Bristol Public Schools

Office of Teaching & Learning

Department	Mathematics
Department Philosophy	*Students learn by doing math, solving problems in mathematical and real-world contexts, and constructing arguments using precise language.* The Bristol mathematics curricula embeds this *learn-by-doing* philosophy by focusing on high expectations for all students and providing students with opportunities that build conceptual understanding, computational and procedural fluency, and problem solving through the use of a variety of strategies, tools, and technologies. The mathematics curriculum is responsive to the individual needs of students, while providing a structure tied to the Common Core State Standards in Connecticut. The *learn-by-doing* philosophy develops mathematically literate and productive students who can effectively and efficiently apply mathematics in their lives to make informed decisions about the world around them by doing math. To be mathematically literate, one must understand major mathematics concepts, possess computational facility, and have the ability to apply these understandings to situations in daily life. Making connections between mathematics and other disciplines is key to the appropriate application of mathematics skills and concepts to solve problems. The ability to read, discuss, and write within the discipline of mathematics is an integral skill that supports mathematical understanding, reasoning and communication. The opportunity to think critically and creatively to solve problems is important to deepen mathematical knowledge and foster innovation. A rich hands-on mathematical experience is essential to provide the foundational knowledge and skills that prepare students to be mathematically literate, productive citizens.
Course	Grade 4 Mathematics
Grade Level	Grade 4
Pre-requisites	Grade 3

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M-Major Cluster, S-Supporting Cluster, A-Additional Cluster

District Learning Expectations and Standards	Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6	Unit 7	Unit 8	Unit 9 (optional)
Operations and Algebraic Thinking									
Use the four operations with whole numbers to solve problems.									
4.OA.A.1 Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.					M				
4.OA.A.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.1					M				M
4.OA.A.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.	M				M	M			M
Gain familiarity with factors and multiples.									
4.OA.B.4 Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number.	S								

Determine whether a given whole number in the range 1-100 is prime or composite.									
Generate and analyze patterns.									
4.OA.C.5 Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.						A			
Number and Operations in Base Ten									
Generalize place value understanding for multi-digit whole numbers.									
4.NBT.A.1 Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.				M					
4.NBT.A.2 Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.				M					
4.NBT.A.3 Use place value understanding to round multi-digit whole numbers to any place.				M					
Use place value understanding and properties of operations to perform multi-digit arithmetic.									
4.NBT.B.4 Fluently add and subtract multi-digit whole numbers using the standard algorithm**.**				M		M			M

4.NBT.B.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.						M			M
4.NBT.B.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.						M			M
Number and Operations - Fractions									
Extend understanding of fraction equivalence and ordering.									
4.NF.A.1 Explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.		M	M						M
4.NF.A.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.		M							M
Build fractions from unit fractions.									
4.NF.B.3 Understand a fraction a/b with a > 1 as a sum of fractions 1/b.			M		M				M
4.NF.B.3.A Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.			M						

4.NF.B.3.B Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.			M						
4.NF.B.3.C Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.			M						
4.NF.B.3.D Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.			M						
4.NF.B.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.			M						M
4.NF.B.4.A Understand a fraction a/b as a multiple of 1/b. For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).			M						
4.NF.B.4.B Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)			M						
4.NF.B.4.C Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?			M						
Understand decimal notation for fractions, and compare decimal fractions.									

4.NF.C.5 Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.2 For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.			M	M					
4.NF.C.6 Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.				M					
4.NF.C.7 Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.				M					
Measurement and Data									
Solve problems involving measurement and conversion of measurements.									
4.MD.A.1 Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...					S				
4.MD.A.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.					S	S		S	

4.MD.A.3 Apply the area and perimeter formulas for rectangles in real world and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.					S	S		S	
Represent and interpret data.									
4.MD.B.4 Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.					S				
Geometric measurement: understand concepts of angle and measure angles.									
4.MD.C.5 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:							A		
4.MD.C.5.A An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles.							A		
4.MD.C.5.B An angle that turns through n one-degree angles is said to have an angle measure of n degrees.							A		
4.MD.C.6 Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.							A		
4.MD.C.7 Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems,							A	A	

e.g., by using an equation with a symbol for the unknown angle measure.									
Geometry									
Draw and identify lines and angles, and classify shapes by properties of their lines and angles.									
4.G.A.1 Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.							A	A	
4.G.A.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.								A	
4.G.A.3 Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.								A	

UNIT 1: FACTORS AND MULTIPLES

Illustrative Mathematics Unit Focus: Students apply understanding of multiplication and area to work with factors and multiples.

Essential Questions:
How can we show mathematical situations in word problems?
How do we decide what operation to use when solving a real-world problem?
Why do we decompose whole numbers into factor pairs?

Unit Pacing: 13 days (7 required lessons, 4 flex, 2 assessment and reaction)

UNWRAPPED STANDARDS

Grade Level Standard	Standard Progression	Concepts (Big Ideas/ Understandings)	Academic Vocabulary (Standard Based)
4.OA.A.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.	Present multi-step word problems with whole numbers and whole-number answers using the four operations. Students should know which operations are needed to solve the problem. Drawing pictures or using models will help students understand what the problem is asking. They should check the reasonableness of their answer using mental computation and estimation strategies. Numbers can be those in the Grade 4 standards, but the number of steps should be no more than three and involve only easy and medium difficulty addition and subtraction problems.	We can show mathematical situations in word problems using numbers, symbols, and operations signs, or by making tables or drawings. Recognizing how a real-world situation fits into a common operation category helps to solve the problem. Estimation strategies, including rounding, can be used to determine the reasonableness of answers. An unknown can be in any position of a multiplicative comparison problem.	Operations Equations Estimation Unknown quantity Multi-step Reasonableness
4.OA.B.4 Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors.	A factor is a number that is multiplied by another number to yield a product. Two and eight are factors of sixteen. A multiple is the product of a whole number and any other whole number. Sixteen is a multiple of eight 1, 3, 7, and 21 are factor pairs of 21. Students find factor pairs of numbers to 100. Students should use basic fact recall and number sense to identify factor pairs. Factoring procedures should be avoided. Explicit instruction of divisibility rules is	A number can be multiplicatively decomposed into factor pairs and expressed as a product of these factor pairs. Any whole number is a multiple of each of its factors. A prime number has only two factors: one and itself (only one factor pair).	Factor Product Multiples Odd Even Prime Composite

	not appropriate, however student discovery of these rules could be an enrichment within this standard for students who show proficiency with factors, multiples, and prime/composite numbers. Students need to develop strategies for determining if a number is prime or composite, in other words, if a number has a whole number factor that is not one or itself. Starting with a number chart of 1 to 20, use multiples of prime numbers to eliminate later numbers in the chart. For example, 2 is prime but 4, 6, 8, 10, 12,… are composite. Encourage the development of rules that can be used to aid in the determination of composite numbers. For example, other than 2, if a number ends in an even number (0, 2, 4, 6 and 8), it is a composite number.	A composite number has more than two factors (more than one factor pair).	

UNIT 1: FACTORS AND MULTIPLES				
How can we show mathematical situations in word problems? How do we decide what operation to use when solving a real-world problem? Why do we decompose whole numbers into factor pairs?				
CCSS Standards #	**Learning Targets**	**Summative Assessment Strategy**	**Lesson Progression and Connection to Math Practices**	**Common Learning Experiences and Assessments**
Section A: Understand Factors and Multiples				
4.OA.B.4	I can find all factor pairs for a whole number between 1 and 100. I can determine if a whole number between 1 and 100 is a multiple of a particular one digit number. I can determine if a number between 1-100 is prime or composite.	X Selected Response X Constructed Response Performance X Observation	**Lesson Progression:** Students are introduced to factors and multiples by building on area and multiplication concepts learned in grade 3. To revisit the concept of area, students build rectangles using tiles, given specific side lengths. They then discuss possible areas for rectangles with only one side length named. These activities help students think intuitively about factors and multiples before they learn the terminology. As the section progresses, students use tiles to build rectangles given a certain area. This task allows students to engage with the commutative property of multiplication in that the orientation of their rectangle does not change the area. They also learn that the side lengths of the rectangles they create represent the factor pairs of the given area values. As the lessons progress, students discover that there are sometimes multiple possible factor pairs for a given value, and other times there is only one possible factor pair, 1 and itself. Finally, they identify the values as prime or composite numbers based on the number of rectangles that can be made using the given area. The section closes with an optional game day, which allows students to be introduced to centers. The centers in this lesson focus on multiplication facts, building on from the grade 3 fluency standards.	**Mandatory Lessons/Activities:** iM Lessons 1, 2, 3

Pacing:	4 days		**Math Practices:** SMP 3, 4, 5, 6, 7	**Assessments:** Cool-downs: 1, 2 and 3 Checkpoint A
Section B: Find Factor Pairs and Multiples				
4.OA.A.3 4.OA.B.4	I can solve real-world problems involving all four operations.	X – Selected Response X – Constructed Response Performance X – Observation	**Lesson Progression:** Students deepen their understanding of factors and multiples by applying what they have learned to games and contextual situations. Through the tasks, students reason about factors and multiples by looking for patterns and making use of structure. They find all of the factor pairs of a number in the range of 1–100 and can determine whether a number within this range is a multiple of given one-digit numbers.	**Mandatory Lessons/Activities:** iM Lessons 5, 6, 7
Pacing:	4 days		**Math Practices:** SMP 1, 2, 3, 4, 5, 6, 7, 8	**Assessments:** Cool-down 5 Checkpoint B

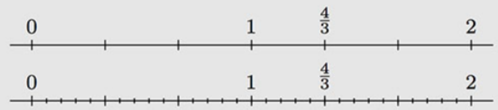
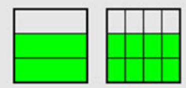
ADDITIONAL CONSIDERATIONS			
COMMON MISCONCEPTIONS	**PRIOR KNOWLEDGE NEEDED TO MASTER STANDARDS FOR THIS UNIT**	**ADVANCED STANDARDS FOR STUDENTS WHO HAVE DEMONSTRATED PRIOR MASTERY**	**OPPORTUNITIES FOR STUDENT-DIRECTED LEARNING WITHIN THE UNIT**
Students have difficulty estimating a two-step problem. Students do not always solve all of the steps needed for a multistep problem. When listing multiples of numbers, students may not list the number itself. Emphasize that the smallest multiple is the number itself. Some students may think that larger	4.OA.A.3: 3.OA.D.8, 4.NBT.A.3, 4.NBT.B.6 4.OA.B.4: 3.OA.C.7	Choose from iM leveled centers and exploration problems to differentiate for students who are ready.	iM Centers District-approved online resources

numbers have more factors. Having students share all factor pairs and how they found them will clear up this misconception.			

