population percentage under the normal curve using	• Find and interpret summary information representing a univariate data set, including measures of central tendency and spread • Find relationships between mean and median of a data set • Justify the placement of means and medians • Relate and develop measures of center and spread	Measures of center & spread, mean, median, outlier, distribution, standard deviation, outliers

<table>
<tr><td></td><td>technology.</td><td></td><td></td></tr>
<tr><td>HSS-ID.B.5
Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.</td><td>As with univariate data analysis, students now take a deeper look at bivariate data, using their knowledge of proportions to describe categorical associations and using their knowledge of functions to fit models to quantitative data, [giving students an opportunity to engage in Standards for Mathematical Practice 7 and 4.] The table below shows statistics from the Center for Disease Control relating HIV risk to age groups. Students should be able to explain the meaning of a row or column total (marginal), a row or column percentage (conditional) or a “total” percentage (joint). They should realize that possible associations between age and HIV risk are best explained in terms of the row or column conditional percentages. Are the comparisons of percentages valid when the first age category is much smaller (in years) than the others?

HIV risk by age groups, in percent of population
<table>
<tr><td></td><td>Age</td><td>18–24</td><td>25–44</td><td>45–64</td><td>Row Total</td></tr>
<tr><td rowspan="3">Not at risk</td><td>Row %</td><td>14.0</td><td>59.6</td><td>26.4</td><td>100.0</td></tr>
<tr><td>Column %</td><td>35.0</td><td>51.7</td><td>27.2</td><td></td></tr>
<tr><td>Total %</td><td>5.6</td><td>23.6</td><td>10.5</td><td>39.6</td></tr>
<tr><td rowspan="3">At risk</td><td>Row %</td><td>17.1</td><td>36.5</td><td>46.4</td><td>100.0</td></tr>
<tr><td>Column %</td><td>65.0</td><td>48.3</td><td>72.8</td><td></td></tr>
<tr><td>Total %</td><td>10.3</td><td>22.0</td><td>28.1</td><td>60.4</td></tr>
<tr><td rowspan="3">Column total</td><td>Row %</td><td>15.9</td><td>45.6</td><td>38.5</td><td>100.0</td></tr>
<tr><td>Column %</td><td>100.0</td><td>100.0</td><td>100.0</td><td>100.0</td></tr>
<tr><td>Total %</td><td>15.9</td><td>45.6</td><td>38.5</td><td>100.0</td></tr>
</table>
Source: Center for Disease Control,
http://apps.nccd.cdc.gov/s_broker/WEATSQL.exe/weat/freq_year.hsql</td><td>● Use relative frequencies to make inferences about associations or trends in data
● Interpret relative frequencies in terms of a subset of a conditioned event</td><td>Bivariant, correlation, frequency, trend</td></tr>
<tr><td>HSS-ID.B.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.</td><td>This particular cluster emphasizes the association of two variables that may be quantitative or categorical. Teachers and students use two-way frequency tables to summarize and analyze categorical data. Similarly, students and teachers use scatter plots and function fitting to describe and represent correlations between two quantitative variables. Problem-solving situation in this domain should include different forms of variability in model fitting, sampling variability, chance variability from sampling and chance variability resulting from assignment to groups in experiments.</td><td></td><td>Bivariate data, scatter plot, line of best fit, frequency table, quantitative, categorical</td></tr>
</table>

[HSS-ID.B.6.a](#) Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.	It is readily apparent to students, after a little experience with plotting bivariate data, that not all the world is linear. The figure below shows the diameters (in inches) of growing oak trees at various ages (in years). A careful look at the scatter plot reveals some curvature in the pattern,which is more obvious in the residual plot, because the older and larger trees add to the diameter more slowly. Perhaps a curved model, such as a quadratic, will fit the data better than a line. The figure below shows that to be the case. 	● Describe how variables are related within context of the situation ● Use technology and other tools to represent scatter plots ● Describe associations in terms of strength and direction ● Discuss extrapolation and its limitations for different contextual and mathematical models	
[HSS-ID.B.6.b](#) Informally assess the fit of a function by plotting and analyzing residuals.	This particular cluster emphasizes the association of two variables that may be quantitative or categorical. Teachers and students use two-way frequency tables to summarize and analyze categorical data. Similarly, students and teachers use scatter plots and function fitting to describe and represent correlations between two quantitative variables. Problem-solving situation in this domain should include different forms of variability in model fitting, sampling variability, chance variability from sampling and chance variability resulting from assignment to groups in experiments.	● Move lines of best fit and/or data points to deduce their interdependence ● Analyze different residual plots for different predictive models ● Recognize that points above the horizontal line on a residual are underestimates and below are overestimates ● Recognize the relationship of patterns in prediction with inappropriate model fitting ● Deduce that points randomly distributed above and below the horizontal line at zero on a residual plot imply appropriate fits for a model	
[HSS-ID.B.6.c](#) Fit a linear function for a scatter plot that suggests a linear association.		● Use technology to make inferences about associations in data sets ● Create a line of best fit and discuss reasons for choosing the line	

		• Use purposeful terminology to imply association rather than causation	
HSS-ID.C.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.	 Source: *Consumer Reports*, January 2002.	• Explain the slope and intercept estimates in terms of problem contexts • Extend connections from linear functions to making connections to contextual situations • Explain reasoning for slope and y-intercept parameters in terms of predicted values	Slope, linear model, variable, increase, decrease, line of best fit
HSS-ID.C.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.	Suppose you want to see if there is a relationship between the cost per slice of supermarket pizzas and the calories per serving. The margin shows data for a sample of 15 such pizza brands, and a somewhat linear trend. A line fitted via technology might suggest that you should expect to see an increase of about 43 calories if you go from one brand to another that is one dollar more in price. But, the line does not appear to fit the data well and the correlation coefficient r... is only about 0.5. Students will observe that there is one pizza that does not seem to fit the pattern of the others, the one with maximum cost. Why is it way out there? A check reveals that it is Amy's Organic Crust & Tomatoes, the only organic pizza in the sample. If the outlier (Amy's pizza) is removed and the discussion is narrowed to non-organic pizzas (as shown in the plot for pizzas other than Amy's),	• Use technology to compare different models and their respective correlation coefficients • Reason and make sense of different correlation coefficients and their relationship to different situations • Interpret points in relationship to the means of the x and y values by graphing a vertical and horizontal line to present the mean of both x and y variables	Correlation, correlation coefficient, line of best fit, association,

	the relationship between calories and price is much stronger with an expected increase of 124 calories per extra dollar spent and a correlation coefficient of 0.8. Narrowing the question allows for a better interpretation of the slope of a line fitted to the data.		
HSS-ID.C.9 Distinguish between correlation and causation.	In situations where the correlation coefficient of a line fitted to data is close to 1 or 1, the two variables in the situation are said to have a high correlation. Students must see that one of the most common misinterpretations of correlation is to think of it as a synonym for causation. A high correlation between two variables (suggesting a statistical association between the two) does not imply that one causes the other. It is not a cost increase that causes calories to increase in pizza, and it is not a calorie increase per se that causes cost to increase; the addition of other expensive ingredients cause both to increase simultaneously. Students should look for examples of correlation being interpreted as cause and sort out why that reasoning is incorrect. Examples may include medications versus disease symptoms and teacher pay or class size versus high school graduation rates.	● Use precision in the wording of relationships between two variables using words such as correlated or associated ● Justify, explain, and provide reasons for the difference between correlation and causation	Correlation, causation, associatio,
HSN-Q.A.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).	Students should be able to sketch each distribution and answer questions about it just from knowledge of these three facts (shape, center, and spread). For either group, about 68% of the data values will be within one standard deviation of the mean. They should also observe that the two measures of center, median and mean, tend to be close to each other for symmetric distributions.	● Experience the use of different measurement tools both digitally and concretely to observe measurement error ● Connect measurement concepts to science and other contexts to show understanding of significant digits and scientific notation ● Use the measurement of the same object multiple times to determine an acceptable level to report	Shape, center, spread, outlier

UNIT 2:Two-Variable Statistics (iM Algebra 1, Unit 3)				
CCSS Standards #	**Learning Targets:** I can	**Assessment Strategy** SR-Selected Response CR-Constructed Response P-Performance O-Observation (behavioral)	**Lesson Progression and Connection to Math Practices**	**Common Learning Experiences and Assessments**
HSS-ID.B.5	What can learn from two-way tables? ● I can calculate missing values in a two-way table. (P) ● I can create a two-way table for categorical data given information in everyday language. (P) ● I can describe what the values in a two-way table mean in everyday language. (CR) ● I can calculate values in a relative frequency table and describe what the values mean in everyday language. (CR) ● I can look for patterns in two-way tables and relative frequency tables to see if there is a possible association between two variables. (P)		● The mathematical purpose of lesson 1 is to create and interpret two-way tables. Students encounter the term categorical variable, which is a variable that represents data which can be divided into groups or categories. In later lessons, students create and interpret two-way tables showing relative frequencies.. ● The mathematical purpose of this lesson 1 is for students to create and interpret relative frequency tables. ● The work of lesson 2 connects to previous work in which students created and interpreted two-way tables. The work of this lesson connects to upcoming work because students will use relative frequency tables to look for associations between categorical variables. ● The mathematical purpose of lesson 3 is to recognize association among variables by analyzing relative frequency tables. The work of this lesson connects to previous work because students created and interpreted relative frequency tables.	iM Lessons 1-3
			● MP1 ● MP6 ● MP7	
HSS-ID.B.6.a HSS-ID.B.6.b	How can we use scatter plots to analyze data?		● The mathematical purpose of lesson 4 is to understand how a linear model is used	iM Lessons 4, 5, 6 (optional)

HSS-ID.B.6.c HSS-ID.C.7 HSN-Q.A.3	• I can describe the rate of change and -intercept for a linear model in everyday language. (CR) • I can draw a linear model that fits the data well and use the linear model to estimate values I want to find. (P) • I can describe the rate of change and -intercept for a linear model in everyday language. (CR) • I can use technology to find the line of best fit. (O) • I can plot and calculate residuals for a data set and use the information to judge whether a linear model is a good fit. (P)	to describe the relationship between two numerical variables, and to use a line of best fit to make predictions. The work of this lesson connects to previous work because students investigated patterns of association in bivariate data in eighth grade and in a previous lesson. • The mathematical purpose of lesson 5 is for students to informally assess the fit of various lines to data, to use technology to find the line of best fit, and to interpret the slope and vertical intercept of the linear model. The work connects to previous work because students created scatter plots and created linear models for the data • The mathematical purpose of lesson 6 is to informally assess the fit of a function by plotting and analyzing residuals. The work of this lesson connects to previous work because students analyzed bivariate data by creating scatter plots and fitting linear functions to the data.	
		• MP2 • MP6 • MP7	
HSS-ID.B.6.a HSS-ID.C.7 HSS-ID.C.8 HSS-ID.C.9	How do we find and interpret correlation coefficients? • I can describe the goodness of fit of a linear model using the correlation coefficient. (CR) • I can match the correlation coefficient with a scatter plot and linear model. (P) • I can describe the strength of a relationship between two variables. (CR) • I can use technology to find the correlation coefficient and explain what the value tells me about a linear model in everyday language. (CR) • I can look for connections between two variables to analyze whether or not there is a causal relationship. (O)	• The mathematical purpose of lesson 7 is for students to find and interpret the correlation coefficient, and to use it to understand the strength of a linear relationship. The work of this lesson connects to previous work because students plotted and analyzed residuals to informally assess the fit of linear models. • The mathematical purpose of lesson 8 is to compute (using technology) and interpret the correlation coefficient for a bivariate, numerical data set. The work of this lesson connects to previous work because students learned how to read	iM Lessons 7, 8, 9

		and interpret the correlation coefficient. • The mathematical purpose of lesson 9 is to understand that the relationship between variables can be, but is not always, a causal relationship. The work of this lesson connects to previous work because students interpreted the relationship between two variables using the correlation coefficient.	
		• MP2 • MP3 • MP4 • MP7	
HSS-ID.B.6 HSS-ID.C.7 HSS-ID.C.8 HSS-ID.C.9	How can we draw conclusions from data? • I can collect data, create a linear model to fit the data, determine if the linear model is a good fit, and use the information from my linear model to answer questions. (P)	• The mathematical purpose of this lesson is for students to collect, summarize, interpret, and draw conclusions from bivariate data using scatter plots, best fit lines, residuals, and correlation coefficients. This connects to previous work because students summarized, represented, interpreted, and drew conclusions from bivariate data using scatter plots, best fit lines, residuals, and correlation coefficients.	iM Lesson 10 is Optional
		• MP1 • MP5	

ADDITIONAL CONSIDERATIONS			
COMMON MISCONCEPTIONS	PRIOR KNOWLEDGE NEEDED TO MASTER STANDARDS FOR THIS UNIT	ADVANCED STANDARDS FOR STUDENTS WHO HAVE DEMONSTRATED PRIOR MASTERY	OPPORTUNITIES FOR STUDENT-DIRECTED LEARNING WITHIN THE UNIT
• Connecting data to real world context	8.SP.A.4 8.SP.A.2 8.SP.A.1 8.F.B.4 8.SP.A.3		

UNIT 3: Conditional Probability (iM Geometry, Unit 8)

Illustrative Mathematics Unit Access (Geometry, Unit 8)

Essential Questions:

- What can I learn from a sample space?
- How can I find a sample space and probability using tables, trees, lists, and Venn Diagrams?
- How are different data displays connected?
- What does it mean for events to be independent?

UNWRAPPED STANDARDS

Grade Level Standard	Standard Progression	Concepts (Big Ideas/ Understandings)	Academic Vocabulary (Standard Based)
HSS-ID.B.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.	As with univariate data analysis, students now take a deeper look at bivariate data, using their knowledge of proportions to describe categorical associations and using their knowledge of functions to fit models to quantitative data, [giving students an opportunity to engage in Standards for Mathematical Practice 7 and 4.] The table below shows statistics from the Center for Disease Control relating HIV risk to age groups. Students should be able to explain the meaning of a row or column total (marginal), a row or column percentage (conditional) or a "total" percentage (joint). They should realize that possible associations between age and HIV risk are best explained in terms of the row or column conditional percentages. Are the comparisons of percentages valid when the first age category is much smaller (in years) than the others?	• Use relative frequencies to make inferences about associations or trends in data • Interpret relative frequencies in terms of a subset of a conditioned event	Bivariant, correlation, frequency, trend

<table>
<tr>
<td></td>
<td>

HIV risk by age groups, in percent of population

<table>
<tr><th></th><th>Age</th><th>18–24</th><th>25–44</th><th>45–64</th><th>Row Total</th></tr>
<tr><td rowspan="3">Not at risk</td><td>Row %</td><td>14.0</td><td>59.6</td><td>26.4</td><td>100.0</td></tr>
<tr><td>Column %</td><td>35.0</td><td>51.7</td><td>27.2</td><td></td></tr>
<tr><td>Total %</td><td>5.6</td><td>23.6</td><td>10.5</td><td>39.6</td></tr>
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<tr><td>Total %</td><td>15.9</td><td>45.6</td><td>38.5</td><td>100.0</td></tr>
</table>

Source: Center for Disease Control,
http://apps.nccd.cdc.gov/s_broker/WEATSQL.exe/weat/freq_year.hsql

</td>
<td></td>
<td></td>
</tr>
<tr>
<td>HSS-CP.A.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).</td>
<td>

Suppose a student is randomly guessing the answers to all four true–false questions on a quiz. The outcomes in the sample space can be arranged as shown in the margin.
[Insert figure here] Probabilities assigned to these outcomes should be equal because random guessing implies that no one outcome should be any more likely than another. By simply counting equally likely outcomes, P(exactly two correct answers) = 6/16 and P(at least one correct answer) = 15/16 = 1 - P (no correct answers). Likewise, P(C on first question) = 1/2 = P(C on second question) as should seem intuitively reasonable. Now, P[(C on first question) and (C on second question)] = 4/16 = 1/4 = 1/2 * ½.