RESOURCES
Kendall Hunt Flourish Blackline masters and materials from Teacher Resource Pack Grid paper, inch tiles, chart paper, glue, tape, scissors, counters, colored pencils, crayons or markers, sticky notes, straightedges

UNIT 2: FRACTION EQUIVALENCE AND COMPARISON
Illustrative Mathematics Unit Focus: Students use visual representations or a numerical process to generate and reason about equivalent fractions, and compare and order fractions with the following denominators: 2, 3, 4, 5, 6, 8, 10, 12, and 100. **Essential Questions:** What are equivalent fractions? When can we compare fractions? What are some strategies we can use to compare fractions? **Unit Pacing: 24 days (17 required lessons, 5 flex, 2 assessment and reaction)**

UNWRAPPED STANDARDS			
Grade Level Standard	**Standard Progression**	**Concepts (Big Ideas/ Understandings)**	**Academic Vocabulary (Standard Based)**
4.NF.A.1: Explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.	Students can use area models and number line diagrams to reason about equivalence. They see that the numerical process of multiplying the numerator and denominator of a fraction by the same number, n, corresponds physically to partitioning each unit fraction piece into n smaller equal pieces. The whole is then partitioned into n times as many pieces, and there are *n* times as many smaller unit fraction pieces as in the original fraction.	Equivalent fractions use different sized fractional parts to describe the same amount, e.g., 1/2 = 2/4 . Two fractions are equivalent (equal) if they are the same size or the same point on a number line. Multiplying the numerator and the denominator by the same number will result in an equivalent fraction. There is a multiplicative relationship between the number of equal parts in a whole and the size of the parts.	Fractions Unit fractions Equivalent Fraction model Numerator Denominator Fraction bars Whole Part Partition Distance Interval Number line

	Using the number line to show that $\frac{4}{3} = \frac{5\times4}{5\times3}$  *$\frac{4}{3}$ is 4 parts when each part is $\frac{1}{3}$, and we want to see that this is also 5×4 parts when each part is $\frac{1}{5\times3}$. Divide each of the intervals of length $\frac{1}{3}$ into 5 parts of equal length. There are 5×3 parts of equal length in the unit interval, and $\frac{4}{3}$ is 5×4 of these. Therefore $\frac{4}{3} = \frac{5\times4}{5\times3} = \frac{20}{15}$.* **Using an area model to show that $\frac{2}{3} = \frac{4\times2}{4\times3}$**  *The whole is the square, measured by area. On the left it is divided horizontally into 3 rectangles of equal area, and the shaded region is 2 of these and so represents $\frac{2}{3}$. On the right it is divided into 4×3 small rectangles of equal area, and the shaded area comprises 4×2 of these, and so it represents $\frac{4\times2}{4\times3}$.*		
4.NF.A.2: Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.	Grade 3 students compare two fractions with the same numerators or the same denominators and justify their comparisons with visual models. Grade 4 students use their understanding of equivalent fractions to compare fractions with different numerators and different denominators. For example, to compare 5/8 and 7/12 [students] rewrite both fractions as 60/96 (= 12x5/12x8) and 56/96 (= 7x8/12x8). Because 60/96 and 56/96 have the same denominator, students can compare them using Grade 3 methods and see that 56/96 is smaller, so 7/12 < 5/8. Students also reason using benchmarks such as 1/2 and 1. For example, they see that 7/8 < 13/12 because 7/8 is less than 1 (and is therefore to the left of 1) but 13/12 is greater than 1 (and is therefore to the right of 1).	Two fractions can be compared when the two fractions refer to the same whole. Comparing two fractions requires thinking about the size of the parts (denominator) and the number of the parts (numerator).	Fractions Equivalent Numerator Denominator Visual fraction model Greater than > Less than < Equal to = Comparison

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<tr><th colspan="5">Section A: Size and Location of Fractions</th></tr>
<tr><td>4.NF.A.1
4.NF.A.2</td><td>I can represent fractions using a variety of models and explain my reasoning.</td><td rowspan="2">X Selected Response
X Constructed Response
Performance
X Observation</td><td>Lesson Progression:
Building on the fraction work in grade 3, this section allows students to revisit the meaning of fractions using concrete physical representations such as fraction strips, and more abstract representations such as tape diagrams and number lines. Focusing on the denominators 2, 3, 4, 5, 6, 8, 10, and 12, students use their understanding of numerators and denominators to reason about the size of fractions, and compare fractions with the same numerators or the same denominators. They also recall the meaning of equivalent fractions. As the section progresses, students discuss and represent the relationship between the sizes of fractions such as ⅕ and 1/10 , or ⅙ and 1/12. Students make sense of the partitions in tape diagrams and number lines, and draw their own representations to demonstrate conceptual understanding of these fractional relationships. They also use their understanding of the relative size of fractions to compare fractions to benchmarks such as ½ and 1.</td><td>Mandatory Lessons/Activities:
iM Lessons 1, 2, 3, 4, 5, 6</td></tr>
<tr><td>Pacing:</td><td>6 days</td><td>Math Practices:
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Cool-downs: 2 and 4
Checkpoint A</td></tr>
<tr><th colspan="5">Section B: Equivalent Fractions</th></tr>
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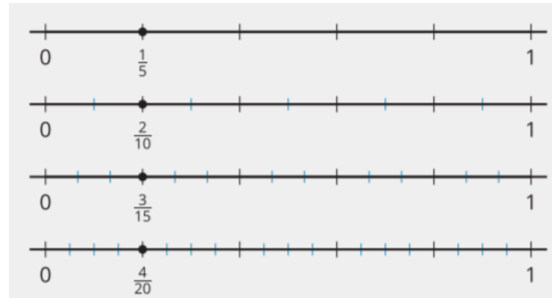
[4.NF.A.1](#)	I can generate equivalent fractions and justify my reasoning.	X Selected Response X Constructed Response Performance X Observation	**Lesson Progression:** Students develop their ability to reason about and generate equivalent fractions. The lessons in this section focus on number lines as a representation, in which students demonstrate that fractions are equivalent when they are located at the same point. As the section progresses, students make sense of a numerical way to determine whether two fractions are equivalent and generate equivalent fractions.  As students analyze patterns in tape diagrams and number lines, they generalize that fraction a ÷ b is equivalent to fraction *(nxa)/(nxb)* . Students will not need to know the algebraic notation of this generalization. Given their understanding of the size of fractions and relationship between fractions, however, they should be able to explain this generalization with fractions that have denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100. Students can then use this generalization to identify and generate equivalent fractions.	**Mandatory Lessons/Activities:** iM Lesson 7,8,9,10, 11
Pacing:	5 days		**Math Practices:** SMP 3, 5, 6, 7	**Assessments:** Cool-downs: 10 Checkpoint B
Section C: Fraction Comparison				
[4.NF.A.1](#) [4.NF.A.2](#)	I can compare two fractions and justify my reasoning.	X Selected Response X Constructed	**Lesson Progression:** Students develop their ability to compare and order fractions with different numerators and different denominators. In the previous sections, they gained experience with representations such	**Mandatory Lessons/Activities:** iM Lessons 12, 13, 14, 15,16, 17

		Response Performance X Observation	as tape diagrams and number lines that helped them reason about the size of fractions. These experiences, as well as their understanding that fraction a/b is equivalent to fraction *(nxa)*/(*nxb)*, help to give students flexibility in making comparisons. For example, students can determine when it is useful to reason about the fractions' distance on a number line, use benchmarks such as or 1, or find a common denominator. They record the results of comparisons with symbols >, =, or <. Throughout this section, students apply these understandings in contexts that call for comparing fractional measurements.	
Pacing:	6 days		**Math Practices:** SMP 1, 3, 5, 6, 7	**Assessments:** Cool-down: 16 Checkpoint C

ADDITIONAL CONSIDERATIONS			
COMMON MISCONCEPTIONS	**PRIOR KNOWLEDGE NEEDED TO MASTER STANDARDS FOR THIS UNIT**	**ADVANCED STANDARDS FOR STUDENTS WHO HAVE DEMONSTRATED PRIOR MASTERY**	**OPPORTUNITIES FOR STUDENT-DIRECTED LEARNING WITHIN THE UNIT**
When representing fractions on a number line: -Students may think that the right side of the number line always represents 1 whole. Fractions may be greater than 1 so the number line may represent other numbers. -Students might count tick marks instead of spaces. -Students might make mistakes in comparing or equivalence if they do not create number lines of the same size or do not partition equally. Students think that when generating equivalent fractions they only need to multiply or divide either the numerator or denominator. For example, when determining an equivalent amount of sixths for 1/2 students might multiply the denominator by 3 to get 1/6 , instead of multiplying ½ by 3/3. They are focusing only on the numerator or denominator and not the fraction as a number. Students try to apply whole number understanding when comparing fractions, for example they think that eighths are larger than fourths because 8 is more than 4. Similarly, students may think that 4/8 is more than 2/4 because 8 is bigger than 4 and 4 is bigger than 2.	4.NF.A.1: 4.OA.A.2, 3.NF.A.3 4.NF.A.2: 4.NF.A.1	Choose from iM leveled centers and exploration problems to differentiate for students who are ready.	iM Centers District-approved online resources
RESOURCES			

Kendall Hunt Flourish
Unit card sorts, Unit cool downs
Straight edges, tools for creating a visual display, sticky notes, colored pencils, markers, paper, paper clips, tape (painter's or masking)

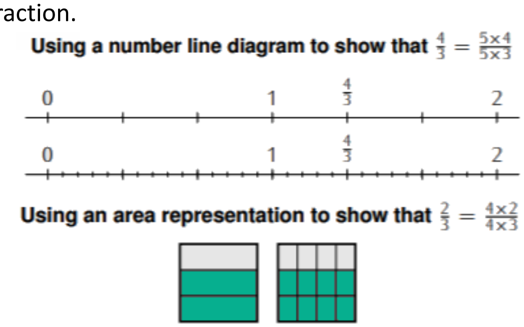
UNIT 3: FRACTION OPERATIONS

Illustrative Mathematics Unit Focus: Students start their work on fraction multiplication by learning that a fraction *a/b* is a product of a whole number *a* and a unit fraction *1/b*, or *a/b=a x 1/b*. Later, they generalize that the product of a whole number *n* and a fraction *a/b* is equal to the fraction *nxa/b*, or *n x a/b = (nxa)/b*. They then move to addition and subtraction of fractions and mixed numbers with like denominators, where they compose and decompose fractions and mixed numbers to perform these operations. Finally, students use their understanding of equivalence from a previous unit to add tenths and hundredths. Throughout the unit, students represent and solve problems involving multiplication, addition, and subtraction of fractions.

Essential Questions:
When can we add or subtract fractions and decimals?
What strategies can help us add and subtract fractions?
Why do we decompose fractions?
What are equivalent fractions and decimals?

Unit Pacing: 24 days (17 required lessons, 5 flex, 2 assessment and reaction)

UNWRAPPED STANDARDS

Grade Level Standard	Standard Progression	Concepts (Big Ideas/ Understandings)	Academic Vocabulary (Standard Based)
4.NF.A.1: Explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.	Students can use area models and number line diagrams to reason about equivalence. They see that the numerical process of multiplying the numerator and denominator of a fraction by the same number, n, corresponds physically to partitioning each unit fraction piece into n smaller equal pieces. The whole is then partitioned into n times as many pieces, and there are *n* times as many smaller unit fraction pieces as in the original fraction. 	Equivalent fractions use different sized fractional parts to describe the same amount, e.g., 1/2 = 2/4 . Two fractions are equivalent (equal) if they are the same size or the same point on a number line. Multiplying the numerator and the denominator by the same number will result in an equivalent fraction. There is a multiplicative relationship between the number of equal parts in a whole and the size of the parts.	Fractions Unit fractions Equivalent Fraction model Numerator Denominator Fraction bars Whole Part Partition Distance Interval Number line

4.NF.A.2: Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.	Grade 3 students compare two fractions with the same numerators or the same denominators and justify their comparisons with visual models. Grade 4 students use their understanding of equivalent fractions to compare fractions with different numerators and different denominators. For example, to compare 5/8 and 7/12 [students] rewrite both fractions as 60/96 (= 12x5/12x8) and 56/96 (= 7x8/12x8). Because 60/96 and 56/96 have the same denominator, students can compare them using Grade 3 methods and see that 56/96 is smaller, so 7/12 < 5/8. Students also reason using benchmarks such as 1/2 and 1. For example, they see that 7/8 < 13/12 because 7/8 is less than 1 (and is therefore to the left of 1) but 13/12 is greater than 1 (and is therefore to the right of 1).	Two fractions can be compared when the two fractions refer to the same whole. Comparing two fractions requires thinking about the size of the parts (denominator) and the number of the parts (numerator).	Fractions Equivalent Numerator Denominator Visual fraction model >, <, = Greater than Less than Equal Comparison
4.NF.B.3 : Understand a fraction a/b with a > 1 as a sum of fractions 1/b. 4.NF.B.3.a: Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. 4.NF.B.3.b: Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8. 4.NF.B.3.c: Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. 4.NF.B.3.d: Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using	In Grade 3, students represented whole numbers as fractions. In Grade 4, they will use this knowledge to add and subtract mixed numbers with like denominators using properties of number and appropriate fraction models. It is important to stress that whichever model is used, it should be the same for the same whole. For example, a circular model and a rectangular model should not be used in the same problem. The meaning of addition is the same for both fractions and whole numbers, even though algorithms for calculating the sums can be different. This simple understanding of addition as putting together allows students to see in a new light the way fractions are built up from unit fractions. The same representation that students used in Grade 4 to see a fraction as a point on the number line now allows them to see a fraction as a sum of unit fractions. Just as 5 = 1 + 1 + 1 + 1 + 1, so $\frac{5}{3} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$	We decompose fractions into sums or products of fractions to make computation easier or to simplify expressions. Fractions can be added and subtracted when the wholes are the same size. Fractions with the same denominators can be added and subtracted using visual models, properties of operations, and relationships of addition and subtraction of whole numbers. We decompose fractions into sums or products of fractions to make computation easier or to simplify expressions. Mixed numbers can be written as fractions, e.g., 14/3 = 4 2/3 , and can be added or subtracted in this form.	Fractions Unit fraction Equivalent Numerator Denominator Decompose Ordering Mixed number