Possible outcomes: Guessing on four true–false questions

<table>
<tr><th>Number correct</th><th>Out-comes</th><th>Number correct</th><th>Out-comes</th><th>Number correct</th><th>Out-comes</th></tr>
<tr><td>4</td><td>CCCC</td><td>2</td><td>CCII</td><td>1</td><td>CIII</td></tr>
<tr><td>3</td><td>ICCC</td><td>2</td><td>CICI</td><td>1</td><td>ICII</td></tr>
<tr><td>3</td><td>CICC</td><td>2</td><td>CIIC</td><td>1</td><td>IICI</td></tr>
<tr><td>3</td><td>CCIC</td><td>2</td><td>ICCI</td><td>1</td><td>IIIC</td></tr>
<tr><td>3</td><td>CCCI</td><td>2</td><td>ICIC</td><td>0</td><td>IIII</td></tr>
<tr><td></td><td></td><td>2</td><td>IICC</td><td></td><td></td></tr>
</table>

C indicates a correct answer; I indicates an incorrect answer.

</td>
<td>

- Relate Venn Diagrams and frequency tables to set notations and probabilistic models.
- Represent contextual situations by their complement to solve problems.
- Use set notation to represent contextual situations.
- Relate the complement of one event to the intersection or union of others.

</td>
<td>Complement, Conditional Probability, Disjoint Sets, Independent, Intersection, Sample Space, Union</td>
</tr>
<tr>
<td>HSS-CP.A.2 Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this</td>
<td>Suppose a student is randomly guessing the answers to all four true–false questions on a quiz. The outcomes in the sample space can be arranged as shown in the margin. [Insert first figure here] Probabilities assigned to these outcomes should be equal because random guessing implies that no one outcome should be any more likely than another. P(C on first question) = 1/2 = P(C on second question) as should seem</td>
<td>

- State problems’ independence and dependence contextually.
- Attend to precision in wording of relationships by emphasizing relationships as associated rather than caused.
- Test for independence by

</td>
<td>Complement, Conditional Probability, Disjoint Sets, Independent, Intersection, Sample Space, Union</td>
</tr>
</table>

characterization to determine if they are independent.	intuitively reasonable. Now, P[(C on first question) and (C on second question)] = 4/16 = 1/4 = 1/2 * ½, which shows that the two events (C on first question) and (C on second question) are independent, by the definition of independence (Two events A and B are said to be independent if P(A) * P(B) = P(A and B)). This, too, should seem intuitively reasonable to students because the random guess on the second question should not have been influenced by the random guess on the first. Students may contrast the quiz scenario above with the scenario of choosing at random two students to be leaders of a five-person working group consisting of three girls (April, Briana, and Cyndi) and two boys (Daniel and Ernesto). The first name chosen indicates the discussion leader and the second the recorder, so order of selection is important. The 20 outcomes are displayed in the margin. [Insert second figure here] Here, the probability of selecting two girls is: P(two girls selected) = 6/20 = 3/10 whereas P(girl selected on first draw) = 12/20 = 3/5 = P(girl selected on second draw). Because 3/5 * 3/5 ≠ 3/10, these two events are not independent. The selection of the second person does depend on the selection of the first when the same person cannot be selected twice.	comparing the joint probability of an event with the product of the two events occuring marginally. ● Challenge ideas of mathematical and statistical independence.	

Possible outcomes: Guessing on four true–false questions

Number correct	Out-comes	Number correct	Out-comes	Number correct	Out-comes
4	CCCC	2	CCII	1	CIII
3	ICCC	2	CICI	1	ICII
3	CICC	2	CIIC	1	IICI
3	CCIC	2	ICCI	1	IIIC
3	CCCI	2	ICIC	0	IIII
		2	IICC		

C indicates a correct answer; I indicates an incorrect answer.

Selecting two students from three girls and two boys

Number of girls	Outcomes	
2	AB	BA
2	AC	CA
2	BC	CB
1	AD	DA
1	AE	EA
1	BD	DB
1	BE	EB
1	CD	DC
1	CE	EC
0	DE	ED

HSS-CP.A.3 Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.	Another way of viewing independence is to consider the conditional probability of an event A given an event B, P(A|B), as the probability of A in the sample space restricted to just those outcomes that constitute B. In the table of outcomes for guessing on the true-false questions, P(C on second question | C on first question) = 4/8 = ½ = P(C on second) and students see that knowledge of what happened on the first question does not alter the probability of the outcome on the second; the two events are independent. In the selecting students scenario, the conditional probability of a girl on the second selection, given that a girl was selected on the first is P(girl on second | girl on first) = 6/12 = ½ and P(girl on second) = 3/5. So, these two events are again seen to be dependent. The outcome of the second draw does depend on what happened at the first draw.	• Contextually interpret probability of different events bringing to light the implications of independence in conditioned events. • Use joint and marginal probabilities to justify reasoning for the rule of conditional probability. • Symbolize events by assigning labels to their description and using notations from probability to define their operations.	Complement, Conditional Probability, Disjoint Sets, Independent, Intersection, Sample Space, Union

Possible outcomes: Guessing on four true–false questions

Number correct	Outcomes	Number correct	Outcomes	Number correct	Outcomes
4	CCCC	2	CCII	1	CIII
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C indicates a correct answer; I indicates an incorrect answer.

Selecting two students from three girls and two boys

Number of girls	Outcomes	
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1	BE	EB
1	CD	DC
1	CE	EC
0	DE	ED

HSS-CP.A.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and	In real world applications the probabilities of events are often approximated by data about those events. For example, the percentages in the table for HIV risk by age group can be used to approximate probabilities of HIV risk with respect to age or age with respect to HIV risk for a randomly selected adult from the U.S. population of adults. Emphasizing the conditional nature of the row and column percentages: P(adult is age 18 to 24 | adult is at risk) = 0.171 whereas P(adult is at risk | adult is age 18 to 24) = 0.650. Comparing the latter to P(adult is at risk | adult is age 25 to 44) = 0.483 shows that the conditional distributions change from column to column, reflecting dependence and an association between age category and HIV risk.	• Use data to interpret the association of different events by comparing conditional probabilities or testing for mathematical independence. • Use frequency tables to describe a sample space of a population. • Use frequency tables to argue and validate arguments of association by relating marginal and joint events.	Complement, Conditional Probability, Disjoint Sets, Independent, Intersection, Sample Space, Union

<table>
<tr>
<td>English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.</td>
<td>
HIV risk by age groups, in percent of population
<table>
<tr><th></th><th>Age</th><th>18–24</th><th>25–44</th><th>45–64</th><th>Row Total</th></tr>
<tr><td rowspan="3">Not at risk</td><td>Row %</td><td>14.0</td><td>59.6</td><td>26.4</td><td>100.0</td></tr>
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</table>
Source: Center for Disease Control,
http://apps.nccd.cdc.gov/s_broker/WEATSQL.exe/weat/freq_year.hsql
</td>
<td></td>
<td></td>
</tr>
<tr>
<td>HSS-CP.A.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.</td>
<td>Students can gain practice in interpreting percentages and using them as approximate probabilities from study data presented in the popular press. Quite often the presentations are a little confusing and can be interpreted in more than one way. For example, two data summaries from USA Today are shown below. What might these percentages represent and how might they be used as approximate probabilities?</td>
<td>
Use conditional probability to make decisions and justify claims of relationships to contextual situations.
Translate and explain conditional probability notation contextually.
Solve word problems related to conditional probability and explain their solutions contextually.
Use marginal distributions in frequency tables to explain conditional probability.
</td>
<td>Complement, Conditional Probability, Disjoint Sets, Independent, Intersection, Sample Space, Union</td>
</tr>
<tr>
<td>HSS-CP.B.6 Find the conditional probability of A given B as the fraction of B’s outcomes that also belong to A, and interpret the answer in terms of the model.</td>
<td>Suppose two fair six-sided number cubes are rolled, giving rise to 36 equally likely outcomes.
Outcomes for specified events can be diagrammed as sections of the table, and probabilities calculated by simply counting outcomes. This type of example is one way to review information on conditional probability and introduce the addition and multiplication rules.

For example, defining events:
A is “you roll numbers summing to 8 or more”
B is “you roll doubles”

and counting outcomes leads to
P(A) = 15/36
P(B) = 6/36
P(A and B) = 3/36, and</td>
<td>
Identify why two probabilities P(A) and P(A|B) are not always equal.
Connect previous ideas using two way frequency tables, Venn Diagrams, and/or counting principles to solve problems.
Explain conditional solutions in terms of outcomes that belong to an observed set of outcomes.
</td>
<td>Combination, Conditional Probability, Disjoint Sets, Factorial, Frequency, Fundamental Counting Principle, Independent, Joint Frequency, Marginal Frequency, Permutation, Uniform Probability Model</td>
</tr>
</table>

<table>
<tr>
<td></td>
<td>P(B|A) = 3/15, the fraction of A's 15 outcomes that also fall in B.

Possible outcomes: Rolling two number cubes
<table>
<tr><th></th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th></tr>
<tr><td>1</td><td>1, 1</td><td>1, 2</td><td>1, 3</td><td>1, 4</td><td>1, 5</td><td>1, 6</td></tr>
<tr><td>2</td><td>2, 1</td><td>2, 2</td><td>2, 3</td><td>2, 4</td><td>2, 5</td><td>2, 6</td></tr>
<tr><td>3</td><td>3, 1</td><td>3, 2</td><td>3, 3</td><td>3, 4</td><td>3, 5</td><td>3, 6</td></tr>
<tr><td>4</td><td>4, 1</td><td>4, 2</td><td>4, 3</td><td>4, 4</td><td>4, 5</td><td>4, 6</td></tr>
<tr><td>5</td><td>5, 1</td><td>5, 2</td><td>5, 3</td><td>5, 4</td><td>5, 5</td><td>5, 6</td></tr>
<tr><td>6</td><td>6, 1</td><td>6, 2</td><td>6, 3</td><td>6, 4</td><td>6, 5</td><td>6, 6</td></tr>
</table>
</td>
<td></td>
<td></td>
</tr>
<tr>
<td>HSS-CP.B.7 Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.</td>
<td>Suppose two fair six-sided number cubes are rolled, giving rise to 36 equally likely outcomes.
Outcomes for specified events can be diagramed as sections of the table, and probabilities calculated by simply counting outcomes. This type of example is one way to review information on conditional probability and introduce the addition and multiplication rules.

For example, defining events:
A is "you roll numbers summing to 8 or more"
B is "you roll doubles"

Now, by counting outcomes
P(A or B) = 18/36
or by using the Addition Rule
P(A or B) = P(A) + P(B) - P(A and B)
= 15/36 + 6/36 – 3/36
= 18/36.</td>
<td>• Discover and build understanding the relationships of the addition rule to Venn Diagrams and frequency tables.
• Use the intersection and union of events to describe the probability of different situations.
• Use the probability rule to solve problems directly and indirectly through formula manipulation.
• Connect Venn Diagrams and frequency tables to the addition rule by justifying and critiquing the reasoning of themselves and their peers.</td>
<td>Combination, Conditional Probability, Disjoint Sets, Factorial, Frequency, Fundamental Counting Principle, Independent, Joint Frequency, Marginal Frequency, Permutation, Uniform Probability Model</td>
</tr>
</table>

Possible outcomes: Rolling two number cubes

	1	2	3	4	5	6
1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

UNIT 3: Conditional Probability (iM Geometry, Unit 8)				
CCSS Standards #	**Learning Targets:** I can	**Assessment Strategy** SR-Selected Response CR-Constructed Response P-Performance O-Observation (behavioral)	**Lesson Progression and Connection to Math Practices**	**Common Learning Experiences and Assessments**
HSS-CP.A.1 HSS-CP.A.4 HSS-ID.B.5	What can I learn from a sample space? How can I find a sample space and probability using tables, trees, lists, and Venn Diagrams? ● I can find or estimate probability using a model or data from a chance experiment. (P) ● I can identify chance experiments. (O) ● I can find the sample space for chance experiments. (CR) ● I can model situations using probability. (CR) ● I can use sample space to calculate probability. (SR/CR) ● I can create organized lists, tables, and tree diagrams and use them to calculate probabilities. (CR) ● I can use information in a two-way table to find relative frequencies and to estimate probability. (SR/CR)		The mathematical purpose of the first lesson is to revisit the idea of chance events and the concept of probability. The probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. The work of this lesson connects to previous work because students investigated chance processes and developed, used, and evaluated probability models in Grade 7. The work of this lesson connects to upcoming work because students will understand independence and conditional probability and use them to interpret data. The mathematical purpose of the second lesson is to make connections to prior knowledge about probability and to use probability to interpret data. The work of this lesson connects to previous work because students learned about chance events and probability in previous grades. The work of this lesson connects to upcoming work because students will describe and use the sample space for chance experiments to calculate the probability of compound events. The mathematical purpose of the third lesson is to describe and use the sample space for chance experiments to calculate probabilities for compound events. The work of this lesson connects to previous work because students made connections to prior knowledge about probability and used probability to interpret data. The work of this lesson connects to upcoming work because students will use two-way tables and relative frequency tables to determine probabilities for some events. When students choose to use an organized list, table, or tree diagram to determine the sample space,	iM Lessons 1 (optional), 2, 3, 4

		they are using appropriate tools strategically. The mathematical purpose of the fourth lesson is to make connections between two-way tables and relative frequency tables and to use the tables to determine probabilities for some events. The work of this lesson connects to previous work because students used sample spaces to calculate probabilities of compound events. The work of this lesson connects to upcoming work because students will use tables and Venn diagrams to determine probabilities for some events. When students use two-way tables to estimate probabilities they are seeing and making use of structure.	
		● MP3 ● MP5 ● MP6 ● MP7	
HSS-CP.A.1 HSS-CP.B.7	How can I find a sample space and probability using tables, trees, lists, and Venn Diagrams? ● I can use tables and Venn diagrams to represent sample spaces and to find probabilities. (SR/CR) ● I can use the addition rule to find probabilities. (SR/CR)	The mathematical purpose of the fifth lesson is to describe events composed of other events and to calculate the probability of events using information represented in tables and in Venn diagrams. The work of this lesson connects to previous work because students used two-way tables and two-way frequency tables to calculate probabilities for some events. The work of this lesson connects to upcoming work because students will learn to use the addition rule to calculate probabilities. When students compare probabilities calculated through experimentation to probabilities calculated by using sample spaces they are reasoning abstractly and quantitatively. The mathematical purpose of the sixth lesson is for students to apply the addition rule and to interpret the answer using the model. The work of this lesson connects to previous work because students calculated probabilities using information represented in tables and Venn diagrams. The work of this lesson connects to upcoming work because students will use probability to	iM Lessons 5, 6