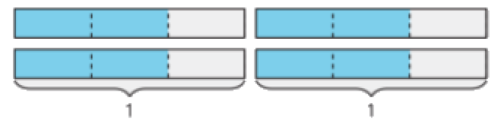
visual fraction models and equations to represent the problem.	Because 5/3 is the total length of five thirds. Armed with this insight, students decompose and compose fractions with the same denominator. They add fractions with the same denominator	When solving word problems, students need to attend to the underlying quantities.	
4.NF.B.4: Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. 4.NF.B.4.a: Understand a fraction a/b as a multiple of 1/b. For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4). 4.NF.B.4.b: Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.) 4.NF.B.4.c: Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?	Understanding of multiplication of whole numbers is extended to multiplying a fraction by a whole number. Allow students to use fraction models and drawing to show their understanding. Present word problems involving multiplication of a fraction by a whole number. Have students solve the problems using visual models and write equations to represent the problems.	The idea of the numerator as a multiplier can be used when a fraction is being multiplied by a whole number, e.g., Just as 5/8 = 5× 1/8 , 5 groups of 3/8 equals 5 × 3/8 = (5×3)× 1/8 which equals 15/8 . Arrays, number lines, fraction strips, or sets can be used to find the solution to multiplying a whole number by a fraction. The relationship between multiplication and division is applied to fractions just as it is applied to whole numbers.	Fractions Whole number Multiple Fraction model
4.NF.C.5: Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. For	Grade 4 students learn to add decimal fractions by generating an equivalent fraction with the same denominator, in preparation for general fraction addition in Grade 5:	Any fraction with a denominator of 10 can be renamed as a fraction with a denominator of 100 using equivalent fractions.	Fractions Whole number Multiple Equivalent fraction

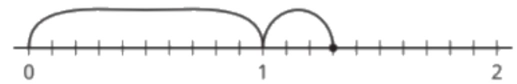
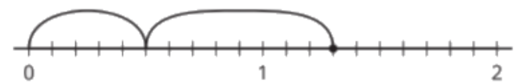
example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.	3/10 + 27/100 = 30/100 + 27/100 = 57/100. They can interpret this as saying that 3 dimes together with 27 cents make 57 cents.	Fractions can be added and subtracted when the wholes are the same size. Fractions with the same denominators can be added and subtracted using visual models, properties of operations, and relationships of addition and subtraction of whole numbers. We decompose fractions into sums or products of fractions to make computation easier or to simplify expressions.	Numerator Denominator

UNIT 3: FRACTION OPERATIONS

When can we add or subtract fractions and decimals?
What strategies can help us add and subtract fractions?
What are equivalent fractions and decimals?
Why do we decompose fractions?

CCSS Standards #	Learning Targets	Summative Assessment Strategy	Lesson Progression and Connection to Math Practices	Common Learning Experiences and Assessments
Section A: Equal Groups of Fractions				
4.NF.B.4 4.NF.B.4.a 4.NF.B.4.b 4.NF.B.4.c	I can represent and solve fraction multiplication problems involving a fraction and a whole number. I can decompose a fraction in more than one way.	X — Selected Response X — Constructed Response Performance X — Observation	**Lesson Progression:** Students extend their earlier understanding of multiplication as equal groups of whole numbers of objects to now include equal groups of fractional pieces. How many do you see? How do you see them? Students begin by reasoning about groups containing unit fractions. For instance, they interpret the 3 plates with half an orange each as 3 x ½ , which is 3/2. Later, they also reason about groups of non-unit fractions and write expressions to represent the quantities. For instance, 5 groups of ¾ can be expressed as 5 x ¾ or 15/4. As the section progresses, they recognize that the product has a numerator that is equal to the product of the whole number (the number of equal groups of discrete tape diagrams) and the numerator of the fractional factor (the number of shaded pieces in one of the discrete tape diagrams), and that the denominator (or the size of each piece) doesn't change.	**Mandatory Lessons/Activities:** iM Lessons 1, 2, 3, 4, 5, 6

			$4 \times \frac{2}{3} = \frac{8}{3}$  These diagrams also help students see that in some instances, a fraction can be represented by more than one multiplication expression. Using the associative property, students can reason that, for example, 8/3 = 8 x ⅓ , which is also equivalent to 4 x 2 x ⅓ and 2 x 4 x ⅓ , and is therefore equivalent to 4 x ⅔ and 2 x 4/3 , respectively. By circling the diagram in various ways, students can visualize the different combinations of groups and understand their equivalence. These images will become increasingly useful in later grades when they make sense of fractions as quotients. Later in the section, students apply what they have learned about fraction multiplication to solve problems and show their reasoning using diagrams and equations.	
Pacing:	6 days		**Math Practices:** SMP 1, 2, 3, 4, 5, 6, 7	**Assessments:** Cool Downs: 5, 6 Section Checkpoint
Section B: Addition and Subtraction of Fractions				
4.NF.B.3 4.NF.B.3.a 4.NF.B.3.b 4.NF.B.3.c 4.NF.B.3.d 4.NF.B.4.c	I can add and subtract fractions and mixed numbers with like denominators. I can decompose a fraction in more than one way.	X | Selected Response X | Constructed Response | Performance X | Observation	**Lesson Progression:** Students decompose a fraction into sums of smaller fractions with the same denominator. They then use this understanding to add and subtract fractions and mixed numbers. Students begin this section by thinking about a fractional quantity in a situation as a product of a whole number and a fraction, or as a sum of fractions with the same denominator, or both. This allows for a transition to thinking about a fraction as a number that can be decomposed into smaller fractions. Students then represent different ways to decompose a	**Mandatory Lessons/Activities:** iM Lessons 7, 8, 9, 10, 11, 12

			fraction by drawing "jumps" on number lines and writing different equations. $\frac{13}{10} = \frac{10}{10} + \frac{3}{10}$  $\frac{13}{10} = \frac{5}{10} + \frac{8}{10}$  Number lines, which previously supported students in adding and subtracting whole numbers, are also used to support them in adding and subtracting fractions. In decomposing fractions, students also gain two other insights. First, they see that a fraction greater than 1 can be composed into a whole number and a fraction, and then be expressed as a mixed number. Second, they see that decomposing a whole number into a sum can help us add and subtract fractions with the same denominator. Students then solve various problems involving addition and subtraction of fractions (including mixed numbers), decomposing and rearranging them as needed.	
Pacing:	6 days		**Math Practices:** SMP 1, 2, 3, 6, 7, 8	**Assessments:** Cool Downs: 8, 9, 12 Section Checkpoint
Section C: Addition of Tenths and Hundredths				
[4.NF.A.1](#) [4.NF.A.2](#) [4.NF.B.3.c](#) [4.NF.B.3.d](#) [4.NF.B.4](#) [4.NF.B.4.c](#) [4.NF.C.5](#)	I can add and subtract fractions with like denominators. I can generate equivalent fractions and justify my reasoning. I can decompose a fraction in more than one way.	X — Selected Response X — Constructed Response Performance	**Lesson Progression:** Students apply their understanding of fraction equivalence to add tenths and hundredths. In the previous unit, students learned that . They use this reasoning to add tenths and hundredths by generating equivalent fractions. They also use what they learned in the previous section to strategically use decomposition and the associative and commutative properties to add three or more	**Mandatory Lessons/Activities:** iM Lessons 14, 15, 16, 17, 18

		X | Observation	tenths and hundredths, including mixed numbers. This section ends with an optional problem solving lesson to allow students to apply what they have learned about multiplication, addition, and subtraction of fractions and mixed numbers.	
Pacing:	6 days		**Math Practices:** SMP 1, 2, 3, 6, 7, 8	**Assessments:** Cool-down 14 Checkpoint C

ADDITIONAL CONSIDERATIONS			
COMMON MISCONCEPTIONS	**PRIOR KNOWLEDGE NEEDED TO MASTER STANDARDS FOR THIS UNIT**	**ADVANCED STANDARDS FOR STUDENTS WHO HAVE DEMONSTRATED PRIOR MASTERY**	**OPPORTUNITIES FOR STUDENT-DIRECTED LEARNING WITHIN THE UNIT**
Students think that it does not matter which model to use when finding the sum or difference of fractions. They may represent one fraction with a rectangle and the other fraction with a circle. They need to know that the models need to represent the same whole.	4.NF.A.1: 4.OA.A.2, 3.NF.A.3 4.NF.A.2: 4.NF.A.1 4.NF.B.3: 3.NF.A.1, 3.NF.A.2, 4.NF.A.1 4.NF.B.4: 3.NF.A.1, 3.OA.A.1, 3.OA.A.3, 4.OA.A.2 4.NF.C.5: 4.NF.A.1, 4.NF.B.3	Choose from iM leveled centers and exploration problems to differentiate for students who are ready.	iM Centers District-approved online resources
RESOURCES			
Kendall Hunt Flourish Blackline masters and materials from Teacher Resource Pack Measuring Cups, tools for creating a visual display, large playing bricks, sticky notes, coins, rulers (inches)			

UNIT 4: DECIMAL FRACTIONS AND LARGE NUMBERS

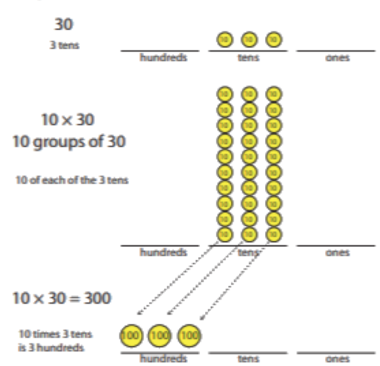
Illustrative Mathematics Unit Focus: Students learn about and use the relationship between multiplication and division, place value understanding and the properties of operations to multiply divide whole numbers within 100. They also represent and solve two-step word problems using the four operations.

Essential Questions:
How is our number system organized?
How can understanding place value help us?
How do the properties of operations make computation simpler?
How do we decide what operation to use when solving a real-world problem?
What are equivalent fractions and decimals?

Unit Pacing: 33 days (23 required lessons, 8 flex, 2 assessment and reaction)

UNWRAPPED STANDARDS

Grade Level Standard	Standard Progression	Concepts (Big Ideas/ Understandings)	Academic Vocabulary (Standard Based)
4.NF.C.5: Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.2 For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.	Grade 4 students learn to add decimal fractions by converting them to fractions with the same denominator, in preparation for general fraction addition in Grade 5: 3/10 + 27/100 = 30/100 + 27/100 = 57/100. They can interpret this as saying that 3 dimes together with 27 cents make 57 cents.	Any fraction with a denominator of 10 can be renamed as a fraction with a denominator of 100 using equivalent fractions. Fractions can be added and subtracted when the wholes are the same size. Fractions with the same denominators can be added and subtracted using visual models, properties of operations, and relationships of addition and subtraction of whole numbers. We decompose fractions into sums or products of fractions to make computation easier or to simplify expressions.	Fractions Whole number Multiple Equivalent fraction Numerator Denominator
4.NF.C.6: Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.	Fractions with denominators equal to 10, 100, etc, such as 27/10, 27/100, etc. can be written by using a decimal point as 2.7 or 0.27. The number of digits to the right of the decimal point indicates the number of zeroes in the denominator, so that	The place value system of whole numbers can be expanded to represent numbers less than 1.	Fractions Decimal Decimal notation Number line Tenths

	2.70=270/100 and 2.7=27/10.	A fraction with a denominator of 10 or 100 can be written using decimal notation. A number can be written as a fraction, e.g., 17/100, or as a decimal, e.g., 0.17. A decimal point or horizontal bar can be used to show where the unit is located, e.g., 35/100 = 0.35.	Hundredths
4.NF.C.7:Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.	Students compare decimals using the meaning of a decimal as a fraction, making sure to compare fractions with the same denominator. For example, to compare 0.2 and 0.09, students think of them as 0.20 and 0.09 and see that 0.20 > 0.09 because 20/100 > 9/100.	Decimals can only be compared when the decimals being compared refer to the same whole. Decimals written as tenths or hundredths can be compared using equivalent fractions	Decimals Hundredths Comparison Greater than, > Less than, < Equal to, =
4.NBT.A.1: Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.	In the base-ten system, the value of each place is 10 times the value of the place to the immediate right. Because of this, multiplying by 10 yields a product in which each digit of the multiplicand is shifted one place to the left. 10 × 30 **represented as** 3 **tens each taken** 10 **times**  Each of the 3 [groups of] tens becomes a hundred and moves to the left. In the product, the three in the tens place of 30 is shifted one place to the left to represent three hundreds. In 300 divided by 10 the 3 is shifted one place to the right in the quotient to represent three tens.	Our number system is a base-ten system. A given place value is ten times greater than the value of the place to its right (500 is ten times greater than 50).	Multi-digit Place value Ones Tens Hundreds Thousands Ten Thousands Hundred Thousands Millions Ten times Multiplication