		determine whether or not two events are independent. Students encounter the addition rule which states that given events A and B, P(A or B)= P(A) + P(B) - P(A and B). When students use the addition rule to get an answer and then interpret the meaning of their answer in a context then they are reasoning abstractly and quantitatively. When students have to fix or find the error and explain the correct reasoning, they are constructing viable arguments and critiquing the reasoning of others.	
		• MP2 • MP3	
HSS-CP.A.2 HSS-CP.A.3 HSS-CP.A.4 HSS-CP.A.5 HSS-CP.B.6	How are different data displays connected? What does it mean for events to be independent? • I can estimate probabilities, including conditional probabilities, from two-way tables. (SR/CR) • I can use probabilities and conditional probabilities to decide if events are independent. (CR) • I can collect data and use it to estimate probabilities. (P) • I can use probabilities to decide if events are independent. (CR)	The mathematical purpose of lesson seven is for students to understand the difference between independent and dependent events in terms of probability. Students encounter the term independent events which is defined as two events from the same experiment for which the probability of one event is not affected by whether the other event occurs or not, and the term dependent events which is defined as two events from the same experiment for which the probability of one event depends on the result of the other event. The work of this lesson connects to previous work because students applied the addition rule to solve problems. The work of this lesson connects to upcoming work because students will solve problems involving conditional probability. When students use probability to analyze a situation to determine whether or not the events are dependent or independent they are looking for and making use of structure. The mathematical purpose of lesson eight is for students to make sense of conditional probability, the relationship between conditional probabilities and the probability that two events both occur, and to use conditional probability to investigate independence. Students encounter the term conditional probability in this lesson which is defined as the probability that one event occurs under the condition that another event occurs. The work of this lesson connects to previous work because students use	iM Lessons 7, 8, 9, 10

		probability to investigate dependence and independence. The work of this lesson connects to upcoming work because students will create and use two-way tables to estimate conditional probabilities. When students do the same type of calculation several times and observe that P(A and B) = P(A|B) * P(B) for events A and B they are looking for and expressing regularity in repeated reasoning. The mathematical purpose of lesson nine is for students to create and use two-way tables to estimate conditional probabilities, and to decide if events are independent. The work of this lesson connects to previous work because students investigated conditional probability and independence. The work of this lesson connects to upcoming work because students will continue to use probability to recognize dependent and independent events, and to practice finding probabilities for dependent and independent events. The mathematical purpose of lesson ten is to find and use probabilities to recognize dependent and independent events. The work of this lesson connects to previous work because students used two-way tables to investigate independence. The work of this lesson connects to upcoming work because students will determine independence of events and calculate conditional probabilities using data collected in class.	
		• MP2 • MP6 • MP7 • MP8	
HSS-CP.A.2 HSS-CP.A.3 HSS-CP.A.5 HSS-CP.B.6	What can I learn from a sample space? How can I find a sample space and probability using tables, trees, lists, and Venn Diagrams? How are different data displays connected? What does it mean for events to be independent?	This lesson is optional. It gives students the opportunity to put into practice what they have learned from this unit, but may be safely skipped if there is a shortage of time. The mathematical purpose of this lesson is to determine the independence of events and calculate conditional	iM Lesson 11 (optional)

	• I can determine independence of events and estimate conditional probabilities using data. (P)	probabilities using data collected in class. The work of this lesson connects to previous work because students found and used probabilities to recognize dependent and independent events. The work of this lesson connects to upcoming work because students will make inferences and justify conclusions. When students explain and justify how they determined probabilities, and ask each other questions they are constructing viable arguments and critiquing the reasoning of others.	
		• MP3	

ADDITIONAL CONSIDERATIONS			
COMMON MISCONCEPTIONS	**PRIOR KNOWLEDGE NEEDED TO MASTER STANDARDS FOR THIS UNIT**	**ADVANCED STANDARDS FOR STUDENTS WHO HAVE DEMONSTRATED PRIOR MASTERY**	**OPPORTUNITIES FOR STUDENT-DIRECTED LEARNING WITHIN THE UNIT**
• Students often calculate the probability from the total instead of subsets. • Students have trouble identifying the difference between independent and dependent events. • When using the addition rule they forget to subtract the overlap. • Some students may create a tree diagram but may not understand how to quantify the number of outcomes in the sample space. • Some students may struggle with the difference between "and" and "or" when used in a situation. • Some students may have difficulty understanding the	• 7.SP.C.5 • 7.SP.C.6 • 7.SP.C.7 • 7.SP.C.8 • 7.SP.C.8.b • 8.SP.A.4		

newly-introduced notation. ● Some students may not know how to represent probability as a percentage.			

UNIT 4: Similarity (iM Geometry, Unit 3)

Illustrative Mathematics Unit Access (Geometry, Unit 3)

Essential Questions:

- How do you apply the properties of dilations?
- What are similarity transformations?
- What similarities exist among right triangles?

UNWRAPPED STANDARDS			
Grade Level Standard	**Standard Progression**	**Concepts Big Ideas/ Understandings)**	**Academic Vocabulary (Standard Based)**
HSG-C.A.1 Prove that all circles are similar.	An advantage of the transformational approach to similarity is that it allows for a notion of similarity that extends to all figures rather than being restricted to figures composed of line segments. For example, consider a dilation of a circle whose center is the center of the dilation. Every point on the circle moves the same distance away, because they were originally all at the same distance from the center. Thus the new figure is also a circle. This reasoning can also be reversed to show that any two circles with the same center are similar. Furthermore, because any circle can be translated so that its center coincides with the center of any other circle, we can see that all circles are similar.	● Transformations can be used to show that all circles are similar.	Arc Measure, Central angle, Chord, Circumcenter, Circumscribed, Circumscribed angle, Incenter, Inscribed, Inscribed angle, Radius,
HSG-MG.A.2 Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).	In this standard, students explore contexts involving density, a compound unit comparing some quantity per unit of area or volume.	● Analyzing and understanding the compound units of density problems is essential and can suggest solution strategies for mathematical modeling problems. ● Draw conclusions about real-world situations and assess their reasonableness.	Density
HSG-MG.A.3	A wide range of problems might be addressed under this	● Use geometry to model real	Density

Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).	standard, related to different standards from throughout this conceptual category. Students' focus should be designing a solution that meets the constraints of a real-world context.	world situations in order to design solutions to real world problems. ● Justify the decision made to demonstrate that the resulting design addresses the contexts.	
HSG-CO.A.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).	Students should develop a general understanding of what a transformation is, exploring a range of both rigid and non-rigid motions.	● Rigid Transformations preserve distance and angle measures, but dilations only preserve angle measures. ● Transformations are Functions	Corresponding parts, Dilation, Geometric transformation, Image, Pre-image, rigid motion, Similarity transformation
HSG-CO.C.10 Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.	These theorems build on those from the previous standard and should again build on student conjectures about the relationships described in the standards.	Transformations can be used to Prove theorems about triangles.	Interior angle of a triangle, Base angle of an isosceles triangle, Midpoint of a segment, Median of a triangle, Theorem,
HSN-Q.A.1 Use units as a way to	Reasoning quantitatively includes knowing when and how to convert units in computations, such as when adding and	Understanding and attending to the units in a problem can help	Unit

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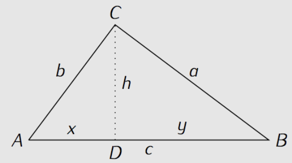
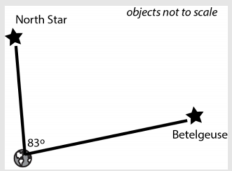
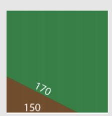
understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.	subtracting quantities that measure the same attribute but are expressed in different units and other computations with measurements in different units or converting units for derived quantities such as density and speed. Reasoning quantitatively can also include analyzing the units in a calculation to reveal the units of the answer.	interpret the problem and suggest a solution strategy.	
HSN-Q.A.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	Quantitative reasoning includes choosing an appropriate level of accuracy when reporting quantities. For example, if the doctor measures your height as 73 inches and your weight as 210 pounds, then your Body Mass Index (BMI) is (weight in pounds)/(height in inches 2) x 703 = (210)/(732) x 703 ≈ 27.7031 ≈ 28. There is no point in reporting a value more precise than 28 here, because any value between 25 and 30 is considered overweight.	• The number of significant digits to use depends on the measurement error and measurement variation of a piece of data.	Significant Digits, Precision, Accuracy
HSG-SRT.A.1 Verify experimentally the properties of dilations given by a center and a scale factor: HSG-SRT.A.1.a A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. HSG-SRT.A.1.b The dilation of a line segment is longer or shorter in the ratio given by the scale factor.	In high school, [students] observe basic properties of dilations. For example, they observe experimentally that a dilation takes a line to another line which is parallel to the first, or identical to it if the line is through the center of dilation. This important fact is used repeatedly in later work. They also observe that under dilation the length of any line segment—not only segments with an endpoint at the center—is scaled by the scale factor of the dilation.	• Dilations given by a center and a scale factor have defined properties.	Scale factor, Dilation,
HSG-SRT.A.2 Given two figures, use the definition of similarity in terms of similarity	The traditional notion of similarity applies only to polygons. Two such figures are said to be similar if corresponding angles are congruent and corresponding lengths are related by a constant scale factor. If similarity is defined in terms of	Two figures can be shown to be similar by using Similarity transformations.	criteria for triangle similarity, similar figures, similarity transformations, corresponding pairs of angles, proportionality of

transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.	transformations, then this understanding is a consequence of the definition rather than being a definition itself.		corresponding pairs of sides
HSG-SRT.B.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.	There is also another advantage of [the transformation] approach to congruence and similarity. Because most of the theorems in plane geometry before the introduction of similarity depend only on the three triangle congruence criteria, once these have been established, it is possible to transition into the traditional way of proving theorems at this point, without further use of basic rigid motions, if so desired. The use of dilations to treat similarity can likewise be limited to the initial stage if so desired.	• Develop facility in using triangle congruence and similarity to explain geometry relationships. • Continue to use geometric transformations as a way to visualize the relationships they find.	criteria for triangle similarity, similar figures, similarity transformations
HSG-SRT.C.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.	Because all right triangles have a common angle, the right angle, the AA criterion becomes, in the case of right triangles, an “A criterion”; that is, two right triangles are similar if they have an acute angle in common. This observation is the key to defining a trigonometric ratio for a single acute angle.	• Understand the foundation of trigonometry and similarity using the AA similarity criterion. • Know the definitions of trigonometric ratios using right triangles. • Are able to find missing sides and angles of a right triangle given other sides and angles.	criteria for triangle similarity, similar figures, similarity transformations,
HSG-SRT.A.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.	Students can see that two figures which are similar according to the traditional notion are also similar according to the transformation definition by deriving the AA criterion for similarity of triangles.	• Understand that when two pairs of corresponding angles in a pair of triangles are congruent, the two triangles must be similar.	criteria for triangle similarity, similar figures, similarity transformations, corresponding angles

HSG-SRT.B.4 Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.	**The Pythagorean Theorem using similarity**  Given a right triangle ABC with right angle at C, drop an altitude from C to AB to decompose the triangle into two smaller triangles. Using the facts that the sum of the angles at C is $90°$ and the sum of the angles in each triangle is $180°$, we see that $\angle DAC$ is congruent to $\angle DBC$. Also, all three triangles have a right angle. So, by the AA criterion for similarity, $\triangle ACD$ and $\triangle CBD$ are similar to $\triangle ABC$, so $\frac{a}{c} = \frac{y}{a}$ and $\frac{b}{c} = \frac{x}{b}$ and therefore $a^2 + b^2 = c(x + y) = c^2.$	● The criteria for similar triangles and similarity transformations can be used to prove that segments found in pairs of triangles are proportional.	criteria for triangle similarity, similar figures, similarity transformations, corresponding angles

HSG-SRT.C.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.	**Modeling with trigonometry** **Vector graphic** A digital artist using a computer drawing program clicked on a diagonal line segment and saw that it measured 216 units horizontally and 45 units vertically. How many units long was the line segment? If the artist wants to rotate the line segment to be vertical, what angle of rotation could be used? Give your answers to the nearest hundredth. *Answers.* 220.64 units long; 11.77° clockwise or 78.23° counter-clockwise. **Flight of the bumblebee** A bumblebee sitting on a tulip wanted to fly over to a daffodil located 100 meters due west. The bumblebee did fly in a straight line, but it mistakenly flew in a direction 5 degrees south of west. The bumblebee passed to the south of the daffodil—how far south? *Answer.* 8.75 meters or about 9 meters. **Star distance** Viewed from Earth, the North Star and Betelgeuse are separated by an angle of about 83 degrees. If the North Star is about 430 light years from Earth, and Betelgeuse is about 640 light years from Earth, then about how far apart are the North Star and Betelgeuse (measured in light years)?  *Answer.* About 726 light years. *Comment.* This could be solved using a G-SRT.10(+) strategy (Law of Cosines), or via G-SRT.8 by dropping a perpendicular to make two right triangles. **Crop Loss** One corner of a soybean field wasn't irrigated, and no soybeans could be harvested from that part of the field. How much money was lost if soybeans sold for $12 a bushel that year and an acre of irrigated land yields 54 bushels of soybeans? Note, 1 acre is approximately 4000 square meters.  *Answer*: $972 or approximately $1000. See `http://tinyurl.com/mtcn6zq`.	● Represent applied problems using right triangles. ● Identify relevant trigonometric ratios and apply them to solve problems.	criteria for triangle similarity, similar figures, similarity transformations, trigonometric ratios
HSA-CED.A.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example,	There are situations where an equation is used to describe the relationship between a number of different quantities. For example, Ohm's Law V = IR relates the voltage, current, and resistance of an electrical circuit. An equation used in this way is sometimes called a formula. It is perhaps best to avoid using the terms "variable", "parameter", or "constant" when working	● Solving an equation for a specific variable can make repeated calculations more efficient.	Linear Equation, Quadratic Equation

rearrange Ohm’s law V = IR to highlight resistance R.	with this formula, because there are six different ways it can be viewed as defining one quantity as a function of the other with a third held constant.		

<table>
<tr><th colspan="5">UNIT 5: Similarity (iM Geometry, Unit 3)</th></tr>
<tr><th>CCSS Standards #</th><th>Learning Targets: I can</th><th>Assessment Strategy
SR-Selected Response
CR-Constructed Response
P-Performance
O-Observation (behavioral)</th><th>Lesson Progression and Connection to Math Practices</th><th>Common Learning Experiences and Assessments</th></tr>
<tr><td rowspan="2">HSG-CO.A.2
HSG-MG.A.3
HSG-SRT.A.1
HSG-SRT.A.2
HSG-SRT.B.5
HSG-MG.A.2
HSN-Q.A.1
HSG-SRT.A.3
HSG-CO.C.10
HSG-SRT.B.4</td><td colspan="2">How do you apply the properties of dilations?
(lessons 1 - 5)</td><td rowspan="2">In lesson 1 students review the definition of scale factor by comparing an example and a non-example of a scaled image. We define scale factor as the factor by which every length in an original figure is increased or decreased when making a scaled copy. In this lesson students begin to identify the properties of dilations.
In lesson 2 students practice finding an unknown value in a proportional relationship while completing part of the modeling cycle to figure out how to create a scale drawing of the solar system. This lesson serves both as practice with proportional relationships and extension into an interesting context.
In lesson 3 students have more opportunities to practice drawing dilations precisely by reasoning about the definition of a dilation. Students construct and compare several examples of dilations, and then measure corresponding lengths and distances in the dilated and original figure. Through experimentation, students conjecture that all distances and lengths in the scaled figure are increased or decreased according to the same ratio given by the scale factor.
In lesson 4, students verify experimentally and assert that dilations take angles to congruent angles, and use this assertion to create a convincing argument that if dilations take angles to congruent angles, they must take lines to parallel lines. Students also examine what happens to lines through the center of dilation under dilation. They reason based on the definition of dilation that those lines don't change under dilation.
In lesson 5, students use the definition of dilation to</td><td rowspan="2">iM Lessons 1, 2, 3, 4, 5</td></tr>
<tr><td colspan="2">I can dilate a figure given a scale factor and center. (CR)
I can calculate the lengths of parts of a scaled drawing. (CR)
I know that when figures are dilated by a scale factor of k, all lengths in the figure are multiplied by k. (CR)
I can explain what happens to lines and angles in a dilation. (O)
I can explain why the segment connecting the midpoints of two sides of a triangle is parallel to the third side and half the length of the third side. (O, CR)</td></tr>
</table>

		prove that the triangles formed by connecting midpoints of two sides are dilations of the original triangle. Then they deduce properties of these segments using a new line of argument: proving that two figures are dilations of one another is a way to prove the figures have properties of dilations.	
		● MP6, MP4, MP3	
HSG-SRT.A.1 HSG-SRT.A.2 HSG-SRT.A.3 HSG-C.A.1 HSG-SRT.C.6 HSG-SRT.B.4 HSG-SRT.B.5 HSN-Q.A.1	What are similarity transformations? (lessons 6 - 12) ● I can write similarity statements. (CR) ● I know the definition of similarity. (O) ● I know the relationships between corresponding sides and angles in similar triangles. (O) ● I can explain why a segment parallel to one side of a triangle divides the other sides proportionally. (O) ● I can find scale factors and use them to solve problems. (CR)	● In lesson 6, students develop an informal understanding of how to use rigid transformations and dilations to show the similarity of any pair of triangles with all pairs of corresponding angles congruent and all pairs of corresponding side lengths proportional. Students look at specific examples and try to come up with a generalized method that works for any pair of triangles that meets the criteria. ● In lesson 7, students use rigid transformations and dilations to reason about and make generalizations about similarity of triangles and other types of figures. Students build on the work they did in earlier lessons, using their repeated reasoning to come up with a generalized sequence of rigid motions and dilations that they can justify will always take two triangles with congruent angles and proportional sides onto one another. ● In lesson 8, students conjecture and reason about whether all shapes in a certain category must be similar, such as whether all circles, or all rectangles, are similar. Then they prove that all equilateral triangles are similar, and all circles are similar. (eliminated?) ● In lesson 9, students use the Angle-Side-Angle Triangle Congruence Theorem as the basis for proving the Angle-Angle Triangle Similarity Theorem. Students have the opportunity to build intuition about the Angle-Angle Triangle Similarity Theorem before proving it. The lesson culminates with students exploring how many angles are needed, and which angles are needed, to be sure we can use the	iM Lessons 6, 7, 9 (modified), 10 (modified), 11, 12

		Angle-Angle Triangle Similarity Theorem to prove two triangles are similar. (eliminating lesson?) ● In lesson 10, students see why the Side-Side-Side Triangle Congruence Theorem implies the Side-Side-Side Triangle Similarity Theorem, and why the Side-Angle-Side Triangle Congruence Theorem implies the Side-Angle-Side Triangle Similarity Theorem. (eliminating lesson?) ● In lesson 11, they prove that a line parallel to one side of a triangle splits the other two sides proportionally. They can now conclude that a line is parallel to one side of a triangle if and only if it splits the other two sides proportionally. ● In lesson 12, students have an additional opportunity to practice finding unknown values in proportional relationships using contextual examples. The problems preview using the Pythagorean Theorem, which is a key idea in a subsequent lesson.	
		● MP7, MP8	
HSG-SRT.B.4 HSG-SRT.B.5 HSG-SRT.C.8	What similarities exist among right triangles? (lessons 13-15) ● I can find similar triangles formed by the altitude to the hypotenuse in a right triangle. (CR) ● I can solve problems involving similar right triangles. (CR)	● Lesson 13 sets the stage for a later proof by supporting students to make sense of the similar triangles formed by drawing the altitude to the hypotenuse of a right triangle. Students also get a chance to practice using the Pythagorean Theorem, equivalent ratios, and scale factors to find unknown side lengths in similar right triangles. ● In lesson 14, students learn another way to prove the Pythagorean Theorem based on the fact that right triangles (and only right triangles) can be decomposed into two similar versions of themselves. ● Lesson 15 invites students to practice the strategies they have used to find unknown lengths in similar right triangles. A key strategy for students to practice in this lesson is using proportional relationships within similar triangles to find unknown lengths in other triangles. In a subsequent unit students will study trigonometry through similar right triangles, and the ratios in trigonometry tables are ratios within	iM Lessons 13, 15

		triangles.	
		• MP2, MP1, MP6	
[HSG-SRT.B.5](#) [HSN-Q.A.3](#)	Putting it all together (lesson 16) • I can solve and interpret problems involving similar right triangles. (P/CR)	• Lesson 16 includes several summative application activities. There are two optional activities that involve going outside or to a room with very high ceilings to use indirect measurement to measure an object. In the first optional activity, students use mirrors (or shallow bowls of water) and in the second, they choose their own tools. The other activities focus on similar triangles and scale factors.	iM Lesson 16
		• MP5	

COMMON MISCONCEPTIONS	PRIOR KNOWLEDGE NEEDED TO MASTER STANDARDS FOR THIS UNIT	ADVANCED STANDARDS FOR STUDENTS WHO HAVE DEMONSTRATED PRIOR MASTERY	OPPORTUNITIES FOR STUDENT-DIRECTED LEARNING WITHIN THE UNIT
• Students often view transformations as a “motion”. Teachers should encourage the students to think of a transformation as a function. • Students may have difficulty expressing their thinking in more formal ways. Encourage precision in students’ communication.	7.RP.A.1 7.G.A.1 7.G.A.4 7.G.B.6 8.G.A.1 8.G.A.3 8.F.A.1 8.G.A.5		

UNIT 5: Right Triangle Trigonometry (iM Geometry, Unit 4)

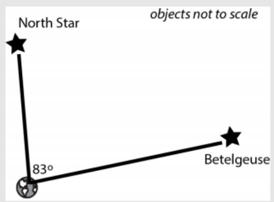
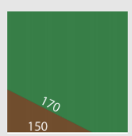
[Illustrative Mathematics Unit Access](Geometry, Unit 4)**(Geometry, Unit 4)**

Essential Questions:

- How are similarity and trigonometry related?
- How can you use trigonometry to solve real world problems?

UNWRAPPED STANDARDS			
Grade Level Standard	**Standard Progression**	**Concepts Big Ideas/ Understandings)**	**Academic Vocabulary (Standard Based)**
HSG-GMD.A.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.	Students are asked to justify why formulas hold. While formal proofs generally require higher mathematics involving calculus, informal proofs can be developed to advance the plausibility of the formulas.	● Consider why the various formulas work, using drawings and models as needed. ● Use similarity to define pi and develop the formula for the circumference of a circle. ● Use a limit argument to develop the area of the circle. ● Use Cavalieri's Principle to explain why the formulas for the volume of a cylinder, pyramid, and cone work. ● Relate the volumes using various solids with the same dimensions.	Cavalieri's Principle, Dissection
HSG-SRT.B.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.	There is also another advantage of [the transformation] approach to congruence and similarity. Because most of the theorems in plane geometry before the introduction of similarity depend only on the three triangle congruence criteria, once these have been established, it is possible to transition into the traditional way of proving theorems at this point, without further use of basic rigid motions, if so desired. The use of dilations to treat similarity can likewise be limited to	● Develop facility in using triangle congruence and similarity to explain geometry relationships. ● Continue to use geometric transformations as a way to visualize the relationships they find.	criteria for triangle similarity, similar figures, similarity transformations

	the initial stage if so desired.		
HSG-SRT.C.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.	Because all right triangles have a common angle, the right angle, the AA criterion becomes, in the case of right triangles, an "A criterion"; that is, two right triangles are similar if they have an acute angle in common. This observation is the key to defining a trigonometric ratio for a single acute angle.	● Understand the foundation of trigonometry and similarity using the AA similarity criterion. ● Know the definitions of trigonometric ratios using right triangles. ● Are able to find missing sides and angles of a right triangle given other sides and angles.	criteria for triangle similarity, similar figures, similarity transformations, trigonometric ratios
HSG-SRT.C.7 Explain and use the relationship between the sine and cosine of complementary angles.	This standard addresses a particular relationship between the sine and cosine ratios.	● Observe that the sine of an angle is equal to the cosine of its complementary angle. ● Show that this relationship holds using the definition of sine and cosine.	criteria for triangle similarity, similar figures, similarity transformations, trigonometric ratios

HSG-SRT.C.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.	**Modeling with trigonometry** **Vector graphic** A digital artist using a computer drawing program clicked on a diagonal line segment and saw that it measured 216 units horizontally and 45 units vertically. How many units long was the line segment? If the artist wants to rotate the line segment to be vertical, what angle of rotation could be used? Give your answers to the nearest hundredth. *Answers.* 220.64 units long; 11.77° clockwise or 78.23° counter-clockwise. **Flight of the bumblebee** A bumblebee sitting on a tulip wanted to fly over to a daffodil located 100 meters due west. The bumblebee did fly in a straight line, but it mistakenly flew in a direction 5 degrees south of west. The bumblebee passed to the south of the daffodil—how far south? *Answer.* 8.75 meters or about 9 meters. **Star distance** Viewed from Earth, the North Star and Betelgeuse are separated by an angle of about 83 degrees. If the North Star is about 430 light years from Earth, and Betelgeuse is about 640 light years from Earth, then about how far apart are the North Star and Betelgeuse (measured in light years)?  *Answer.* About 726 light years. *Comment.* This could be solved using a G-SRT.10(+) strategy (Law of Cosines), or via G-SRT.8 by dropping a perpendicular to make two right triangles. **Crop Loss** One corner of a soybean field wasn't irrigated, and no soybeans could be harvested from that part of the field. How much money was lost if soybeans sold for $12 a bushel that year and an acre of irrigated land yields 54 bushels of soybeans? Note, 1 acre is approximately 4000 square meters.  *Answer*: $972 or approximately $1000. See `http://tinyurl.com/mtcn6zq`.	• Represent applied problems using right triangles. • Identify relevant trigonometric ratios and apply them to solve problems.	criteria for triangle similarity, similar figures, similarity transformations, trigonometric ratios

HSG-MG.A.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).	Any mathematical object that represents a situation from outside mathematics and can be used to solve a problem about that situation is a mathematical model. Modeling often involves making simplifying assumptions that ignore some features of the situation being modeled. In geometry, in order to study how the illuminated percentage of the moon's surface varies during a month, students might represent the moon as a rotating sphere, half black and half white.	• Use geometric shapes to model real world situations. • Use properties of those shapes to draw conclusions about the real world situations.	density
HSG-MG.A.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).	A wide range of problems might be addressed under this standard, related to different standards from throughout this conceptual category. Students' focus should be on designing a solution that meets the constraints of a real-world context.	• Use geometry to model real world situations in order to design solutions to real world problems. • Justify the decision made to demonstrate that the resulting design addresses the contexts.	density
HSN-Q.A.2 Define appropriate quantities for the purpose of descriptive modeling.	In modeling situations (MP.4), defining the key quantity of interest might be part of the task. For example, in a situation that involves crop productivity, a student might choose to examine the number of tons of fertilizer per acre as the variable of interest. In a situation that involves content development for a web site, a choice might arise as to whether the number of posts per day or the number of words per day is the key productivity variable.	• Select and properly use an existing quantity for a real world context. • Create an appropriate quantity for a real world context. • Explain the meaning of different quantities in a problem and its solution.	Real numbers
HSN-Q.A.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	Quantitative reasoning includes choosing an appropriate level of accuracy when reporting quantities. For example, if the doctor measures your height as 73 inches and your weight as 210 pounds, then your Body Mass Index (BMI) is (weight in pounds)/(height in inches 2) x 703 = (210)/(732) x 703 ≈ 27.7031 ≈ 28. There is no point in reporting a value more precise than 28 here, because any value between 25 and 30 is considered overweight.* (See reference under Figure #).	• Experience the use of different measurement tools both digitally and concretely to observe measurement error. • Connect measurement concepts to science and other contexts to show understanding of significant digits and scientific notation. • Use the measurement of the same object multiple times to determine an acceptable level to report.	Real numbers

UNIT 5: Right Triangle Trigonometry (iM Geometry, Unit 4)				
CCSS Standards #	**Learning Targets:** I can	**Assessment Strategy** SR-Selected Response CR-Constructed Response P-Performance O-Observation (behavioral)	**Lesson Progression and Connection to Math Practices**	**Common Learning Experiences and Assessments**
[HSG-MG.A.3](#) [HSG-SRT.B.5](#) [HSG-SRT.C.6](#) [HSN-Q.A.2](#) [HSN-Q.A.3](#)	How are similarity and trigonometry related? How can you use trigonometry to solve real world problems? ● I can explain why knowing one acute angle in a right triangle determines the ratio of the side lengths. (O) ● I can determine the side lengths of triangles with 45, 45, and 90 degree angles. (CR) ● I can determine the side lengths of triangles with 30, 60, and 90 degree angles. (CR) ● I can build a table of ratios of side lengths of right triangles. (CR) ● I can use a table of ratios of side lengths of right triangles to estimate unknown angle measures. (CR) ● I can use a table of ratios of side lengths of right triangles to estimate unknown side lengths. (CR)		The goal of lesson one is for students to recognize that the ratio of the legs of a right triangle with a given acute angle is fixed. They are building connections to similar right triangles in the previous unit. Students should leave the lesson wondering how the people who wrote the Americans with Disabilities Act guidelines knew what the ratio of two sides of a right triangle would be for a given angle, or what the angle would be for a given ratio. **Note on language: In previous courses, students may have learned that a ratio is an association between two or more quantities. However in more advanced work, such as this course, ratio is commonly used as a synonym for quotient. This expanded use of the word ratio first came into play in the previous unit on similarity. This usage continues in this unit.** Lessons two and three give students an opportunity to apply their reasoning about similar figures and their skill with the Pythagorean Theorem to make sense of the relationship and understand how they might use a reference diagram to solve problems. Special right triangle relationships give students the experience of knowing an angle measurement and a single side length and being able to figure out the other sides, which previews trigonometry. In addition, knowing some ratios of side lengths for certain angle measures provides a reference point as students begin filling in tables of trigonometric ratios. As students solve multiple problems involving finding the	iM Lessons 1, 2, 3, 4, 5

		lengths of diagonals in similar rectangles, and then squares, they have an opportunity to use repeated reasoning to generalize that the diagonal of a rectangle is always the scale factor times the diagonal of a similar unit rectangle. Students have an opportunity to reason abstractly and quantitatively as they both measure lengths in, and solve analytically, 30-60-90 triangles. In Lesson four students start by measuring side lengths and calculating ratios in several different-sized triangles with the same angle measures. This reinforces students' understanding of similarity. Do not name these ratios yet; the long descriptions are important for students to build understanding. The decision to put the columns in the order that will eventually be named "cosine, sine, tangent" is purposeful. Because cosine represents the x-coordinate in the unit circle, while sine represents the y-coordinate, tables with cosine first correctly correspond to the (x,y) coordinates that students will see later. As students measure side lengths and compute ratios there is an opportunity to discuss measurement error and the relationships between precision in measurement and precision in values calculated with those measurements. In this unit, we recommend rounding side lengths to the nearest tenth and angle measures to the nearest degree. Students are instructed to calculate the ratios of side lengths based on measured lengths to the hundredths place, and, when using digital tools, to use ratios calculated to the thousandths place. When students examine the class table they might notice that: ● angles with larger measures have larger ratios of the opposite side to the adjacent side or hypotenuse ● the larger the ratio of the opposite side to the hypotenuse, the smaller the ratio of the adjacent side to the hypotenuse	

		- the ratio of adjacent side to hypotenuse is equal to the ratio of opposite side to hypotenuse for complementary angles, or angles which sum to 90 degrees. **These observations will be topics in subsequent lessons, so students need not justify their conjectures at this point.** In a previous unit, students found unknown side lengths in right triangles using similarity. In Lesson 5, this is the first time students are expected to use trigonometric ratios rather than a specific similar triangle to find unknown side lengths in right triangles. This is not an easy concept for students, who must grapple with: - recognizing that the right triangle table has useful information for solving the problem - being given ratios within triangles, rather than being able to easily find a scale factor between two triangles - ratios given as a decimal value, rather than a ratio relationship like - the algebraic reasoning required to find a side length that will result in a specified ratio The goal of this lesson is for students to continue to build the understanding that an acute angle measure in a right triangle determines the ratios of its side lengths, and vice versa. Students first use their right triangle table to estimate angle measures given a right triangle with three known side lengths. Then they use the ratios they calculated to find unknown side lengths precisely. Students are not expected to master this skill in this lesson. The goal is to discover the multiple ways students may have for thinking about using the ratios they've computed, and to understand how students connect those ratios to the right triangles they represent. **Throughout this lesson, resist the urge to name any trigonometric ratios.** Continue to refer to descriptions such as, "the ratio of the opposite leg to the adjacent leg."	