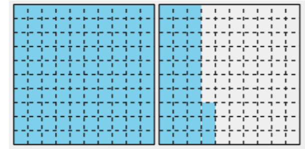
4.NBT.A.2: Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.	To read numerals between 1,000 and 1,000,000, students need to understand the role of commas. Each sequence of three digits made by commas is read as hundreds, tens, and ones, followed by the name of the appropriate base-thousand unit (thousand, million, billion, trillion, etc.). Thus, 457,000 is read "four hundred fifty seven thousand." The same methods students used for comparing and rounding numbers in previous grades apply to these numbers, because of the uniformity of the base-ten system.	Understanding place value enables us to represent, compare order and round numbers and perform computations. Numbers can be expressed in standard form, word form, and expanded form.	Multi-digit Compare Place value Digit Expanded form Written form Word form Greater than, > Less than, < Equal to, = Parentheses
4.NBT.A.3: Use place value understanding to round multi-digit whole numbers to any place.	When rounding, the goal is to find the closest multiple of 10, 100, 1000, etc. Rounding to the unit represented by the leftmost place is typically the sort of estimate that is easiest for students and often is sufficient for practical purposes. Rounding to the unit represented by a place in the middle of a number may be more difficult for students (the surrounding digits are sometimes distracting).	Rounding helps us solve problems mentally and assess the reasonableness of an answer.	Place value Digit Multi-digit Rounding Whole numbers Ones Ten(s) Hundred(s) Thousand(s) Ten Thousand(s) Hundred Thousand(s) Million(s)
4.NBT.B.4: Fluently add and subtract multi-digit whole numbers using the standard algorithm.	A crucial theme in multi-digit arithmetic is encouraging students to develop strategies that they understand, can explain, and can think about, rather than merely follow a sequence of directions that they don't understand. In grades 2 and 3, students have seen and used a variety of strategies and materials to deepen their understanding of addition and subtraction algorithms. The goal is for them to understand all the steps in an algorithm, and they should be able to explain the meaning of each digit. For example, a 1 can represent one, ten, one hundred, and so on. For multi-digit addition and subtraction in Grade 4, the goal is also fluency, which means students must be able to carry out the calculations efficiently and	There are different algorithms that can be used to add or subtract. Fluency is being efficient, accurate, and flexible with strategies.	Add Subtract Sum DIfference Standard algorithm Multi-digit

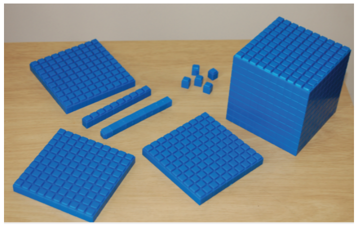
	accurately. Start with a student's understanding of a certain strategy, and then make intentional, clear-cut connections for the student to the standard algorithm. This allows the student to gain understanding of the algorithm rather than just memorize certain steps to follow.		

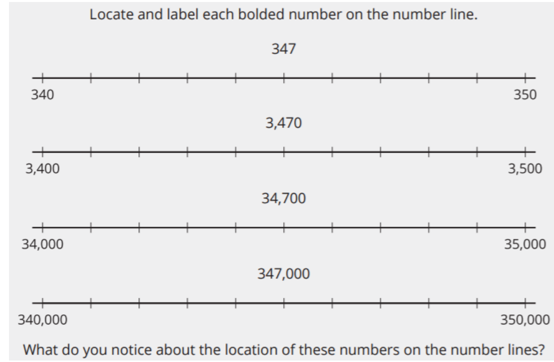
UNIT 4: DECIMAL FRACTIONS AND LARGE NUMBERS

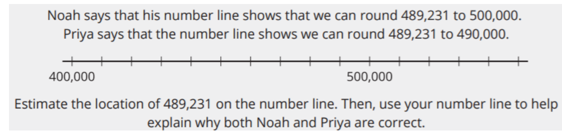
How is our number system organized?
How can understanding place value help us?
How do the properties of operations make computation simpler?
How do we decide what operation to use when solving a real-world problem?
What are equivalent fractions and decimals?

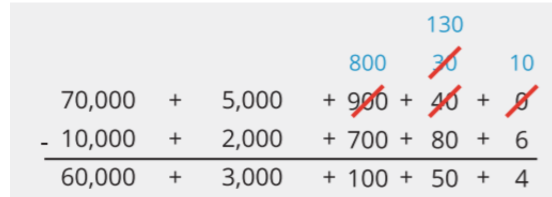
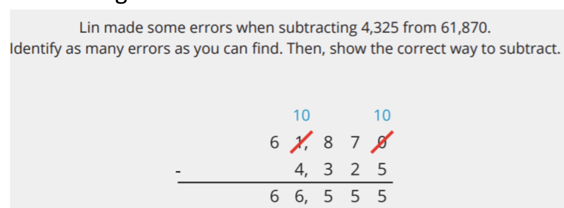
CCSS Standards #	Learning Targets	Summative Assessment Strategy	Lesson Progression and Connection to Math Practices	Common Learning Experiences and Assessments
Section A: Decimal Notation With Tenths and Hundredths				
4.NF.C.5 4.NF.C.6 4.NF.C.7	I can read, write, and represent decimals through hundredths. I can compare two decimals through hundredths.	X Selected Response X Constructed Response Performance X Observation	**Lesson Progression:** In prior units, students reasoned about fraction equivalence to understand the size of tenths and hundredths and to see that there are 10 hundredths in 1 tenth. This section introduces decimal notation as a way to represent tenths and hundredths. Students begin by reading, writing, and representing decimal fractions using decimal notation. They determine the fraction represented by the shaded region of a unit square partitioned into 100 small squares. Each unit square grid represents 1 whole unit, so each small square of each grid represents 1/100, and 10 small squares represent **1/10.**	**Mandatory Lessons/Activities:** iM Lessons 1, 2, 3, 4, 5

			 Although students may shade a grid in any pattern, the grids in this section are intentionally shaded from left to right, to reflect the digits in a decimal. For example, the number 1.33 is represented by shading one full unit square grid, 3 columns of the next unit square grid, and 3 small squares of in the adjacent column. The structure of the unit square grid enables students to reason about equivalence and see that 10/100 = 1/10 and 0.10 = 0.1. These equations help students make generalizations about other equivalent tenths and hundredths (for instance, 0.20 = 0.2, 0.50 = 0.5, and so on). Later in the section, students compare and order decimals based on size. They locate and label decimals on a number line, using benchmarks such as 0.5, and an understanding of the relationship between tenths and hundredths. Students then compare decimals using the symbols <, >, and =.	
Pacing:	5 days		**Math Practices:** SMP 1, 2, 3, 4, 5, 6	**Assessments:** Cool-downs: 1, 3, 4 Checkpoint A
Section B: Place-Value Relationships Through 1,000,000				
[4.NBT.A.1](#) [4.NBT.A.2](#)	I can explain the patterns found in place value. I can read, write, and represent numbers to 1,000,000.	X – Selected Response X – Constructed Response – Performance X – Observation	**Lesson Progression:** Students develop an understanding of the place value of multi-digit whole numbers through 1,000,000. In the first few lessons, students use base-ten blocks and base-ten diagrams to represent and make sense of numbers to the ten-thousands place and then the hundred thousands place. Starting from what they know, students count by 10, 100, and 1,000 to learn how to say, read, write multi-digit numbers.	**Mandatory Lessons/Activities:** iM Lessons 6, 7, 8, 9, 10, 11

			Students will quickly see that the use of base-ten blocks as tools to represent large numbers is limiting. To represent numbers that are more than four-digits long using base-ten blocks, they change the value of the smallest block to 10. For example, the blocks in this image represent 1,325 if the smallest block represents 1, but the value of the blocks becomes 13,250 or 10 times as much if the smallest block represents 10.  Students learn that they can use this adjustment if they are to use base-ten blocks to represent numbers to the hundred-thousands place. For example, the number 109,990 can be represented with 10 large cube blocks, 9 large square blocks, 9 long rectangular blocks, and 9 small cube blocks. Although the materials available in the classroom may not allow students to actually build such a representation, they should be encouraged to describe or draw a base-ten diagram to represent the blocks needed. As students analyze and draw base-ten diagrams and write multi-digit numbers in expanded form, they begin to understand the value of each digit in a multi-digit number. They describe the “ten times” relationship between the value of each place and that of the place to its right, and represent this relationship with multiplication and division equations. For example, students recognize that the value of the 3 in 347,000 is ten times the value of the 3 in 34,700 because 30,000 x 10 = 300,000. Similarly, the value of the 4 in 347,000 is ten times the value of the 4 in 34,700 because 4,000 = 40,000 ÷ 10. Students also see this “ten times” relationship as	

			they locate numbers on a number line. With the number lines partitioned into intervals of 10, students realize that by multiplying the number by ten, it remains in the same relative position when the endpoints also increase by a factor of 10.  The use of the number line also helps students prepare to compare, order and round numbers in the next section.	
Pacing:	6 days		**Math Practices:** SMP 3, 5, 6, 7	**Assessments:** Cool-down: 10 Checkpoint B
Section C: Compare, Order, and Round Within 1,000,000				
4.NBT.A.2 4.NBT.A.3	I can compare two multi-digit numbers using the symbols <, >, =. I can round numbers to any place.	X Selected Response X Constructed Response Performance X Observation	**Lesson Progression:** Students compare and order numbers within 1,000,000. They also extend their understanding of rounding from numbers within 1,000 to numbers within 1,000,000. In the first few lessons, students place multi-digit numbers on a number line with increasing levels of precision. They then compare large numbers, and they state these comparisons using the symbols >, =, and <. In comparing numbers, including those that are missing digits in some places, they reason and make generalizations about the size of numbers and the significance of the value of the digits.	**Mandatory Lessons/Activities:** iM Lessons 12, 13, 14, 15, 16, 17

			Is it possible to fill in the two blanks with the same digit to make: 1. 4□, 300 less than 3□, 400? 2. □4, 300 less than □3, 400? The use of the number line also elicits students’ work from grade 3 about what it means to be “close to a number” and to round a number. Students round numbers within 1,000,000 to the nearest multiples of 10, 100, 1,000, 10,000 and 100,000.  Students extend the conventions they used in grade 3 when rounding numbers that fall on the midpoint of two consecutive multiples of 10, 100, or 1,000. For example, 25 rounded to the nearest multiple 10 is 30 by convention, even though 25 is the same distance away from 20 and from 30. Students apply this convention as they round numbers within 1,000,000 to any place. To apply their understanding to solve contextual problems and consider the benefits and limitations of rounding large numbers in different situations.	
Pacing:	6 days		**Math Practices:** SMP 1, 2, 3, 5, 6, 7	**Assessments:** Cool-downs: 12, 16 Checkpoint C
Section D: Add and Subtract Within 1,000,000				
4.NBT.A.2 4.NBT.A.3 4.NBT.B.4 4.NF.B.3.c	I can add and subtract multi-digit whole numbers using the standard algorithm.	X Selected Response X Constructed Response Performance X Observation	**Lesson Progression:** In grade 3, students use various representations and strategies—including the standard algorithm—to add and subtract within 1,000. To build on these strategies, and to clarify the connection between place value and the standard algorithm, students first use expanded form to add and subtract multi digit numbers. The form prompts students to find partial sums and differences, reason about place value, and think preemptively about what happens when a	**Mandatory Lessons/Activities:** iM Lessons 18, 19, 20, 21, 22, 23

			subtraction problem requires regrouping (decomposing a unit in a place into 10 of the unit in the place to its right).  In later lessons, students encounter examples in which regrouping occurs more than one time in order to complete the subtraction, and learn how to notate the regrouping. For example, they consider cases in which large numbers without zeros are subtracted from numbers with zeros, and they also examine common errors made when subtracting.  By the end of grade 4, students become fluent with the procedures of the standard algorithm for addition and subtraction within 1,000,000. The centers that align with this section, as well as practice problems and explorations can be used beyond this unit to support the development of fluency with the standard algorithm.	
Pacing:	6 days		**Math Practices:** SMP 1, 2, 3, 4, 5, 6, 7, 8	**Assessments:** Cool-downs: 18, 19, 20 Checkpoint D