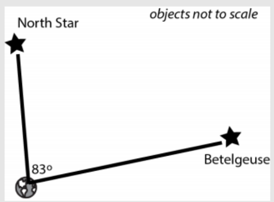
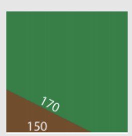
		If students note that the descriptions are long or wonder if there are names for these calculations, explain that mathematicians do have names for them that they'll learn soon. The purpose of continuing to use the long descriptions is so that students understand what the ratios mean and where they come from.	
		• MP1 • MP2 • MP3 • MP4 • MP7 • MP8	
HSG-SRT.C.6 HSG-SRT.C.7 HSG-SRT.C.8 HSN-Q.A.2 *HSG-GMD.A.1*	How can you use trigonometry to solve real world problems? • I can use cosine, sine, and tangent to find side lengths of right triangles. (CR) • I can use cosine, sine, and tangent to find the height of an object. (P) • I can explain why sin(ϴ)=cos(90-ϴ). (CR) • I can use arccosine, arcsine, and arctangent to find angle measures in right triangles. (CR) • I can use trigonometry to solve problems. (CR) • I can explain how to use regular polygons to approximate the value of π. (O)	In lesson six students learn the names of the trigonometric ratios they have been using in the right triangle table and how to look them up on the calculator. Cosine is the ratio of the length of the adjacent leg to the length of the hypotenuse for a given acute angle in a right triangle. Sine is the ratio of the length of the opposite leg to the length of the hypotenuse. Tangent is the ratio of the length of the opposite leg to the length of the adjacent leg. The decision to write the trigonometric ratios in the order “cosine, sine, tangent” is purposeful. Because cosine represents the x-coordinate in the unit circle, while sine represents the y-coordinate, tables with cosine first correctly correspond to the (x,y) coordinates. This will not align with the SOHCAHTOA mnemonic, but it is not expected that students memorize these definitions. They can use their reference chart or the right triangle table for reference at any time. In lesson seven students will reinforce the idea that trigonometry is based on right triangles. So far students have only encountered trigonometry with triangles where the angle is given, so at this point they need one side length and one acute angle measure to calculate the remaining side lengths and angle measures. Later in this	iM Lessons 6, 7, 8, 9, 10, 11

		unit they will learn how to compute angle measures of right triangles given only side lengths. In lesson eight, students do some calculations to remind them of their previous conjectures and then prove $\sin(\Theta)=\cos(90-\Theta)$. In lesson nine students find an angle given two side lengths of a right triangle. They learn to use a calculator to look up the angle corresponding to a ratio of sides and then apply that skill to a variety of problems and a context. As students grapple with the idea of using sides to find angles, they will need to move from concrete to abstract. The arctangent, arccosine, or arcsine of a ratio tells them an angle measure in any right triangle with that particular ratio of side lengths. Students continue to reason abstractly and quantitatively as they apply these ideas to the ladder scenario in the next activity, especially if they do the measurements themselves. Some calculators and texts use arctan and others use $\tan^{-1}$. In lesson ten students apply the concepts of trigonometry to different situations. In lesson ten students build off their concrete calculations from the previous lesson to write a generalized formula for the perimeter of a polygon inscribed in a circle of radius 1.	
		• MP1 • MP2 • MP6 • MP8	

COMMON MISCONCEPTIONS	PRIOR KNOWLEDGE NEEDED TO	ADVANCED STANDARDS FOR STUDENTS	OPPORTUNITIES FOR

	MASTER STANDARDS FOR THIS UNIT	WHO HAVE DEMONSTRATED PRIOR MASTERY	STUDENT-DIRECTED LEARNING WITHIN THE UNIT
• Students may need a reminder of the definition of altitude. • Students may struggle to identify opposite and adjacent legs. Prompt students to highlight the angle in question to accurately identify the sides. • Students may need to be encouraged to draw a diagram for the problem if one is not provided. • If students struggle prompt them to draw a right triangle and label one of the acute angles ϴ. Ask them what the measure of the other acute angle (90-ϴ). Then prompt students to label the sides with any variables.	• 6.G.A.1 • 7.G.A.1 • 7.G.B.4 • 7.G.B.6 • 7.RP.A.2 • 7.EE.B.4 • 8.G.B.6 • 8.G.B.7 • 8.G.C.9		

UNIT 6: Solid Geometry (iM Geometry, Unit 5)			
Illustrative Mathematics Unit Access (Geometry, Unit 7) **Essential Questions:** ● How do you find a cross section, dilation or calculate the scaled area of a solid? ● What happens when you scale a solid? ● How do you find the volume of prisms and cylinders? ● How do you find the volume of a pyramid?			
UNWRAPPED STANDARDS			
Grade Level Standard	**Standard Progression**	**Concepts Big Ideas/ Understandings)**	**Academic Vocabulary (Standard Based)**

HSG-SRT.C.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.	**Modeling with trigonometry** **Vector graphic** A digital artist using a computer drawing program clicked on a diagonal line segment and saw that it measured 216 units horizontally and 45 units vertically. How many units long was the line segment? If the artist wants to rotate the line segment to be vertical, what angle of rotation could be used? Give your answers to the nearest hundredth. *Answers.* 220.64 units long; 11.77° clockwise or 78.23° counter-clockwise. **Flight of the bumblebee** A bumblebee sitting on a tulip wanted to fly over to a daffodil located 100 meters due west. The bumblebee did fly in a straight line, but it mistakenly flew in a direction 5 degrees south of west. The bumblebee passed to the south of the daffodil—how far south? *Answer.* 8.75 meters or about 9 meters. **Star distance** Viewed from Earth, the North Star and Betelgeuse are separated by an angle of about 83 degrees. If the North Star is about 430 light years from Earth, and Betelgeuse is about 640 light years from Earth, then about how far apart are the North Star and Betelgeuse (measured in light years)?  *Answer.* About 726 light years. *Comment.* This could be solved using a G-SRT.10(+) strategy (Law of Cosines), or via G-SRT.8 by dropping a perpendicular to make two right triangles. **Crop Loss** One corner of a soybean field wasn't irrigated, and no soybeans could be harvested from that part of the field. How much money was lost if soybeans sold for $12 a bushel that year and an acre of irrigated land yields 54 bushels of soybeans? Note, 1 acre is approximately 4000 square meters.  *Answer*: $972 or approximately $1000. See `http://tinyurl.com/mtcn6zq`.	• Represent applied problems using right triangles. • Identify relevant trigonometric ratios and apply them to solve problems.	criteria for triangle similarity, similar figures, similarity transformations, trigonometric ratios

HSG-GMD.A.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.	Construct viable arguments for why the formulas for volume learned in middle grades work.	● Consider why the various formulas work, using drawings and models as needed. ● Use similarity to define pi and develop the formula for the circumference of a circle. ● Use a limit argument to develop the area of the circle. ● Use Cavalieri's Principle to explain why the formulas for the volume of a cylinder, pyramid, and cone work. ● Relate the volumes using various solids with the same dimensions	*Cavalieri's Principle, Dissection*
HSG-GMD.A.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.	Use Volume Formulas for cylinders, pyramids, cones, and spheres to solve problems	● Problems involving real world objects can be solved by modeling using the properties of geometric solids	
HSG-GMD.B.4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.	Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.	● Geometric solids can be formed by rotating two-dimensional figures. ● The cross-sections of three dimensional objects form a variety of two dimensional shapes.	Cross-section of a solid, Rotational Solid
HSG-MG.A.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).	Any mathematical object that represents a situation from outside mathematics and can be used to solve a problem about that situation is a mathematical model. Modeling often involves making simplifying assumptions that ignore some features of the situation being modeled. In geometry, in order to study how the illuminated percentage of the moon's surface varies during a month, students might represent the moon as a rotating sphere, half black and half white.	● Use geometric shapes to model real world situations. ● Use properties of those shapes to draw conclusions about the real world situations.	density

HSG-MG.A.2 Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).	In this standard, students explore contexts involving density, a compound unit comparing some quantity per unit of area or volume.	● Analyzing and understanding the compound units of density problems is essential and can suggest solution strategies for mathematical modeling problems. ● Draw conclusions about real-world situations and assess their reasonableness.	Density
HSG-MG.A.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).	A wide range of problems might be addressed under this standard, related to different standards from throughout this conceptual category. Students' focus should be designing a solution that meets the constraints of a real-world context.	● Use geometry to model real world situations in order to design solutions to real world problems. ● Justify the decision made to demonstrate that the resulting design addresses the contexts.	Density
HSA-CED.A.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	[Students use complex equations—including equations arising from linear and quadratic expressions, and simple rational and exponential expressions—to model] relationships between quantities with equations in two variables. All the standards in the Creating Equations group carry a modeling star, denoting their connection with the Modeling category in high school. This connotes not only an increase in the complexity of the equations studied, but an upgrade of the student's ability in every part of the modeling cycle.	● Create an equation, inequality or system to model a situation with two variables. ● Choose correctly from among linear functions, exponential functions and others as appropriate for modeling a situation. ● Explain their reasoning in steps when creating an equation, inequality, or system of equations or inequalities.	Constraints, linear equation, quadratic equation, system of equations
HSA-SSE.A.1.a Interpret parts of an expression, such as terms, factors, and coefficients. HSA-SSE.A.1.b Interpret complicated expressions by viewing one or more of their parts as a	The middle grades standards in Expressions and Equations build a ramp from arithmetic expressions in elementary school to more sophisticated work with algebraic expressions in high school. As the complexity of expressions increases, students continue to see them as being built out of basic operations; they see expressions as sums of terms and products of factors.	● Students see that complicated expressions are built up out of simpler ones. ● Explain what factor, coefficient, term, and like term mean in the context of expressions, ● Identify individual parts of an expression as a single entity to make use of the structure of	Terms, Factors, Coefficients, Expression,

single		the expression	
HSF-IF.C.7.b Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.	Functions are often studied and understood as families, and students should spend time studying functions within a family, varying parameters to develop an understanding of how the parameters affect the graph of a function and its key features. Within a family, the functions often have commonalities in the shapes of their graphs and in the kinds of features that are important for identifying functions more precisely within a family. This standard indicates which function families should be in students' repertoires, detailing which features are required for several key families. [F-IF.C.7] is an overarching standard that covers the entire range of a student's high school experience. In most of the other function families [those that are not linear, quadratic, or exponential] students are expected to graph simple cases without technology, and more complex ones with technology.	● Identify from graphs and use when graphing the characteristics of square root and cube root functions (shape, intercepts, and end behavior). ● Explain an efficient process for graphing piecewise functions and for determining their domain and range.	Maximum, Minimum,
HSN-Q.A.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.	Reasoning quantitatively includes knowing when and how to convert units in computations, such as when adding and subtracting quantities that measure the same attribute but are expressed in different units and other computations with measurements in different units or converting units for derived quantities such as density and speed. Reasoning quantitatively can also include analyzing the units in a calculation to reveal the units of the answer. This can help reveal a mistake if, for example, the answer comes out to be a distance when it should be a speed (MP.2). Students should specify units when defining variables and attend to units when writing expressions and equations (MP.6).	● Solve contextual problems and multi-step problems and explain how units were used to understand the problems. ● Determine and/or explain why a specific scale was chosen for a graph. ● When given a formula, determine the correct units to be used with the formula and explain their reasoning.	Unit, formula, scale

UNIT 6: Solid Geometry (iM Geometry, Unit 5)				
CCSS Standards #	**Learning Targets:** I can	**Assessment Strategy** SR-Selected Response CR-Constructed Response P-Performance O-Observation (behavioral)	**Lesson Progression and Connection to Math Practices**	**Common Learning Experiences and Assessments**
HSG-GMD.B.4 HSG-GMD.A.1 HSF-IF.C.7.b HSA-CED.A.2 HSA-SSE.A.1.a	How do you find a cross section, dilation or calculate the scaled area of a solid? (Lessons 1 - 5) ● I can draw the two-dimensional shape that creates a particular three-dimensional solid when rotated using a given axis. (CR) ● I can identify the three-dimensional solid created by rotating a two-dimensional figure using a linear axis. (SR) ● I can identify the three-dimensional shape that generates a set of cross sections. (SR) ● I can visualize and draw multiple cross sections of a three-dimensional figure. (CR) ● I know that a pyramid's cross sections are dilations of its base with scale factors ranging from 0 to 1. (O) ● I know that when figures are dilated by a scale factor of k, their areas are multiplied by k^2. (CR) ● I can use square root graphs and do calculations to interpret the relationships between scale factors and areas. (CR)		● In lesson 1, students visualize solids of rotation created from rotating two-dimensional figures using an axis of rotation. These visualization skills will be useful in later lessons when students need to analyze two-dimensional figures that are cross sections of three-dimensional figures. For example, sometimes it is necessary to consider vertical cross sections of cones or pyramids to solve problems involving volume. ● In lesson 2, students analyze cross sections, or the intersections between planes and solids, by slicing three-dimensional objects. Next, they identify three-dimensional solids given parallel cross-sectional slices. In addition, they revisit solid geometry vocabulary terms from earlier grades: sphere, prism, cylinder, cone, pyramid, and faces. Understanding the relationship between solids and their parallel cross sections will be critical to understanding Cavalieri's Principle in later lessons. Cavalieri's Principle will be applied to the development of the formula for the volume of pyramids and cones ● In lesson 3, students create dilations of a rectangle and suspend them to resemble cross sections of a pyramid. They learn that given a pyramid's base, its cross sections are dilations of the base with scale factors between 0 and 1. These activities help build the spatial visualization skills and familiarity with cross sections they will need to derive volume formulas later in the unit. For example, students will later use dilations and cross sections to conclude that pyramids of the same height and with bases of equal area have equal volumes, regardless of the particular	iM Lessons 1, 2, 3, 4, 5

		shapes of the bases. ● In lesson 4, students analyze the result of scaling on area. This concept will be essential to creating a volume formula for pyramids later in the unit. Additionally, students will encounter the graph representing the equation $y = \sqrt{x}$ in a geometric context when, in upcoming lessons, they analyze the relationship between scaled areas and factors of dilation. This will build on work students did in grade 8 evaluating square roots of small perfect squares and determining that $\sqrt{2}$ is irrational. ● In lesson 5, students practice working with areas of scaled figures, connecting them to cross sections and encountering a common misconception. Then, they create a graph representing $y = \sqrt{x}$and use it to answer questions about a situation. They analyze the shape of the graph to better understand the relationship between an area and the scale factor needed to achieve it.	
		● MP6, MP2	
HSA-SSE.A.1 HSF-IF.C.7.b HSN-Q.A.1 HSA-CED.A.2 HSG-MG.A.3 HSN-Q.A.1 HSG-GMD.A.1 HSG-GMD.B.4	What happens when you scale a solid? (Lessons 6 - 8) ● I know that when a solid is dilated by a scale factor of *k*, its surface area is multiplied by k^2 and its volume is multiplied by k^3. (CR) ● I can create and describe graphs that show relationships between volumes and scale factors. (CR) ● I can work backwards from a volume or surface area scaling to find a scale factor. (CR) ● I can calculate scale factors for lengths, surface areas, and volumes if I'm given any 1 of the 3 factors. (CR)	● In lesson 6, students continue their analysis of the connections between geometric measurement and dimension by studying the results of dilating a three-dimensional solid. In grade 6, students learned that the formulas $V = lwh$ and $V = Bh$ can be used to find the volume of a right rectangular prism. In this lesson's warmup, students revisit that concept and apply it to cubes. They extend their knowledge of dilated areas to surface areas, and they investigate the relationship between scale factor and volume to find that dilating by a scale factor of *k* multiplies the volume by k^3. This will lead to students creating and interpreting a graph representing in an upcoming lesson. ● In lesson 7, students work backwards from the volumes of original and scaled solids to calculate scale factors. To illustrate the relationship between	iM Lessons 6, 7, 8

		volume and scale factor, students create a graph of the cube root equation $y = \sqrt[3]{x}$ based on a situation arising from a geometric context. They use the graph to analyze rates of change in scale factor for different volume inputs. Then, they solve a design problem, using cube roots and square roots to find particular scale factors. This builds on work from grade 8, in which students evaluated cube roots of small perfect cubes and used rational approximations of irrational numbers. ● In lesson 8, students apply their knowledge of the relationships between original and scaled figures. Students decide what information is necessary to move backwards and forwards between lengths, surface areas, and volumes of an original solid and its dilation. They communicate with each other to obtain the data they need to solve a problem.	
		● MP1, MP2	
HSA-SSE.A.1.b HSG-GMD.A.1 HSG-GMD.A.3 HSG-GMD.B.4 HSG-MG.A.1 HSG-SRT.C.8	How do you find the volume of prisms and cylinders? (Lessons 9 - 11) ● I can calculate volumes of solids that are composed of cylinders. (CR) ● I can explain how finding the volume of a prism relates to finding the volume of a cylinder. (O) ● I know that if two solids have equal-area cross sections at all heights, they have the same volumes. (CR) ● I can calculate volumes of right and oblique prisms and cylinders and figures composed of prisms and cylinders. (CR)	● In lesson 9, students recall how to calculate the volume of a cylinder, using informal arguments to compare the volume of a cylinder to the volume of a prism that has an equal height and area of its base. Students start to think about volume in 1-unit layers. This lays a foundation for the informal derivation of the pyramid formula, which will rely on consideration of area of cross sections. Finally, students apply cylinder volume calculations to a solid of rotation. ● In lesson 10, students develop the idea of oblique versus right solids. They analyze volumes of two prisms: one right and one oblique, but of equal height and with bases that have equal area. They conclude the volumes of the two prisms are equal. This leads to the introduction of Cavalieri's Principle, or the idea that solids of equal height have the same volume if their cross sections have equal area at all heights. This concept is needed for understanding volumes of oblique solids and will also be used to develop the	iM Lessons 9, 10, 11

		formula for the volume of a pyramid in upcoming lessons. ● Lesson 11 gives students the opportunity to practice and apply what they have learned about volumes of prisms. They use trigonometry and the Pythagorean Theorem to find missing measurements, and they use decomposition to find volumes of more complex solids.	
		● MP2, MP7	
HSG-GMD.A.1 HSG-GMD.A.3 HSG-MG.A.1 HSG-MG.A.3 HSG-GMD.B.4	How do you find the volume of a pyramid? (Lessons 12 - 15) ● I can explain the relationships between pyramids, cones, prisms, and cylinders. (O) ● I can explain why the volume formula for pyramids and cones is $V = \frac{1}{3}Bh$. (O) ● I can calculate volumes of pyramids and cones. (CR) ● I can work backward from a given volume to find possible dimensions of a pyramid or cone. (CR) ● I can use the Pythagorean Theorem and trigonometry to help calculate volumes of prisms, cylinders, cones, and pyramids, including solids of rotation. (CR)	● In lesson 12, students analyze relationships between prisms and pyramids. They categorize pyramids and cones as solids with a single base and an apex or central vertex, as opposed to cylinders and prisms which have 2 congruent bases. They extend their understanding of the terms right and oblique to apply to cones and some pyramids. Then, students assemble a triangular prism out of 3 triangular pyramids, and they make initial observations about these solids. The pyramids they build will be used in the next lesson to develop the formula for the volume of a pyramid. ● In lesson 13, students use informal arguments to show that the volume of any pyramid is one-third the volume of the prism with equal height and congruent base. They use cross section and dilation arguments from earlier lessons to show that Cavalieri's Principle applies to the two solids, and therefore the volumes are equal. Because any pyramid or cone can be shown to have the same volume as a triangular pyramid with the same height and a base with equal area, the expression $\frac{1}{3}Bh$ extends to all pyramids and cones. ● In lesson 14, students calculate the volume of several pyramids, and they work backward to find possible dimensions of a pyramid given its volume. ● In lesson 15, students apply their knowledge of pyramid, cone, prism, and cylinder volume formulas.	iM Lessons 12, 13, 14, 15

		Trigonometry and the Pythagorean Theorem are incorporated as additional tools. Students practice thinking through the methods they will use to solve a problem before starting their calculations. Finally, they calculate the volume of a solid of rotation composed of a cylinder and a cone.	
		• MP6, MP1, MP7	
HSN-Q.A.1 HSF-IF.C.7.b HSG-MG.A.1 HSG-MG.A.2 HSG-MG.A.3	Putting it All Together (Lessons 16 - 18) • I can use surface area and volume relationships to solve problems. (CR) • I can solve problems involving density and volume. (CR) • I know that the density of an object is the ratio between its mass and its volume. (O) • I can use cube root and square root graphs to solve geometric problems. (CR)	• The purpose of lesson 16 is for students to strengthen their understanding of geometric measurement and dimension by analyzing relationships between surface area and volume in application problems. Students will explore properties of shapes that maximize surface area for a given volume. Then they study surface area to volume ratios of animals with different shapes and sizes. • In lesson 17, students apply volume problem-solving concepts to situations involving density, which is defined as the mass of a substance per unit volume. Density is an important concept for many modeling applications. • In lesson 18, students analyze the graphs of cube root and square root equations in geometric contexts. This work builds on earlier lessons in the unit that analyzed square root and cube root graphs. Students also use concepts of rearranging formulas to highlight a quantity of interest and understanding the nature of the intersection points of the graphs of two equations.	iM Lessons 16, 17, 18
		• MP1, MP6, MP4	

COMMON MISCONCEPTIONS	PRIOR KNOWLEDGE NEEDED TO MASTER STANDARDS FOR THIS UNIT	ADVANCED STANDARDS FOR STUDENTS WHO HAVE DEMONSTRATED PRIOR	OPPORTUNITIES FOR STUDENT-DIRECTED LEARNING WITHIN

		MASTERY	THE UNIT
● Students may have difficulty with multiple-step problems. ● Students may struggle with problems that involve different units of measure within the same problem.	8.G.C.9 7.G.A.3 7.G.B.3 7.G.B.4 8.EE.C.8 8.F.B.5		

UNIT 7: Coordinate Geometry (iM Geometry Unit 6)

Illustrative Mathematics Unit Access (Geometry, Unit 6)

Essential Questions:
- What does it mean for an image to be congruent or similar?
- How can I find the equation of a circle?
- What can I learn from the structure of an equation or graph?

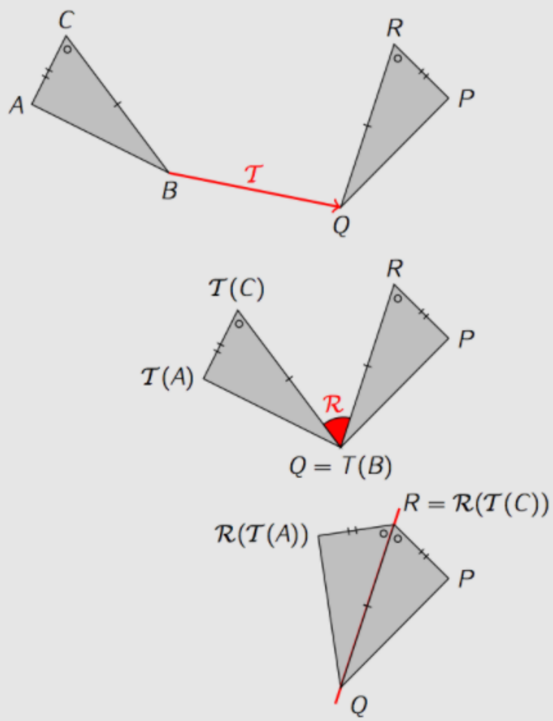
UNWRAPPED STANDARDS

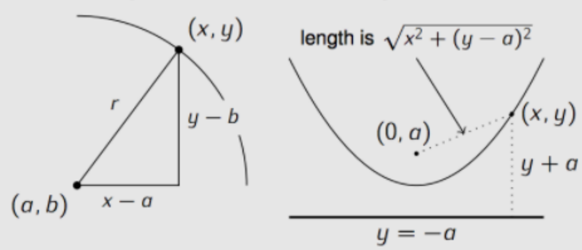
Grade Level Standard	Standard Progression	Concepts Big Ideas/ Understandings	Academic Vocabulary (Standard Based)
HSN-Q.A.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.	Reasoning quantitatively includes knowing when and how to convert units in computations, such as when adding and subtracting quantities that measure the same attribute but are expressed in different units and other computations with measurements in different units or converting units for derived quantities such as density and speed. Reasoning quantitatively can also include analyzing the units in a calculation to reveal the units of the answer.	Understanding and attending to the units in a problem can help interpret the problem and suggest a solution strategy.	Unit
HSA-SSE.A.1.a Interpret parts of an expression, such as terms, factors, and coefficients.	The middle grades standards in Expressions and Equations build a ramp from arithmetic expressions in elementary school to more sophisticated work with algebraic expressions in high school. As the complexity of expressions increases, students continue to see them as being built out of basic operations; they see expressions as sums of terms and products of factors.	● Students see that complicated expressions are built up out of simpler ones. ● Explain what factor, coefficient, term, and like term mean in the context of expressions, ● Identify individual parts of an expression as a single entity to make use of the structure of the expression	Terms, Factors, Coefficients, Expression,
HSA-SSE.A.2 Use the structure of an expression	Seeing structure in expressions entails a dynamic view of an algebraic expression, in which potential rearrangements and	● Explain, in their own words, how specific structures are	Maximum, Minimum, Quadratic Expression, Term, Zero of a

to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.	manipulations are ever present. An important skill for college readiness is the ability to try possible manipulations mentally without having to carry them out, and to see which ones might be fruitful and which not.	seen in different expressions. ● Rewrite expressions using structure to identify important components of the expression (where zeros may occur) or to end behavior.	function, coefficient, distributive property of addition over multiplication, equivalent equations, equation, exponent, expression, factor
HSA-SSE.B.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.	The Standards emphasize purposeful transformation of expressions into equivalent forms that are suitable for the purpose at hand. The Standards avoid talking about simplification, because it is often not clear what the simplest form of an expression is, and even in cases where that is clear, is in not obvious that the simplest form is desirable for a given purpose. [T]here are three commonly used forms for a quadratic expression: • Standard form, e.g., $x^2 - 2x - 3$ • Factored form, e.g., $(x + 1)(x - 3)$ • Vertex form (a square plus or minus a constant), e.g. $(x - 1)^2 - 4$ Rather than memorize the names of these forms, students need to gain experience with them and their different uses. The traditional emphasis on simplification as an automatic procedure might lead students to automatically convert the second two forms to the first, rather than convert an expression to a form that is useful in a given context.	● Explain how they used equivalent forms of quadratic expressions to determine important components of a quadratic function (and its graph). ● Solve contextual problems using equivalent forms of expressions. For example, students may find extrema, end behavior, growth factors, or decay factors.	Maximum, Minimum, Quadratic Expression, Term, Zero of a function, coefficient, distributive property of addition over multiplication, equivalent equations, equation, exponent, expression, factor, geometric series
HSA-CED.A.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	[Students use complex equations—including equations arising from linear and quadratic expressions, and simple rational and exponential expressions—to model] relationships between quantities with equations in two variables. All the standards in the Creating Equations group carry a modeling star, denoting their connection with the Modeling category in high school. This connotes not only an increase in the complexity of the equations studied, but an upgrade of the student's ability in every part of the modeling cycle.	● Create an equation, inequality or system to model a situation with two variables. ● Choose correctly from among linear functions, exponential functions and others as appropriate for modeling a situation. ● Explain their reasoning in steps when creating an equation, inequality, or system of equations or inequalities.	Constraints, linear equation, quadratic equation, system of equations

HSA-REI.C.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line y = –3x and the circle $x^2 + y^2 = 3$.	Another important method of solving systems is the method of substitution. Again this can be understood in terms of simultaneity; if (x, y) satisfies two equations simultaneously, then the expression for y in terms of x obtained from the first equation should form a true statement when substituted into the second equation. Since a linear equation can always be solved for one of the variables in it, this is a good method when just one of the equations in a system is linear.	● Solve a system consisting of a linear equation and a quadratic equation graphically. ● Solve a system consisting of a linear equation and a quadratic equation algebraically. ● Describe systems consisting of a linear equation and a quadratic equation that have no solution, one solution, or two solutions.	Intersection, Inverse, Matrix, Quadratic Equations, System of Equations
HSG-CO.A.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.	Students in high school start to formalize the intuitive geometric notions they developed in Grades 6–8. For example, in Grades 6–8 they worked with circles and became familiar with the idea that all the points on a circle are the same distance from the center. In high school, this idea underlies the formal definition of a circle: given a point O and a positive number r, a circle is the set of all points P in the plane such that $\|OP\|$= r. This definition will be important in proving theorems about circles, for example the theorem that all circles are similar.	● Challenge existing ideas of basic geometric terms in order to provide more precise definitions. ● Build understanding of formal definitions for later use in geometric proof.	Circle, Corresponding parts, Definition, geometric transformation, rigid motion, symmetric, symmetry, undefined notions or terms
HSG-CO.A.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).	Students should develop a general understanding of what a transformation is, exploring a range of both rigid and non-rigid motions.	● Rigid Transformations preserve distance and angle measures, but dilations only preserve angle measures. ● Transformations are Functions	Circles, Corresponding parts, Dilation, Geometric transformation, Image, Pre-image, rigid motion, Similarity transformation
HSG-CO.A.5 Given a geometric figure and a rotation, reflection, or	In Grade 8, students described sequences of rigid motions informally and in terms of coordinates. An important step forward in high school is to give precise descriptions of	● Develop accurate methods for carrying out the rigid motions using a variety of tools.	Circle, corresponding parts, definition, geometric transformation, rigid motion,

translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.	sequences of rigid motions that carry one figure onto another. Each rigid motion must be specified: For each rotation, a specific point and angle must be given; each translation is determined by a pair of points; and each reflection by a specific line, known as the line of reflection. These points, lines and angles must be described in terms of the two figures.	● Look for patterns in what happens when carrying out sequences of rigid motions. ● Find rigid motions needed to map a pre-image to a particular image.	symmetric, symmetry, undefined notions or terms