ADDITIONAL CONSIDERATIONS			
COMMON MISCONCEPTIONS	**PRIOR KNOWLEDGE NEEDED TO MASTER STANDARDS FOR THIS UNIT**	**ADVANCED STANDARDS FOR STUDENTS WHO HAVE DEMONSTRATED PRIOR MASTERY**	**OPPORTUNITIES FOR STUDENT-DIRECTED LEARNING WITHIN THE UNIT**
Students use whole number thinking when working with decimals. For example, students may think that the more digits after the decimal point, the greater the value, i.e. 0.25 is greater than 0.3. Students may have misconceptions about writing numerals from verbal descriptions. Numbers like one thousand do not cause a problem; however a number like one thousand two causes problems for students. Many students will understand the 1000 and the 2 but then instead of placing the 2 in the ones place, students will write the numbers as they hear them, 10002 (ten thousand two). Students often assume that the first digit of a multi-digit number indicates the magnitude of a number. The assumption is made that 954 is greater than 1002 because students are focusing on the first digit instead of the number as a whole. Students need to be aware of the greatest place value. Students may not have a conceptual understanding of place value so they would think 561 – 147 = 426, because they subtract the 7 in 147 from the 1 in 561 instead of regrouping.	4.NF.C.5: 4.NF.A.1, 4.NF.B.3 4.NF.C.7: 4.NF. A.2, 4.NF.C.6 4.NBT.A.1: 3.NBT.A.1 4.NBT.A.2: 4.NBT.A.1 4.NBT.A.3: 3.NBT.A.1, 4.NBT.A.1, 4.NBT.A.2 4.NBT.B.4:3.NBT.A.2, 4.NBT.A.1	Choose from iM leveled centers and exploration problems to differentiate for students who are ready.	iM Centers District-approved online resources

RESOURCES
Kendall Hunt Flourish Blackline masters and materials from Teacher Resource Pack colored pencils, Base-ten blocks, dot stickers, grid paper

UNIT 5: MULTIPLICATIVE COMPARISON AND MEASUREMENT
Illustrative Mathematics Unit Focus: In this unit, students deepen their understanding of multiplication and of measurement. **Essential Questions:** How can we use multiplication to compare quantities? How do we convert units of time or measure? Why do we collect, organize, represent and analyze data? **Unit Pacing: 26 days (20 required lessons, 4 flex, 2 assessment and reaction)**

UNWRAPPED STANDARDS

Grade Level Standard	Standard Progression	Concepts (Big Ideas/ Understandings)	Academic Vocabulary (Standard Based)
4.OA.1 Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.	In a multiplicative comparison, the underlying question is 'what factor would multiply one quantity' in order to result in the other?'	Comparisons can be additive or multiplicative depending on the mathematical situation. In multiplicative comparisons, the relationship between quantities is described in terms of how many times larger one is than the other	Multiplication Equation Multiplicative Comparison Times as many as
4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.	Students need to solve word problems involving multiplicative comparison (product unknown, partition unknown) using multiplication or division as described in the Common Core State Standards Mathematics Glossary, Table 2. They should use drawings or equations with a symbol for the unknown number to represent the problem. Students need to be able to distinguish whether a word problem involves multiplicative comparison or additive comparison (solved when adding and subtracting in Grades 1 and 2).	Comparisons can be additive or multiplicative depending on the mathematical situation. In multiplicative comparisons, the relationship between quantities is described in terms of how many times larger one is than the other.	Multiplication Equation Multiplicative Additive Comparison Times as many as Equation Unknown Symbol

4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.	Numbers can be those in the Grade 4 standards, but the number of steps should be no more than three and involve only easy and medium difficulty addition and subtraction problems. Present multi-step word problems with whole numbers and whole-number answers using the four operations. Students should know which operations are needed to solve the problem. Drawing pictures or using models will help students understand what the problem is asking. They should check the reasonableness of their answer using mental computation and estimation strategies.	Estimation strategies, including rounding, can be used to determine the reasonableness of answers. An unknown can be in any position of a story problem. A remainder in a division problem should be interpreted based on understanding of number and context.	Operations Equation Mental computation Estimation Rounding Dividend Divisor Quotient Remainder Unknown Multistep Reasonableness
4.NBT.B.4: Fluently add and subtract multi-digit whole numbers using the standard algorithm.	A crucial theme in multi-digit arithmetic is encouraging students to develop strategies that they understand, can explain, and can think about, rather than merely follow a sequence of directions that they don't understand. In grades 2 and 3, students have seen and used a variety of strategies and materials to deepen their understanding of addition and subtraction algorithms. The goal is for them to understand all the steps in an algorithm, and they should be able to explain the meaning of each digit. For example, a 1 can represent one, ten, one hundred, and so on. For multi-digit addition and subtraction in Grade 4, the goal is also fluency, which means students must be able to carry out the calculations efficiently and accurately. Start with a student's understanding of a certain strategy, and then make intentional, clear-cut connections for the student to the standard algorithm. This allows the student to gain understanding of the algorithm rather than just memorize certain steps to follow.	There are different algorithms that can be used to add or subtract. Fluency is being efficient, accurate, and flexible with strategies.	Add Subtract Sum DIfference Standard algorithm Multi-digit
4.NBT.B.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on	As with addition and subtraction, students should use methods they understand and can explain. Visual representations such as area and	Properties of operations allow us to reorder, decompose and/or compose numbers in order to make computation simpler.	Multiply Equation Product

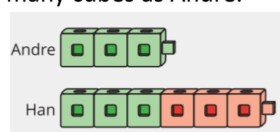
place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	array diagrams that students draw and connect to equations and other written numerical work are useful for this purpose. By reasoning repeatedly about the connection between math drawings and written numerical work, students can come to see multiplication and division algorithms as abbreviations or summaries of their reasoning about quantities. One component of understanding general methods for multiplication is understanding how to compute products of one-digit numbers and multiples of 10, 100, and 1000. This extends work in Grade 3 on products of one-digit numbers and multiples of 10. We can calculate 6 x 700 by calculating 6 x 7 and then shifting the result to the left two places (by placing two zeros at the end to show that these are hundreds) because 6 groups of 7 hundred is 6 x 7 hundreds, which is 42 hundreds, or 4,200.		Area model Array Distributive Property Commutative Property Associative Property Strategies
4.NF.B.3: Understand a fraction a/b with a > 1 as a sum of fractions 1/b.	In Grade 3, students represented whole numbers as fractions. In Grade 4, they will use this knowledge to add and subtract mixed numbers with like denominators using properties of number and appropriate fraction models. It is important to stress that whichever model is used, it should be the same for the same whole. For example, a circular model and a rectangular model should not be used in the same problem. The meaning of addition is the same for both fractions and whole numbers, even though algorithms for calculating the sums can be different. This simple understanding of addition as putting together allows students to see in a new light the way fractions are built up from unit fractions. The same representation that students used in Grade 4 to see a fraction as a point on the number line now allows them to see a fraction as a sum of unit fractions. Just as 5 = 1 + 1 + 1 + 1 + 1, so	We decompose fractions into sums or products of fractions to make computation easier or to simplify expressions. Fractions can be added and subtracted when the wholes are the same size. Fractions with the same denominators can be added and subtracted using visual models, properties of operations, and relationships of addition and subtraction of whole numbers. We decompose fractions into sums or products of fractions to make computation easier or to simplify expressions. Mixed numbers can be written as fractions, e.g., 14/3 = 4 2/3 , and can be added or subtracted in this form. When solving word problems, students need to attend to the underlying quantities.	Fractions Unit fraction Equivalent Numerator Denominator Decompose Ordering Mixed number

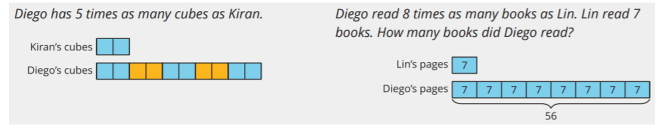
	$\frac{5}{3} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$ Because 5/3 is the total length of five thirds. Armed with this insight, students decompose and compose fractions with the same denominator. They add fractions with the same denominator.		
4.NF.B.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.	Understanding of multiplication of whole numbers is extended to multiplying a fraction by a whole number. Allow students to use fraction models and drawing to show their understanding. Present word problems involving multiplication of a fraction by a whole number. Have students solve the problems using visual models and write equations to represent the problems.	The idea of the numerator as a multiplier can be used when a fraction is being multiplied by a whole number, e.g., Just as 5/8 = 5× 1/8 , 5 groups of 3/8 equals 5 × 3/8 = (5×3)× 1/8 which equals 15/8 . Arrays, number lines, fraction strips, or sets can be used to find the solution to multiplying a whole number by a fraction. The relationship between multiplication and division is applied to fractions just as it is applied to whole numbers.	Fractions Whole number Multiple Fraction model
4.MD.A.1 Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two- column table. For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...	Relating units within the metric system is another opportunity to think about place value. For example, students might make a table that shows measurements of the same lengths in centimeters and meters.	Larger units can be expressed in terms of smaller units. The number of units used to measure an object will depend on the size of the unit of measure. The larger the unit, the smaller the measurement reads; the smaller the unit, the larger the measurement reads. Metric units are related by powers of ten.	Metric System Kilometer Meter Centimeter Millimeter Gram Kilogram Milliliter Liter Table
4.MD.A.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing	Students combine competencies from different domains as they solve measurement problems using all four arithmetic operations: addition, subtraction, multiplication, and division. For example, “How many liters of juice does the class need to have at least 35 cups if each cup	We convert units of time or measure by understanding how the units are related to each other (e.g. one foot is 12 times one inch).	Interval Decimal notation Line diagrams Hours Minutes Elapsed Time

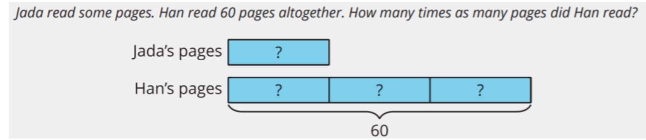
measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.	takes 225 ml?" Students may use tape or number line diagrams for solving such problems (MP1).		Liquid volume Mass Metric System Kilometer Meter Centimeter Millimeter Gram Kilogram Milliliter Liter
4.MD.B.3 Apply the area and perimeter formulas for rectangles in real world and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.	Students used models to find area and perimeter in Grade 3. They need to relate discoveries from the use of models to develop an understanding of the area and perimeter formulas to solve real-world and mathematical problems. By repeatedly reasoning about how to calculate areas and perimeters of rectangles, students can come to see area and perimeter formulas as summaries of all such calculations.	The area of a rectangle can be found by multiplying the lengths of adjacent sides (length and width) of the rectangle. Given an area or a perimeter of a rectangle and one side length, the adjacent side length can be determined.	Perimeter Area Adjacent Strategy
4.MD.B.4 Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.	Grade 4 students learn elements of fraction equivalence and arithmetic, including multiplying a fraction by a whole number and adding and subtracting fractions with like denominators. Students can use these skills to solve problems, including problems that arise from analyzing line plots. For example, with reference to the line plot above, students might find the difference between the greatest and least values in the data. (In solving such problems, students may need to label the measurement scale in eighths so as to produce like denominators. Decimal data can also be used in this grade.)	Data can be organized and represented in a picture graph, a bar graph, or a line plot. Symbols used in line plots should be consistently spaced and sized for visual accuracy. Information presented in a graph can be used to solve problems involving the data in the graph.	Line plot Interpret Data

UNIT 5: MULTIPLICATIVE COMPARISON AND MEASUREMENT

How can we use multiplication to compare quantities?
How do we convert units of time or measure?
Why do we collect, organize, represent and analyze data?

CCSS Standards #	Learning Targets	Summative Assessment Strategy	Lesson Progression and Connection to Math Practices	Common Learning Experiences and Assessments
Section A: Multiplicative Comparison				
4.NBT.B.5 4.OA.A.1 4.OA.A.2 4.OA.A.3	I can analyze, describe, represent, and solve multiplicative comparison situations.	X Selected Response X Constructed Response Performance X Observation	**Lesson Progression:** In earlier grades, students compared quantities by solving additive comparison problems. In an additive comparison, the underlying question is “How many more?” In this section and throughout the unit, students compare quantities by solving multiplicative comparison problems. In a multiplicative comparison, the underlying question is “How many times as many? For example, the image shows that Andre has 3 cubes and Han has 6 cubes. Students know from prior grades that Han has 3 more cubes than Andre. In this section, students learn that they can describe this comparison by saying, “Han has 2 times (or twice) as many cubes as Andre.”  In the beginning, students describe multiplicative comparisons that involve small factors and familiar situations (such as comparing blocks). They build on familiar multiplicative comparison language (“twice,” or “twice as many”). The goal is to focus on the language and representations for describing multiplicative comparisons. The representations progress from concrete (actual cubes) to discrete (diagrams showing cubes, or tape diagrams where each section of the diagram represents one object). Then, as they encounter larger factors and more abstract situations, students interpret and use abstract tape diagrams where each section of the diagram is labeled to	**Mandatory Lessons/Activities:** iM lesson 1, 2, 3, 4, 5, 6

<table>
<tr><td></td><td></td><td></td><td>represent any quantity.