	Proving congruence  Suppose that the corresponding sides and corresponding angles of $\triangle ABC$ and $\triangle PQR$ are equal. First translate $\triangle ABC$ along the line segment BQ, so that $\mathcal{T}(B) = Q$. Then rotate clockwise about Q through the angle $\angle \mathcal{T}(C)QR$. Because translation and rotation preserve distance, we have $R = \mathcal{R}(\mathcal{T}(C))$. Now reflect across the line through R and Q. Because the rigid motions preserve angles, the line through R and P coincides with the reflection of the line through R and $\mathcal{R}(\mathcal{T}(A))$, and because they preserve distance the point P coincides with the reflection of $\mathcal{R}(\mathcal{T}(A))$. Now all three points of the triangle coincide, so we have produced a sequence of rigid motions that maps $\triangle ABC$ onto $\triangle PQR$, and they are therefore congruent.		
HSG-SRT.B.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships	There is also another advantage of [the transformation] approach to congruence and similarity. Because most of the theorems in plane geometry before the introduction of similarity depend only on the three triangle congruence criteria, once these have been established, it is possible to	• Develop facility in using triangle congruence and similarity to explain geometry relationships. • Continue to use geometric	criteria for triangle similarity, similar figures, similarity transformations

in geometric figures.	transition into the traditional way of proving theorems at this point, without further use of basic rigid motions, if so desired. The use of dilations to treat similarity can likewise be limited to the initial stage if so desired.	transformations as a way to visualize the relationships they find.	
HSG-C.A.2 Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.	This standard addresses a number of common theorems involving circles and arcs. STudents will benefit from first having an opportunity to explore various properties of circles to make conjectures about how they are related and then to justify why they are true.	• Explore various properties related to circles. • Form conjectures about the relationships they find. • Develop justifications for why their conjectures work.	Arc measure, central angles, cord, circumference, circumscribed, circumscribed angle, incenter, inscribed, inscribed angle, radius
HSG-GPE.A.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.	**Equations for circles and parabolas**  Applying the Pythagorean Theorem to the triangle on the left yields the equation for a circle, $(x-a)^2+(y-b)^2=r^2$. On the right is parabola, defined geometrically by the condition that a point on the parabola is equidistant from the focus (at $(0, a)$ and the directrix (the line $y=-a$). Setting these two distances equal and squaring both sides yields $x^2+(y-a)^2=(y+a)^2$, which reduces to the familiar equation $y=(1/4a)x^2$.	• Use the Pythagorean THeorem or distance formula to find the equation of a circle with a given center and radius. • See the relationship between the distance formula and the Pythagorean Theorem. • Develop the general formula for the equation of a circle. • Given the general form for a circle, compare the square to find its center and radius.	Distance formula, ellipse, hyperbola, parabola
HSG-GPE.B.4 Use coordinates to prove simple geometric theorems algebraically. For example,	[When working with two points on a line, and the right triangle whose hypotenuse is the line segment between the two points and whose legs are parallel to the axes] the Pythagorean Theorem applies. The length of the hypotenuse is the distance	• Identify properties and how they can be expressed using coordinates. • Demonstrate why a property	Distance formula, slope criterion for parallel lines, slope criterion for perpendicular lines

prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).	between the two points, and the lengths of the legs can be calculated as differences between the coordinates. The algebraic manifestation of the Pythagorean Theorem is the formula for the distance, d, between the two points: d=√(x2-x1)2+(y2-y1)2. Students can use the distance formula to prove simple facts about configurations of points in the plane.	holds using algebra to show the relationships among the coordinates.	
[HSG-GPE.B.5](#) Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).	The relationship between the slopes of parallel and perpendicular lines is a nice example of the interplay between geometry and algebra.	● Recognize that the slopes of parallel lines are equal, since they are related by a translation. ● Recognize that the slopes of perpendicular linear are negative reciprocals, since they are related by a 90° rotation. ● Apply criteria of parallel and perpendicular lines to solve problems.	Distance formula, slope criterion for parallel lines, slope criterion for perpendicular lines
[HSG-GPE.B.6](#) Find the point on a directed line segment between two given points that partitions the segment in a given ratio.	Once again, the relationship discussed in this standard can be visualized as a transformation - in this case, a dilation. For example, suppose we want to find point D that divides the directed segment from A to B in a 1:3 ratio. Then triangle ADE is a dilation of triangle ABC using a ¼ scale factor, since AB will be 4 parts total, and the initial segment of 1 part serves as the image. Since AC=4 and BC=2, then AE=¼*4, and DE = ¼*2. Thus, the coordinates of E will be (1+1,2) and D will be (1+1, 2+½). This approach can be generalized algebraically. Express the ratio a:b as a/(a+b) to show the fraction of the segment, which will be the scale factor from the whole segment to the part from the initial point of the directed segment, which will also be the center of dilation. To find the coordinates of the image, multiply the scale factor by the directed distances in the x and y directions, found by subtracting the endpoints of the segment, and add the resulting distance to the initial endpoint, the image point will divide the directed segment as desired.	● Use dilations to find the coordinates of a point that divide a directed segment in a given ratio.	Distance formula, slope criterion for parallel lines, slope criterion for perpendicular lines
[HSG-GPE.B.7](#) Use coordinates to compute	This standard is fairly self-evident. For example, consider the rectangle. Using the distance formula, AB=, and AD = $\sqrt{40}$=2	● Use the distance formula to find relevant lengths and apply	Distance formula, slope criterion for parallel lines, slope criterion for

perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.	$\sqrt{10}$. Thus, the perimeter will be $2(\sqrt{10} + 2\sqrt{10}.) = 6\sqrt{10}$ Likewise, the area will be $\sqrt{10} * 2\sqrt{10} = 20$.	formulas for finding perimeter and area of polygons whose vertices are given using coordinates. ● Decompose more complex figures whose vertices are given using coordinates into familiar shapes that are easier to work with.	perpendicular lines

UNIT 7: Coordinate Geometry (iM Geometry Unit 6)				
CCSS Standards #	**Learning Targets:** I can	**Assessment Strategy** SR-Selected Response CR-Constructed Response P-Performance O-Observation (behavioral)	**Lesson Progression and Connection to Math Practices**	**Common Learning Experiences and Assessments**
HSG-CO.A.2 HSG-CO.A.5 HSG-SRT.B.5	What does it mean for an image to be congruent or similar? ● I can prove triangles are congruent using coordinates. (P) ● I can reflect, rotate, and translate figures in the coordinate plane. (O) ● I can use coordinate transformation notation to take points in the plane as inputs and give other points as outputs. (CR) ● I can determine whether a transformation produces congruent or similar images (or neither). (SR/CR)		● Lesson 1 connects ideas from several previous units and extends them to the coordinate plane. In grade 8, students applied the Pythagorean Theorem to find the distance between two points in a coordinate system. Here, students calculate side lengths and angle measures, proving triangles are congruent. Students also draw and specify sequences of rigid transformations in the plane. The goal is to prepare students to see transformations as functions using a new coordinate transformation notation that they will encounter in upcoming lessons. The notion of using the Pythagorean Theorem to calculate distances is a foundational idea that will reoccur in several lessons. ● In lesson 2, students use coordinate transformation notation such as (x, y) ⟶ (x+1, y+2). This notation supports the understanding of	iM Lessons 1, 2, 3

		transformations as functions while also providing a novel context to have students examine rigid transformations, similarity transformations, and transformations that do not fit any vocabulary students have learned. ● The goal of lesson 3 is for students to describe transformations in the coordinate plane, with a focus on identifying transformations as producing congruent or similar figures (or neither). Students will need to write their own rules for given transformations.	
		● MP2 ● MP7	
HSG-CO.A.1 HSG-GPE.A.1 HSG-GPE.B.4 HSA-SSE.A.2 HSA-SSE.B.3	How can I find the equation of a circle? ● I can derive an equation for a circle in the coordinate plane. (P) ● I understand how squared binomials relate to the equation of a circle. (O) ● I can complete the square to find the center and radius of a circle. (CR)	● In lesson 4, students repeatedly test whether points are on a circle by finding the distance between the points and the circle's center. This lesson requires more algebraic fluency than the previous lessons in this course, as do the next several lessons. Depending on students' familiarity and comfort with the algebra skills involved, it might make sense to spend more than one day on some lessons. ● In grade 6, students applied the distributive property to produce equivalent expressions. In lesson 5, students build on that, looking for regularity in repeated reasoning to identify and factor trinomials that are perfect squares. Then, students rewrite perfect square trinomials in factored form to find a circle's center. This will lead to completing the square to identify a circle's radius and center in upcoming activities. ● In lesson 6, students complete the square to find the center and radius of circles. First, they look at a partially complete problem. They analyze what's already been done and decide what steps must be taken to finish. Then, they execute the process on their own with no scaffolding.	iM Lessons 4, 5, 6

		- MP1 - MP7 - MP8	
HSA-SSE.A.1.a HSG-GPE.B.4 HSG-GPE.B.5 HSG-GPE.B.6 HSG-GPE.B.7 HSN-Q.A.1 HSG-CO.A.2 HSG-C.A.2 HSA-CED.A.2 HSA-REI.C.7	What can I learn from the structure of an equation or graph? - I can use the definition of slope to write the equation for a line in point-slope form. (CR) - I can prove that the slopes of parallel lines are equal. (CR) - I can use slopes of parallel lines to solve problems. (CR) - I can prove that the slopes of perpendicular lines are opposite reciprocals. (O) - I can use slopes of perpendicular lines to solve problems. (CR) - I can gather information about a line and write its equation. (CR) - I can use a graph to find the intersection points of a line and a circle. (CR) - I can use coordinates of figures to prove geometric theorems. (O) - I can calculate the coordinates of a point on a line segment that partitions the segment in a given ratio. (CR)	- In grade 8, students used similar triangles to explain why the slope *m* is the same between any two distinct points on a non-vertical line in the coordinate plane, and they derived the equation $y = mx + b$ for a line intercepting the vertical axis at *b*. In lesson 9, students develop the **point-slope form** of a linear equation: $y - k = m(x - h)$. Students will be writing equations of lines in the next several lessons, and intercepts will not always be readily available. Point-slope form will require the least algebraic manipulation and allow students to focus on geometric properties. Slope calculations are an important part of this lesson. - In previous courses, students studied slopes of lines, and in previous units, students studied parallel lines. In lesson 10, students connect these ideas and prove that non-vertical parallel lines have equal slopes. They begin class by noticing the slopes of translated lines are equal and recalling that translated lines are parallel. Then they deconstruct a proof of the slope criterion for parallel lines and explain each step. In this process, they are taking a compact mathematical statement and constructing a viable argument (MP3) to communicate the ideas more clearly to themselves and their peers. Once students are convinced parallel lines have equal slopes, they apply this theorem to write equations and prove a quadrilateral is a parallelogram. - In previous courses, students studied slopes of lines. In previous units, students studied perpendicular lines. In lesson 11, students connect these ideas and prove that non-vertical and non-horizontal perpendicular lines have slopes with opposite reciprocals—that is, that	iM Lessons 9, 10, 11, 12, 13, 14, 15

		they have a product of -1. Students begin by rotating a figure 90 degrees. Then they collect data about the slopes of the perpendicular lines in the original figure and the rotated image. They use their data to make a conjecture about slopes of perpendicular lines. Finally, they have an opportunity to construct a viable argument (MP3) by proving their conjecture is true for all lines. ● In lesson 12, students have the opportunity to attend to precision in thinking and language (MP6) as they determine what information is needed to graph a line and ask precise questions to elicit that information. Then, they apply concepts of parallel and perpendicular lines to conclude that if two lines are both perpendicular to the same line, they must be parallel. ● In lesson 13, students combine concepts of geometry and algebra to find solutions to a system of equations consisting of a linear and a quadratic equation. Students begin by considering the number of ways in which a circle and a line can intersect. Then, they solve 2 simple systems graphically, using algebraic methods to verify their estimates of the solutions. Finally, they write their own equations that meet certain constraints. ● In lesson 14, students use coordinates to make conjectures and prove simple geometric theorems algebraically. They begin with some informal reasoning in a “Which One Doesn’t Belong” prompt. In the next activity, students use slopes to classify a quadrilateral. Then, they use inductive reasoning to observe a pattern and make a conjecture which will be generalized in a subsequent unit. Students have an opportunity to attend to precision in mathematical language (MP6) as they write and refine their conjectures. At the end of the lesson, students critique each other’s reasoning (MP3) about properties of the quadrilaterals from the warm-up.	

		• In lesson 15, students partition segments in given ratios. They begin by calculating midpoints, then attempt more difficult ratios in the subsequent activities. They're introduced to a weighted average and asked to connect weighted averages to segment partitioning. Finally, they link segment partitioning to concepts of similarity transformations from earlier units. In each case, students are presented with a problem with little direction and expected to make sense of the problem and find a strategy on their own (MP1).	
		• MP1 • MP2 • MP3 • MP6	

COMMON MISCONCEPTIONS	PRIOR KNOWLEDGE NEEDED TO MASTER STANDARDS FOR THIS UNIT	ADVANCED STANDARDS FOR STUDENTS WHO HAVE DEMONSTRATED PRIOR MASTERY	OPPORTUNITIES FOR STUDENT-DIRECTED LEARNING WITHIN THE UNIT
• If students are stuck, suggest the use of tracing paper. • If students are stuck on finding the measures of the angles, suggest they look at their reference chart for concepts from a prior unit that can help. • If students struggle to keep their work organized, suggest they create a table of inputs and outputs, or create another organizational structure that works for them. • If students suggest that figures are congruent or similar simply because they look that way, tell them that they need to provide more backing for their answer.	• 4.G.A.1 • 4.MD.C.5 • 4.G.A.2 • 6.EE.A.2 • 6.EE.A.3 • 6.EE.B.7 • 6.EE.C.9 • 6.RP.A.3 • 7.EE.A.1 • 7.EE.A.2 • 7.RP.A.1 • 8.EE.A.3 • 8.EE.A.4 • 8.EE.B.5 • 8.EE.B.6 • 8.EE.C.8 • 8.F.A.1 • 8.F.B.4		

What are some ways we can verify that 2 figures are congruent or similar? Remind students that for triangles, they've learned some shortcuts that they can use here. • If students struggle to write a rule, ask them to start by writing out the pattern they see in words. For example, they may write, "The x-coordinate stays the same and the y-coordinate doubles." Then ask how they could put those words into coordinate transformation notation. • If students aren't sure of the definition of a trinomial, remind them that a binomial is an expression that has 2 terms. How might that relate to a trinomial? • Students may struggle to decide whether the coordinates of the circles' centers are positive or negative. Encourage them to rewrite the equation in the form (x-h)2 + (y-k)2 =r2. Remind them that we subtract the coordinates of the center from the given point (x,y) to get the distance between the center and the point. • If students get stuck, suggest that they look at the form for the equation of a circle that's on their reference chart. Ask them why that form is useful and how it relates to what they've done in recent activities. • Students can look in their	• 8.F.B.5 • 8.G.A.1 • 8.G.A.2 • 8.G.A.3 • 8.G.B.8 • 8.SP.A.2 • 8.SP.A.3		

reference chart for ways to prove a quadrilateral is a parallelogram. ● If students aren't sure how to graph lines, remind them they can use slope triangles or they can find intercepts to help graph the line. Ask how many points are needed to draw the graph of a line (just 2). ● Students may need to be reminded that a product is the result of multiplication. ● Some students may state that a quadrilateral is a rectangle simply because it looks like one. Remind these students that we need to back up our reasoning with mathematics. Suggest students review their reference charts for definitions and properties of rectangles. ● If students don't remember the term midpoint, remind them that it is the point that partitions the segment exactly in half.			