Students use multiplication to compare two quantities, using these diagrams to support their reasoning. As the problems become more complex, students also use these diagrams to reason about how division may be used to find a missing factor.

</td><td></td></tr>
<tr><td>Pacing:</td><td>6 days</td><td></td><td>Math Practices:
SMP 1, 2, 3, 4, 5, 6, 7, 8</td><td>Assessments:
Cool-downs: 3 and 4
Checkpoint A</td></tr>
<tr><td colspan="5">Section B: Measurement Conversion</td></tr>
<tr><td>4.MD.A.1
4.MD.A.2
4.OA.A.2
4.OA.A.3</td><td>I can use the relationship between units to make conversions from larger units to smaller units within a given system of measurement.

I can solve measurement word problems using the four operations.</td><td><table><tr><td>X</td><td>Selected Response</td></tr><tr><td>X</td><td>Constructed Response</td></tr><tr><td></td><td>Performance</td></tr><tr><td>X</td><td>Observation</td></tr></table></td><td>Lesson Progression:
Prior to this unit, students have encountered units of measurement in the classroom and in their daily lives. They measured and estimated lengths in centimeters and meters, and solved problems involving capacity and mass. Students have also recognized the number of minutes in an hour and learned to measure intervals of time. In this section, they expand on these concepts of measurement to convert within the same system (metric or customary) from larger units to smaller units. These conversions require an understanding of the multiplicative relationship between units. The section begins by building on the understanding of multiplicative comparison from the previous section and on student knowledge that there are 100 centimeters in 1 meter. To develop a sense of the multiplicative relationship between centimeters and meters, students build a meter from centimeter grid paper. They recognize that 1 meter is 100 times as long as 1 centimeter and use this reasoning to convert any number of meters to centimeters. Later, they build an intuition of the size of 1 kilometer by relating it to multiples of shorter, more-familiar measurements, such as</td><td>Mandatory Lessons/Activities:
iM Lessons 7, 8, 9, 10, 11, 12, 13</td></tr>
</table>

			the length of a basketball court or a soccer field. Throughout the section, students learn the multiplicative relationships between various units of measurement: centimeters, meters, and kilometers, grams and kilograms, milliliters and liters, ounces and pounds, and seconds, hours, and minutes. In each instance, they solve problems to develop a sense of the relative size of the units and convert larger units to smaller units using multiplication.	
Pacing:	7 days		**Math Practices:** SMP 1, 2, 3, 4, 5, 6, 7, 8	**Assessments:** Cool-downs: 7, 8, 9, 10, 12 Checkpoint B
Section C: Measurement Data and Line Plots				
4.MD.B.4 4.NF.B.3 4.NF.B.4	I can create a line plot to display measurement data in fractions. I can use information from a line plot to solve problems.	X — Selected Response X — Constructed Response Performance X — Observation	**Lesson Progression:** Students use measurement data to create and interpret line plots that include units measured in halves, fourths, and eighths. Students are given measurement data to plot on a line plot and analyze, and then prompted to ask and answer questions about the data. Because the data include measurements with different denominators, students rely on their knowledge of fraction equivalence and comparison to accurately plot the data. Prior to this point, students have solved problems that involve adding and subtracting fractions and mixed numbers with like denominators. They have also reasoned about sums and differences of fractions with related denominators (thirds and sixths, for instance). In this section, they apply these understandings of fraction equivalence and operations to solve measurement problems.	**Mandatory Lessons/Activities:** iM Lessons 14, 15, 16
Pacing:	3 days		**Math Practices:** SMP 1, 2, 3, 4, 5, 6, 7, 8	**Assessments:** Cool-downs: 15 Checkpoint C

Section D: Let's Put it to Work: Multiplicative Comparison and Measurement				
4.MD.A.1 4.MD.A.2 4.MD.A.3 4.NBT.B.4 4.NBT.B.5 4.OA.A.2	I can solve real-world problems involving all four operations.	X Selected Response X Constructed Response Performance X Observation	**Lesson Progression:** Students use their understanding of multiplicative comparison and their emerging strategies for converting units of measurement to solve real-world, multi-step problems. They continue to develop their understanding of relative sizes of units within systems as they convert customary and metric units of length, mass, and capacity. Students work with measurement units introduced in earlier sections (pounds, ounces, kilometers, meters, centimeters), some from previous grades (yards, feet, and inches), as well as some new ones (gallons, quarts, and cups). The last two lessons in this section allow students to explore multiplicative relationships in a geometric context. Students analyze the relationship between the side lengths and the perimeters of quadrilaterals. They apply multiplicative reasoning as they solve problems involving measurement conversions.	**Mandatory Lessons/Activities:** iM Lessons 17, 18, 19, 20
Pacing:	5 days		**Math Practices:** SMP 1, 2, 3, 4, 5, 6, 7, 8	**Assessments:** Checkpoint D

ADDITIONAL CONSIDERATIONS			
COMMON MISCONCEPTIONS	**PRIOR KNOWLEDGE NEEDED TO MASTER STANDARDS FOR THIS UNIT**	**ADVANCED STANDARDS FOR STUDENTS WHO HAVE DEMONSTRATED PRIOR MASTERY**	**OPPORTUNITIES FOR STUDENT-DIRECTED LEARNING WITHIN THE UNIT**
Students may confuse multiplicative comparison with additive comparison. The important question for multiplicative comparison problems is which factor to multiply one quantity in order to result in the other quantity. Key words are misleading. Some key words typically mean addition or subtraction, but not always. Consider: There were 4 jackets left on the playground on Monday and 5 jackets left on the playground on Tuesday. How many jackets were left on the playground? "Left" in this problem does not mean subtract. Many problems have no key words. For example, How many legs do 7 elephants have?, does not have a key word. However, students should be able to solve the problem by thinking and drawing a picture or building a model. The most important strategy, when solving a problem, is to make sense of the problem's context and actions. Key words encourage students to ignore meaning and look for a formula. Mathematics is about meaning (Van de Walle, 2012). Students have difficulty estimating a two-step problem. Students do not always solve all of the steps needed for a multistep problem. Students may not be able to identify	4.OA.1: 3.OA.1, 3.OA.3 4.OA.A.2: 3.OA.3 4.OA.A.3: 3.OA.D.8, 4.NBT.A.3, 4.NBT.B.6 4.NBT.B.4: 3.NBT.A.2, 4.NBT.A.1 4.NBT.B.5: 3.NBT.A.2, 3.NBT.A.3, 3.OA.B.5, 3.OA.C.7, 4.NBT.A.1 4.NF.B.3: 3.NF.A.1, 3.NF.A.2, 4.NF.A.1 4.NF.B.4: 3.NF.A.1, 3.OA.1, 3.OA.3, 4.OA.2 4.MD.A.1: 3.MD.A.2, 3.OA.C.7 4.MD.A.2: 4.MD.A.1, 4.NF.C.5, 4.NF.C.6, 4.NF.B.4 4.MD.A.3 : 3.MD.D.8, 3.OA.A.4, 3.MD.C.7 4.MD.B.4: 3.MD.B.4	Choose from iM leveled centers and exploration problems to differentiate for students who are ready.	iM Centers District-approved online resources

which part of the equation is unknown in order to represent it as a variable. Students often do not understand why they need to regroup and just subtract the smaller digit from the larger one. Emphasize place value and the meaning of each of the digits. When converting from a larger unit of measure to a smaller unit, students may divide rather than multiply. Students need to understand that you need more of the smaller units. Students may mistakenly choose to display non-numerical data in a line plot, for example "Favorite Pizza Toppings". When making a line plot, students might not remember to include every number within the range of data.			
RESOURCES			
Kendall Hunt Flourish Blackline masters and materials from Teacher Resource Pack Connecting cubes, index cards, scissors, tape, colored pencils, rulers, yardsticks, tape, sticky notes			

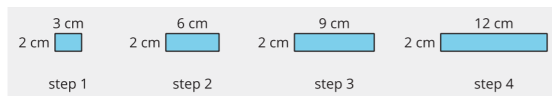
UNIT 6: WHOLE-NUMBER MULTIPLICATION AND DIVISION
Illustrative Mathematics Unit Focus: Students build upon their understanding of factors and multiples to generate and identify features of number patterns. They then learn to multiply and divide multi-digit whole numbers using partial products and partial quotients strategies, and apply this understanding to solve multi-step problems using the four operations. **Essential Questions:** Why do we analyze patterns? How can understanding place value help us? How do the properties of operations make computation simpler? How are multiplication and division related? How do we decide what operation to use when solving a real-world problem? **Unit Pacing: 32 days (24 required lessons, 6 flex, 2 assessment and reaction)**

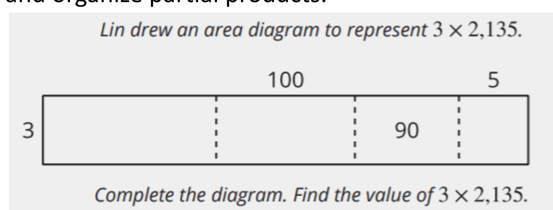
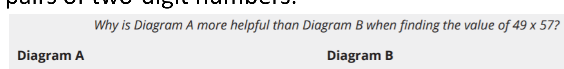
UNWRAPPED STANDARDS			
Grade Level Standard	**Standard Progression**	**Concepts (Big Ideas/ Understandings)**	**Academic Vocabulary (Standard Based)**
4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.	Students need to solve word problems involving multiplicative comparison (product unknown, partition unknown) using multiplication or division as described in the Common Core State Standards Mathematics Glossary, Table 2. They should use drawings or equations with a symbol for the unknown number to represent the problem. Students need to be able to distinguish whether a word problem involves multiplicative comparison or additive comparison.	Comparisons can be additive or multiplicative depending on the mathematical situation. In multiplicative comparisons, the relationship between quantities is described in terms of how many times larger one is than the other.	Multiplication equation Multiplicative comparison

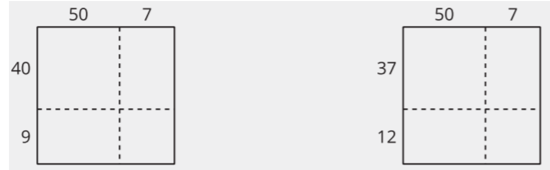
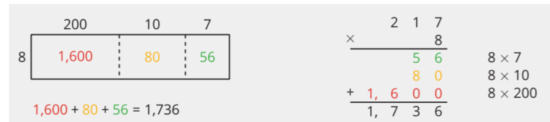
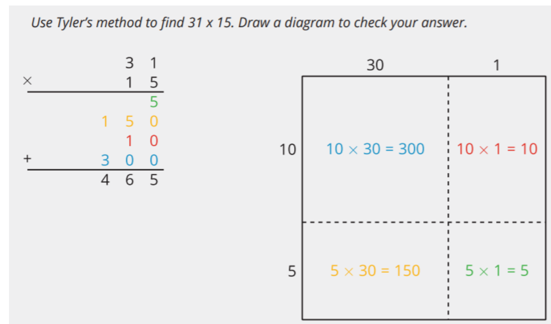
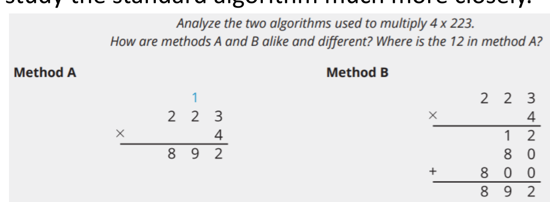
4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.	Present multi-step word problems with whole numbers and whole-number answers using the four operations. Students should know which operations are needed to solve the problem. Drawing pictures or using models will help students understand what the problem is asking. They should check the reasonableness of their answer using mental computation and estimation strategies.	Estimation strategies, including rounding, can be used to determine the reasonableness of answers. An unknown can be in any position of a story problem. Interpreting the remainder is developed over time. They should not be introduced to or held accountable for rules of working with remainders. Students should justify their interpretations. A remainder in a division problem should be interpreted based on understanding of number and context.	Operations Equations Estimation Rounding Dividend Divisor Quotient Remainder Unknown quantity Multistep Reasonableness
4.OA.B.4 Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors.	A factor is a number that is multiplied by another number to yield a product. Two and eight are factors of sixteen. A multiple is the product of a whole number and any other whole number. Sixteen is a multiple of eight 1, 3, 7, and 21 are factor pairs of 21. Students find factor pairs of numbers to 100. Students should use basic fact recall and number sense to identify factor pairs. Factoring procedures should be avoided. Explicit instruction of divisibility rules is not appropriate, however student discovery of these rules could be an enrichment within this standard for students who show proficiency with factors, multiples, and prime/composite numbers.	A number can be multiplicatively decomposed into factor pairs and expressed as a product of these factor pairs. Any whole number is a multiple of each of its factors.	Factor Product Multiple
4.OA.C.5 Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself.	In order for students to be successful later in the formal study of algebra, their algebraic thinking needs to be	Analyzing patterns helps us make predictions, identify trends, and form rules to solve problems.	Number pattern Shape pattern Generate

For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.	developed. Understanding patterns is fundamental to algebraic thinking. Students have experience in identifying arithmetic patterns, especially those included in addition and multiplication tables. Contexts familiar to students are helpful in developing students' algebraic thinking. Students should generate numerical or geometric patterns that follow a given rule. They should look for relationships in the patterns and be able to describe and make generalizations.		Identify Features Rule Analyze
4.NBT.B.4: Fluently add and subtract multi-digit whole numbers using the standard algorithm.	A crucial theme in multi-digit arithmetic is encouraging students to develop strategies that they understand, can explain, and can think about, rather than merely follow a sequence of directions that they don't understand. In grades 2 and 3, students have seen and used a variety of strategies and materials to deepen their understanding of addition and subtraction algorithms. The goal is for them to understand all the steps in an algorithm, and they should be able to explain the meaning of each digit. For example, a 1 can represent one, ten, one hundred, and so on. For multi-digit addition and subtraction in Grade 4, the goal is also fluency, which means students must be able to carry out the calculations efficiently and accurately. Start with a student's understanding of a certain strategy, and then make intentional, clear-cut connections for the student to the standard algorithm. This allows the student to gain understanding of the algorithm rather than just memorize certain steps to follow.	There are different algorithms that can be used to add or subtract. Fluency is being efficient, accurate, and flexible with strategies.	Add Subtract Sum Difference Standard algorithm Multi-digit

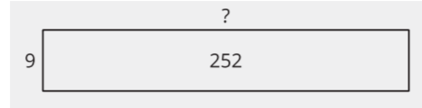
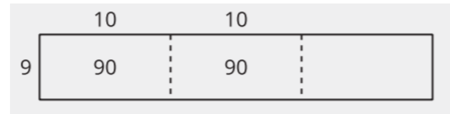
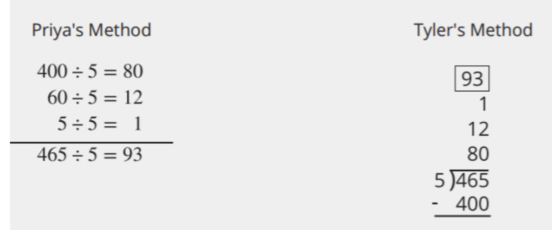
4.NBT.B.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	As with addition and subtraction, students should use methods they understand and can explain. Visual representations such as area and array diagrams that students draw and connect to equations and other written numerical work are useful for this purpose. By reasoning repeatedly about the connection between math drawings and written numerical work, students can come to see multiplication and division algorithms as abbreviations or summaries of their reasoning about quantities.	Understanding place value enables us to represent, compare order and round numbers and perform computations. Properties of operations allow us to reorder, decompose and/or compose numbers in order to make computation simpler. There is a relationship between multiplication and division.	Multiply Equation Area model Rectangular arrays Factor Multiple Product Distributive Property Commutative Property
4.NBT.B.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	General methods for computing quotients of multi-digit numbers and one-digit numbers rely on the same understandings as for multiplication, but cast in terms of division. One component is quotients of multiples of 10, 100, or 1000 and one-digit numbers. For example, 42 divided by 6 is related to 420 divided by 6 and 4200 divided by 6. Students can draw on their work with multiplication and they can also reason that 4200 divided by 6 means partitioning 42 hundreds into 6 equal groups, so there are 7 hundreds in each group.	Understanding place value enables us to represent, compare order and round numbers and perform computations. Properties of operations allow us to reorder, decompose and/or compose numbers in order to make computation simpler. There is a relationship between multiplication and division.	Division Quotient Remainder Dividend Divisor Equation Area model Rectangular arrays

UNIT 6: WHOLE-NUMBER MULTIPLICATION AND DIVISION				
Why do we analyze patterns? How can understanding place value help us? How do the properties of operations make computation simpler? How are multiplication and division related? How do we decide what operation to use when solving a real-world problem?				
CCSS Standards #	**Learning Targets**	**Summative Assessment Strategy**	**Lesson Progression and Connection to Math Practices**	**Common Learning Experiences and Assessments**
Section A: Features of Patterns				
[4.OA.C.5](#)	I can generate a number or shape pattern that follows a given rule.	X Selected Response X Constructed Response Performance X Observation	**Lesson Progression and connections:** Students notice and describe features of patterns. Given the rule of a pattern, they predict the values or features of future terms in a pattern sequence. Rather than extending the pattern one step at a time, which would be inefficient, students use their understanding of operations and place value to make predictions. The section begins with patterns that are more concrete—such as shapes with features that visibly elicit addition or multiplication—and moves toward patterns that involve repeating objects or numbers. It closes with an exploration of patterns in the features—side length, perimeter, and area—of rectangles that change by a rule. The section includes multiple opportunities to build on students' understanding of factors and multiples from an earlier unit.  *If the pattern continues, could 50 represent the side length or area of one of the rectangles? If so, which step? If not, why not?*	**Mandatory Lessons/Activities:** iM Lessons 1, 2, 3, 4
Pacing:	4 days		**Math Practices:** SMP 3, 5, 6, 7, 8	**Assessments:** Cool-downs: 2, 4 Checkpoint A

Section B: Multi-digit Multiplication				
[4.MD.A.2](#) [4.NBT.B.4](#) [4.NBT.B.5](#) [4.OA.A.3](#)	I can represent and solve multi-digit multiplication problems.	X | Selected Response X | Constructed Response | Performance X | Observation	**Lesson Progression and connections:** This section extends students' ability to find products of numbers in base ten. Students learn to multiply one-digit numbers by numbers up to four digits, multiply a pair of two-digit numbers, and to use increasingly more efficient methods. A key thread through the lessons is the idea of decomposing factors—particularly by place value—as a productive way of finding products. Students start exploring this idea with concrete and visual representations—arrays, base-ten diagrams, and rectangular grids. As they break up larger factors to facilitate multiplication, they see the limits of concrete representations, motivating more efficient representations and strategies. Students recognize that rectangular area diagrams offer a practical way to represent multiplication, with the sides of the rectangle representing the factors and the area representing the product. Initially, students see gridded rectangles, with unit squares marking their area. Then, they see diagrams with no grid, but with side lengths that are scaled to represent the factors in the product. As the lessons progress to include larger numbers, the diagrams are more abstract and not drawn to scale. Students use this rectangular diagram as a visual tool to decompose factors by place value and organize partial products.  The benefits of decomposing factors by place value also become more apparent as students multiply pairs of two-digit numbers.  	**Mandatory Lessons/Activities:** iM Lessons 5, 6, 7, 8, 9, 10, 11, 12

			 Later, students encounter a partial-products algorithm, a different way to record the reasoning they used with diagrams. They learn that the partial products can be listed vertically, instead of in the boxes of a rectangular diagram.   The partial-products algorithm offers an opportunity to preview the standard algorithm for multiplication. Students take a peek at how this algorithm records the process of multiplying a multidigit number by a single-digit number where no regrouping is needed. In grade 5, students will study the standard algorithm much more closely.  	
Pacing:	8 days		**Math Practices:** SMP 1, 2, 3, 5, 6, 7, 8	**Assessments:** Cool-downs: 7, 10 Checkpoint B

Section C: Multi-digit Division				
4.MD.A.3 4.NBT.B.6 4.OA.A.3 4.OA.B.4	I can represent and solve multi-digit division problems.	X — Selected Response X — Constructed Response Performance X — Observation	**Lesson Progression and connections:** In grade 3, students made sense of division in relation to multiplication and in terms of equal-size groups. They reasoned about division problems in context and found whole-number quotients from two-digit dividends and one-digit divisors. In this section, students build on and extend these understandings. They find quotients from larger dividends (up to four digits), investigate new division strategies and ways to represent them, and interpret division situations that involve remainders. Students begin by recalling two ways to interpret a division expression. The expression 96 ÷ 8, for instance, can be used to find the number of groups of 8 in 96 items, or to find the size of one group if 96 items are split into 8 groups. Both interpretations are related to a multiplication equation n x 8 = 96, or 8 x n = 96 , so dividing can be understood as finding an unknown factor. Students use their knowledge of factors and multiples to help them do so. Han starts writing multiples of a number. When he reaches 104, he has written 8 numbers. What number is Han writing multiples of? Han gets to 286. How many numbers has he written at that point? Next, students reason about division in the context of finding an unknown side length of a rectangle when its area and the other side length are known. They notice that the missing side length can be found in parts, by decomposing the area of the rectangle into smaller rectangles. This process is the reverse of what they have done with multiplication: instead of decomposing the factors, we can decompose the dividend. A pool has an area of 252 square meters. Its width is 9 meters and its length is a whole number of meters. What is the length of the pool? *To find out, Diego drew this rectangular diagram.*	**Mandatory Lessons/Activities:** iM Lessons 13, 14, 15, 16, 17, 18, 19, 20

			 *He then started to decompose it into smaller rectangles, each with a width of 9 meters. "Nine times 10 is 90. Another 9 times 10 is another 90."*  *Complete Diego's reasoning.* Instead of finding partial products, we can find partial quotients. Even though the dividends could be decomposed in different ways, students once again find that it is often helpful to decompose the numbers by place value. Students learn to use a series of equations and a vertical recording method to organize partial quotients. *Let's compare Priya's and Tyler's work.* 	
Pacing:	8 days		**Math Practices:** SMP 1, 2, 3, 4, 5, 6, 7, 8	**Assessments:** Cool-downs: 14, 16, 18, 20 Checkpoint C
Section D: Let's Put It To Work: Problem Solving With Large Numbers				
4.MD.A.2 4.MD.A.3 4.NBT.B.4 4.NBT.B.5 4.NBT.B.6 4.OA.A.2 4.OA.A.3 4.OA.C.5	I can solve real-world problems involving all four operations.	X Selected Response X Constructed Response Performance	**Lesson Progression and connections:** In the final section of this unit, students engage with a variety of contextual problems that involve multi-digit numbers and require all operations to solve. The problems can be approached in many ways and offer opportunities for students to choose their strategies and representations strategically. Many of them also involve multiple steps, prompting students to practice constructing	**Mandatory Lessons/Activities:** iM Lessons 21, 22, 23, 24

		X Observation	logical reasoning and critiquing the reasoning of others (MP3). *Jada plans to cut up a sheet of poster paper and then rearrange and tape the pieces into a banner that is 8 inches tall and 8 feet long.*  *Does she have enough paper to make the banner?* *What is the area of the paper in square inches?*	
Pacing:	5 days		**Math Practices:** SMP 1, 2, 3, 4, 5, 6, 7, 8	**Assessments:** Cool-downs: 23 Checkpoint D