UNIT 8: Circles (iM Geometry, Unit 7)
Illustrative Mathematics Unit Access **(Geometry, Unit 7)** **Essential Questions:** ● What relationships exist between lines, angles and circles? ● How can polygons and circles relate? ● How do we measure circles?

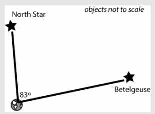
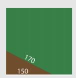
UNWRAPPED STANDARDS

Grade Level Standard	Standard Progression	Concepts (Big Ideas/ Understandings)	Academic Vocabulary (Standard Based)
HSA-SSE.A.1 Interpret expressions that represent a quantity in terms of its context.	The middle grades standards in Expressions and Equations build a ramp from arithmetic expressions in elementary school to more sophisticated work with algebraic expressions in high school. As the complexity of expressions increases, students continue to see them as being built out of basic operations; they see expressions as sums of terms and products of factors. In "Animal Populations'' students compare P + Q and 2P by seeing 2P as P + P. They distinguish between (Q–P)/2 and Q – P/2 by seeing the first as the quotient where the numerator is a difference and the second as a difference where the second term is a quotient. [This example] illustrates how students are able to see complicated expressions as built out of simpler ones.	● Explain, in their own words, what factor, coefficient, term, and like terms mean in the context of expressions. ● Identify factors, coefficients, different terms and like terms in expressions. ● Identify individual parts of an expression as a single entity to make use of the structure of the expression.	Coefficient, distributive property of addition over multiplication, equivalent expressions, equation, exponent, expression, factor

	Animal Populations Suppose P and Q give the sizes of two different animal populations, where $Q > P$. In 1–4, say which of the given pair of expressions is larger. Briefly explain your reasoning in terms of the two populations. 1. $P + Q$ and $2P$ 2. $\frac{P}{P+Q}$ and $\frac{P+Q}{2}$ 3. $(Q - P)/2$ and $Q - P/2$ 4. $P + 50t$ and $Q + 50t$ Task from Illustrative Mathematics. For solutions and discussion, see http://www.illustrativemathematics.org/illustrations/436.		
HSA-SSE.A.1.b Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+r)n as the product of P and a factor not depending on P.	The middle grades standards in Expressions and Equations build a ramp from arithmetic expressions in elementary school to more sophisticated work with algebraic expressions in high school. As the complexity of expressions increases, students continue to see them as being built out of basic operations; they see expressions as sums of terms and products of factors. In "Animal Populations'' students compare P + Q and 2P by seeing 2P as P + P. They distinguish between (Q–P)/2 and Q – P/2 by seeing the first as the quotient where the numerator is a difference and the second as a difference where the second term is a quotient. [This example] illustrates how students are able to see complicated expressions as built out of simpler ones.	• Explain, in their own words, what factor, coefficient, term, and like terms mean in the context of expressions. • Identify factors, coefficients, different terms and like terms in expressions. • Identify individual parts of an expression as a single entity to make use of the structure of the expression.	Coefficient, distributive property of addition over multiplication, equivalent expressions, equation, exponent, expression, factor

	Animal Populations Suppose P and Q give the sizes of two different animal populations, where $Q > P$. In 1–4, say which of the given pair of expressions is larger. Briefly explain your reasoning in terms of the two populations. 1. $P + Q$ and $2P$ 2. $\frac{P}{P+Q}$ and $\frac{P+Q}{2}$ 3. $(Q - P)/2$ and $Q - P/2$ 4. $P + 50t$ and $Q + 50t$ Task from Illustrative Mathematics. For solutions and discussion, see http://www.illustrativemathematics.org/illustrations/436.		
HSA-CED.A.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	[Students use complex equations—including equations arising from linear and quadratic expressions, and simple rational and exponential expressions—to model] relationships between quantities with equations in two variables. All the standards in the Creating Equations group carry a modeling star, denoting their connection with the Modeling category in high school. This connotes not only an increase in the complexity of the equations studied, but an upgrade of the student's ability in every part of the modeling cycle.	● Create an equation, inequality or system to model a situation with two variables. ● Choose correctly from among linear functions, exponential functions and others as appropriate for modeling a situation. ● Explain their reasoning in steps when creating an equation, inequality, or system of equations or inequalities.	Constraints, linear equation, quadratic equation, system of equations
HSG-CO.C.9 Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly	These basic theorems build from basic understandings about lines and angles, such as supplementary angles.	● Make and prove conjectures about situations involving lines and angles. ● Use transformations as a way about those relationships.	Congruent, corresponding parts, rigid motion, theorem

those equidistant from the segment's endpoints.			
[HSG-CO.C.10](#) Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.	These theorems build on students' conjectures about the relationships described in the standards.	● Transformations can be used to Prove theorems about triangles.	Interior angle of a triangle, Base angle of an isosceles triangle, Midpoint of a segment, Median of a triangle, Theorem,
[HSG-SRT.B.5](#) Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.	There is also another advantage of [the transformation] approach to congruence and similarity. Because most of the theorems in plane geometry before the introduction of similarity depend only on the three triangle congruence criteria, once these have been established, it is possible to transition into the traditional way of proving theorems at this point, without further use of basic rigid motions, if so desired. The use of dilations to treat similarity can likewise be limited to the initial stage if so desired.	● Develop facility in using triangle congruence and similarity to explain geometry relationships. ● Continue to use geometric transformations as a way to visualize the relationships they find.	criteria for triangle similarity, similar figures, similarity transformations

HSG-SRT.C.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.	**Modeling with trigonometry** **Vector graphic** A digital artist using a computer drawing program clicked on a diagonal line segment and saw that it measured 216 units horizontally and 45 units vertically. How many units long was the line segment? If the artist wants to rotate the line segment to be vertical, what angle of rotation could be used? Give your answers to the nearest hundredth. *Answers.* 220.64 units long; 11.77° clockwise or 78.23° counter-clockwise. **Flight of the bumblebee** A bumblebee sitting on a tulip wanted to fly over to a daffodil located 100 meters due west. The bumblebee did fly in a straight line, but it mistakenly flew in a direction 5 degrees south of west. The bumblebee passed to the south of the daffodil—how far south? *Answer.* 8.75 meters or about 9 meters. **Star distance** Viewed from Earth, the North Star and Betelgeuse are separated by an angle of about 83 degrees. If the North Star is about 430 light years from Earth, and Betelgeuse is about 640 light years from Earth, then about how far apart are the North Star and Betelgeuse (measured in light years)?  *Answer.* About 726 light years. *Comment.* This could be solved using a G-SRT.10(+) strategy (Law of Cosines), or via G-SRT.8 by dropping a perpendicular to make two right triangles. **Crop Loss** One corner of a soybean field wasn't irrigated, and no soybeans could be harvested from that part of the field. How much money was lost if soybeans sold for $12 a bushel that year and an acre of irrigated land yields 54 bushels of soybeans? Note, 1 acre is approximately 4000 square meters.  *Answer*: $972 or approximately $1000. See `http://tinyurl.com/mtcn6zq`.	• Represent applied problems using right triangles. • Identify relevant trigonometric ratios and apply them to solve problems.	criteria for triangle similarity, similar figures, similarity transformations, trigonometric ratios
HSG-C.A.2 Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.	This standard addresses a number of common theorems involving circles and arcs. STudents will benefit from first having an opportunity to explore various properties of circles to make conjectures about how they are related and then to justify why they are true.	• Explore various properties related to circles. • Form conjectures about the relationships they find. • Develop justifications for why their conjectures work.	Arc measure, central angles, cord, circumference, circumscribed, circumscribed angle, incenter, inscribed, inscribed angle, radius
HSG-C.A.3 Construct the inscribed and circumscribed circles of a triangle, and prove	For this standard, students should be given an opportunity to construct different centers of a triangle- including the incenter, the circumcenter, the centroid and the circumcircle.	• Identify how to find the incenter and circumcenter of a triangle. • Use the incenter and	Arc measure, central angles, cord, circumference, circumscribed, circumscribed angle, incenter, inscribed, inscribed angle, radius

properties of angles for a quadrilateral inscribed in a circle.		circumcenter to construct the incircle and circumcircle of a triangle. ● Recognize that the opposite angles of a quadrilateral inscribed in a circle are supplementary and justify why that is the case.	
HSG-C.B.5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.	The standard includes explorations of various measurements of circles, building from arc measure, which is defined as the measure of the central angle that includes an arc.	● Relate arc length and area of sectors to the fraction of the circle cut off by the corresponding central angle. ● Develop and use general formulas for arc length and sector area. ● Recognize radian as an alternate method to define the measure of an angle based on the arc it cuts off.	Arc, arc length, arc measure, sector
HSG-GMD.A.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.	Students should have seen the formulas in this standard in the middle grades. In this standard, students are asked to justify why these formulas hold.	● Consider why the various formulas work, using drawings and models as needed. ● Use similarity to define pi and develop the formula for the circumference of a circle. ● Use a limit argument to develop the area of the circle. ● Use Cavalieri's Principle to explain why the formulas for the volume of a cylinder, pyramid, and cone work. ● Relate the volumes using various solids with the same dimensions.	Cavalieri's Principle, Dissection
HSG-MG.A.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy	A wide range of problems might be addressed under this standard, related to different standards from throughout this conceptual category. Students' focus should be designing a solution that meets the constraints of a real-world context.	● Use geometry to model real world situations in order to design solutions to real world problems. ● Justify the decision made to	Density

physical constraints or minimize cost; working with typographic grid systems based on ratios).		demonstrate that the resulting design addresses the contexts.	

UNIT 8: Circles (iM Geometry, Unit 7)				
CCSS Standards #	**Learning Targets:** I can	**Assessment Strategy** SR-Selected Response CR-Constructed Response P-Performance O-Observation (behavioral)	**Lesson Progression and Connection to Math Practices**	**Common Learning Experiences and Assessments**
HSG-C.A.2 HSG-SRT.B.5	What relationships exist between lines, angles and circles? ● I know what chords, arcs, and central angles are. ● I can use the relationship between central and inscribed angles to calculate angle measures. ● I know that an inscribed angle is half the measure of the central angle that defines the same arc. ● I can use the relationship between tangent lines and radii to calculate angle measures. ● I know that a line tangent to a circle is perpendicular to the radius drawn to the point of tangency.		● Students are introduced to the vocabulary terms chord (a segment whose endpoints are on a circle), central angle (an angle formed by 2 rays whose endpoints are the center of the same circle), and arc (the portion of a circle between 2 endpoints) in the first lesson in this unit. ● Then, students encounter inscribed angles in circles, or angles formed by 2 chords which share an endpoint. They develop the conjecture that the measure of an inscribed angle is half the measure of the central angle that defines the same arc. Then, taking this conjecture as an assertion, they show that 2 intersecting chords and the segments joining adjacent endpoints of the chords create similar triangles. ● To finish this learning sequence, students construct a line perpendicular to the radius of a circle that goes through the point where the radius intersects the circle. They prove that this line intersects the circle in exactly 1 point. That is, the line is tangent to the circle. Students also prove the converse: If a line is tangent to a circle, it is perpendicular to the radius drawn to the point of tangency. ● MP1, MP2	iM Lessons 1, 2, 3
HSG-C.A.3 HSG-CO.C.10 HSG-CO.C.9, HSG-SRT.B.5	How can polygons and circles relate? ● I can prove a theorem about opposite angles in quadrilaterals inscribed in circles.		● Students begin by exploring an outcome of the Inscribed Angle Theorem in an analysis of cyclic quadrilaterals. These are quadrilaterals that have a circumscribed circle, or a circle that passes through	iM Lesson 4

		each vertex of the quadrilateral.	
		• MP3, MP5	
HSA-SSE.A.1.b HSG-C.B.5 HSG-GMD.A.1 HSA-CED.A.2 HSA-SSE.A.1	How do we measure circles? • I can calculate lengths of arcs and areas of sectors in circles. • I can gather information about a sector to draw conclusions about the entire circle. • I know that when a circle is dilated, some ratios, like the ratio of the circumference to the diameter, stay constant. • I know that the radian measure of an angle whose vertex is the center of a circle is the ratio of the length of the arc defined by the angle to the circle's radius. • I understand the relative sizes of angles measured in radians. • I can calculate the area of a sector whose central angle measure is given in radians. • I know that the radian measure of an angle can be thought of as the slope of the line.	• Students start this learning sequence by investigating circle sectors. A sector is a region of a circle enclosed between 2 radii. • In the next lesson, students work to find missing information about a circle given a variety of inputs. They start by looking for a central angle given an area and radius, then move toward, for example, calculating arc lengths given a sector area and a central angle. • Then, students experience a progression of learning that will build to a definition of radians in a subsequent lesson. The activities in this lesson build intuition about the relationship between arc length and central angle, without yet naming the arc length to radius ratio as radian measure. • Students begin by using the example of a circular progress bar to make the observation that arc length for a particular central angle is dependent on the radius of the circle. Then, students analyze ratios in dilated circles, recalling that all circles are similar. • They also make initial observations about the relationships between arc lengths, radii, and angles, noting that the arc length to radius ratio appears to be constant for the same angle measure across circles of different sizes. • In the next lesson, students prove that the length of the arc intercepted by a central angle is proportional to the radius of the circle. Then, they learn that the ratio of arc length to radius is called the radian measure of an angle. They use string to measure arc length in terms of a circle's radius, and connect the results to the definition of radian angle measurement. • An understanding of radian measure will be necessary in future courses when students extend the domain of trigonometric functions using the unit	iM Lessons 8, 9, 10, 11, 12, 13, 14

		circle. ● In the fourth lesson in this sequence, students strengthen their understanding of the relationship between radian and degree measures. They use double number lines and proportional reasoning to make connections between degrees and radians, then develop their intuitive sense of radian measures by shading sectors of circles with given radian measures. ● It's more important that students build their intuition around radian measurements than it is to develop a formal algorithm to convert between radians and degrees. ● In the final lesson of this sequence, students find sector areas and arc lengths for central angles with radian measure. They justify the formula for the area of a sector, and they observe that radian measure simplifies arc length calculations.	
		● MP8, MP1, MP7, MP2, MP6	

COMMON MISCONCEPTIONS	PRIOR KNOWLEDGE NEEDED TO MASTER STANDARDS FOR THIS UNIT	ADVANCED STANDARDS FOR STUDENTS WHO HAVE DEMONSTRATED PRIOR MASTERY	OPPORTUNITIES FOR STUDENT-DIRECTED LEARNING WITHIN THE UNIT
● Students may be confused about the measure of an inscribed angle.	● 6.EE.A.2 ● 7.EE.A.1 ● 7.EE.A.2 ● 7.EE.B.4 ● 7.G.A.1 ● 7.G.B.4 ● 7.G.B.5 ● 7.G.B.6 ● 7.RP.A.2 ● 8.EE.C.8 ● 8.G.A.5 ● 8.G.B.6 ● 8.G.B.7 ● 8.G.C.9		

	8.F.B.4 8.F.B.5 8.SP.A.2 8.SP.A.3		