ADDITIONAL CONSIDERATIONS			
COMMON MISCONCEPTIONS	**PRIOR KNOWLEDGE NEEDED TO MASTER STANDARDS FOR THIS UNIT**	**ADVANCED STANDARDS FOR STUDENTS WHO HAVE DEMONSTRATED PRIOR MASTERY**	**OPPORTUNITIES FOR STUDENT-DIRECTED LEARNING WITHIN THE UNIT**
Students may assume all patterns have the same rule due to limited exposure. When working with multiplication and division, students often do not think about the importance of place value. They treat each digit in the factor or dividend separately without looking at the value of the entire number. Encourage students to explore different strategies and consider the relationship between multiplication and division. Estimating by using multiplication prior to dividing, helps students see what a reasonable quotient will be. When interpreting remainders, some students do not attend to the context of the situation.	4.OA.A.2: 3.OA.3 4.OA.A.3: 3.OA.D.8, 4.NBT.A.3, 4.NBT.B.6 4.OA.B.4: 3.OA.C.7 4.OA.C.5: 3.OA.D.9 4.NBT.B.4: 3.NBT.A.2, 4.NBT.A.1 4.NBT.B.5: 3.NBT.A.2, 3.NBT.A.3, 3.OA.B.5, 3.OA.C.7, 4.NBT.A.1 4.NBT.B.6: 3.NBT. A.2, 3.OA.B.5, 4.MD.A.2: 4.MD.A.1, 4.NF.C.5, 4.NF.C.6, 4.NF.B.4 4.MD.A.3 : 3.MD.D.8, 3.OA.A.4, 3.MD.C.7	Choose from iM leveled centers and exploration problems to differentiate for students who are ready.	iM Centers District-approved online resources

RESOURCES
Kendall Hunt Flourish Blackline masters and materials from Teacher Resource Pack Connecting cubes, index cards, scissors, tape, colored pencils, rulers, yardsticks, tape, sticky notes

UNIT 7: ANGLES AND ANGLE MEASUREMENT
Illustrative Mathematics Unit Focus: Students learn to draw and identify points, rays, segments, angles, and lines, including parallel and perpendicular lines. Students also learn how to use a protractor to measure angles and draw angles of given measurements, and identify acute, obtuse, right, and straight angles in two-dimensional figures. **Essential Questions:** What does the measure of an angle tell us? **Unit Pacing: 18 days (13 required lessons, 3 flex, 2 assessment and reaction)**

UNWRAPPED STANDARDS			
Grade Level Standard	**Standard Progression**	**Concepts (Big Ideas/ Understandings)**	**Academic Vocabulary (Standard Based)**
4.NBT.B.4: Fluently add and subtract multi-digit whole numbers using the standard algorithm.	A crucial theme in multi-digit arithmetic is encouraging students to develop strategies that they understand, can explain, and can think about, rather than merely follow a sequence of directions that they don't understand. In grades 2 and 3, students have seen and used a variety of strategies and materials to deepen their understanding of addition and subtraction algorithms. The goal is for them to understand all the steps in an algorithm, and they should be able to explain the meaning of each digit. For example, a 1 can represent one, ten, one hundred, and so on. For multi-digit addition and subtraction in Grade 4, the goal is also fluency, which means students must be able to carry out the calculations efficiently and accurately. Start with a student's understanding of a certain strategy, and	There are different algorithms that can be used to add or subtract. Fluency is being efficient, accurate, and flexible with strategies.	Add Subtract Sum Difference Standard algorithm Multi-digit

	then make intentional, clear-cut connections for the student to the standard algorithm. This allows the student to gain understanding of the algorithm rather than just memorize certain steps to follow.		
4.NBT.B.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	As with addition and subtraction, students should use methods they understand and can explain. Visual representations such as area and array diagrams that students draw and connect to equations and other written numerical work are useful for this purpose. By reasoning repeatedly about the connection between math drawings and written numerical work, students can come to see multiplication and division algorithms as abbreviations or summaries of their reasoning about quantities.	Understanding place value enables us to represent, compare order and round numbers and perform computations. Properties of operations allow us to reorder, decompose and/or compose numbers in order to make computation simpler. There is a relationship between multiplication and division.	Multiply Equation Area model Rectangular arrays Product Distributive Property Commutative Property
4.NBT.B.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	General methods for computing quotients of multi-digit numbers and one-digit numbers rely on the same understandings as for multiplication, but cast in terms of division. One component is quotients of multiples of 10, 100, or 1000 and one-digit numbers. For example, 42 divided by 6 is related to 420 divided by 6 and 4200 divided by 6. Students can draw on their work with multiplication and they can also reason that 4200 divided by 6 means partitioning 42 hundreds into 6 equal groups, so there are 7 hundreds in each group.	Understanding place value enables us to represent, compare order and round numbers and perform computations. Properties of operations allow us to reorder, decompose and/or compose numbers in order to make computation simpler. There is a relationship between multiplication and division.	Division Quotient Remainder Dividend Divisor

4.MD.C.5 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: 4.MD.C.5.A An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles. 4.MD.C.5.B An angle that turns through n one-degree angles is said to have an angle measure of n degrees.	As with length, area, and volume, children need to understand equal partitioning and unit iteration to understand angle and turn measure. Whether defined as more statically as the measure of the figure formed by the intersection of two rays or as turning, having a given angle measure involves a relationship between components of plane figures and therefore is a property.	Angles are formed when two rays share a common endpoint; the common endpoint of the rays is called a vertex. Angles are measured in degrees. There are 360 degrees in a circle. One degree is 1/360 of a circle.	Angle Degree Ray Circle Endpoint Geometric shape
4.MD.C.6 Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.	Students with an accurate conception of angle can recognize that angle measure is additive. As with length, area, and volume, when an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Students can then solve interesting and challenging addition and subtraction problems to find the measurements of unknown angles on a diagram in real world and mathematical problems.	The measure of or number of degrees in an angle tells us how far one ray (side) of the angle is rotated from the other. A protractor is a tool used to measure angles.	Angle Degree Measure Protractor
4.MD.C.7 Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.	Students with an accurate conception of angle can recognize that angle measure is additive. As with length, area, and volume, when an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Students can then solve interesting and challenging addition and subtraction problems to find the	Angles can be decomposed into unit angles. (n degrees is *n* one degree angles.)	Angle Degree Additive Sum Decompose Equation Symbol Unknown angle Measure

	measurements of unknown angles on a diagram in real world and mathematical problems.		
4.G.A.1 Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.	Students see points and lines as abstract objects: Lines are infinite in extent and points have location but no dimension. Grids are made of points and lines and do not end at the edge of the paper.	A point is a location in space; it has no length, width, or height. A line is a continuous straight path that extends indefinitely in two opposite directions. A line segment is a continuous straight path between two points. A ray is a continuous straight path that extends indefinitely in one direction from one point.	Point Vertex Line Line segment Ray Intersecting lines Perpendicular lines Parallel lines Angle Acute Angle Obtuse Angle Straight Angle Right Angle Two-dimensional figures

UNIT 7: ANGLES AND ANGLE MEASUREMENT				
What does the measure of an angle tell us?				
Section A: Points, Lines, Segments, Rays, and Angles				
CCSS Standards #	**Learning Targets**	**Summative Assessment Strategy**	**Lesson Progression and Connection to Math Practices**	**Common Learning Experiences and Assessments**
4.G.A.1 4.MD.C.5	I can draw and identify points, lines, rays, segments, angles, and parallel and intersecting lines in geometric figures.	X Selected Response X Constructed Response Performance X Observation	**Lesson Progression:** Students are introduced to some building blocks of geometric figures and the language to describe them. Students start by describing geometric images that contain lines for others to draw and drawing images relying only on others' descriptions. The experience motivates a need for more precise vocabulary to describe geometric parts. Students learn to distinguish points as locations in space, rays as lines that are bounded by one point, and line segments as lines that are bounded by two points. Students are familiar with lines that cross or intersect. Here they identify and then draw some lines—called parallel lines—that never intersect. They also learn that when lines (or parts of lines) do intersect, they form angles, and that the point of intersection is the vertex of each angle. Students practice identifying angles, noticing that angles are ubiquitous around us and can have different sizes.	**Mandatory Lessons/Activities:** iM Lessons 1, 2, 3, 4, 5
Pacing:	5 days		**Math Practices:** SMP 3, 5, 6	**Assessments:** Cool Downs: 2, 3 Section A checkpoint
Section B: The Size of Angles				
4.G.A.1 4.MD.C.5 4.MD.C.5.a	I can measure and draw angles using degrees.	X Selected Response	**Lesson Progression:** Students learn two main ideas: that angles can be measured, with degrees (°) as the unit of	**Mandatory Lessons/Activities:** iM Lessons 6, 7, 8, 9, 10

4.MD.C.5.b 4.MD.C.6 4.MD.C.7 4.NBT.B.4 4.NBT.B.5 4.NBT.B.6	I can draw and identify perpendicular lines or rays. I can solve problems involving unknown angles.	X Constructed Response Performance X Observation	measurement, and that angles are additive. Prior to this point, students have learned that rectangles have square corners. Here they learn that square corners are called right angles and have a measure of 90°. Using right angles as building blocks, they compose increasingly larger angles. They see that when four right angles share the same vertex without overlaps and gaps, they fill up the entire region around that point. They conclude that a full turn around a point is 360°. Students go on to find the size of angles that could be composed into 360° or composed into another known measurement. For example, if six equal angles that share a vertex cover all the regions around that point, each angle must be 60°. Composing and decomposing angles to find their measurements help students see that angles can be added and subtracted.Students also learn to use a protractor to measure angles and to draw angles.	
Pacing:	5 days		**Math Practices:** SMP 1, 2, 3, 5, 6, 7	**Assessments:** Cool Downs: 9 Section B Checkpoint
		Section C: Angle Analysis		
4.G.A.1 4.G.A.2 4.MD.C.6 4.MD.C.7	I can draw and identify acute, obtuse, right, and straight angles in two-dimensional figures. I can solve problems involving unknown angles.	X Selected Response X Constructed Response Performance X Observation	**Lesson Progression:** Students classify angles by their size and identify acute, obtuse, and straight angles. Then, they reason about angle measurements in concrete and abstract contexts. Students write equations to solve problems about angles on the clock and in geometric diagrams.	**Mandatory Lessons/Activities:** iM Lessons 11, 12, 13
Pacing:	4 days		**Math Practices:** SMP 1, 2, 3, 5, 6, 7	**Assessments:** Cool Downs: 11 Section C Checkpoint

ADDITIONAL CONSIDERATIONS			
COMMON MISCONCEPTIONS	**PRIOR KNOWLEDGE NEEDED TO MASTER STANDARDS FOR THIS UNIT**	**ADVANCED STANDARDS FOR STUDENTS WHO HAVE DEMONSTRATED PRIOR MASTERY**	**OPPORTUNITIES FOR STUDENT-DIRECTED LEARNING WITHIN THE UNIT**
Students believe a wide angle with short sides may seem smaller than a narrow angle with long sides because they focus on the length of the rays rather than the spread of the rays. Students are confused as to which number to use when determining the measure of an angle using a protractor because most protractors have a double set of numbers. Students believe that if they do not see a pair of lines intersect, those lines are parallel. Students may not recognize angles larger than 180° as angles.	4.G.A.1: 3.G.A.1 4.G.A.2: 4.G.A.1 4.MD.C.5 4.MD.C.6: 4.M.D.C.5 4.MD.C.7 4.NBT.B.5: 3.NBT.A.2, 3.NBT.A.3, 3.OA.B.5, 3.OA.C.7, 4.NBT.A.1 4.NBT.B.6: 3.NBT.A.2, 3.OA.B.5, 3.OA.B.6, 3.OA.C.7, 4.NBT.A.1, 4.NBT.B.5	Choose from iM leveled centers and exploration problems to differentiate for students who are ready.	iM Centers District-approved online resources

RESOURCES
Kendall Hunt Flourish Blackline masters and materials from Teacher Resource Pack Rulers or straightedges, paper, pattern blocks, Origami paper, Patty paper, protractors, index cards

UNIT 8: PROPERTIES OF TWO-DIMENSIONAL SHAPES
Illustrative Mathematics Unit Focus: Students classify triangles and parallelograms based on the properties of their side lengths and angles, and learn about lines of symmetry in two-dimensional figures. They use their understanding of these attributes to solve problems, including problems involving perimeter and area. **Essential Questions:** How can polygons be compared, sorted and classified? Why is symmetry important? **Unit Pacing: 11 days (7 required lessons, 2 flex, 2 assessment and reaction)**

UNWRAPPED STANDARDS

Grade Level Standard	Standard Progression	Concepts and Disciplinary-Specific Vocabulary	Academic Vocabulary (Standard Based)
4.G.A.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.	Students can use side length to classify triangles as equilateral, equiangular, isosceles, or scalene; and can use angle size to classify them as acute, right, or obtuse. They then learn to cross-classify, for example, naming a shape as a right isosceles triangle. Thus, students develop explicit awareness of and vocabulary for many concepts they have been developing, including points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Such mathematical terms are useful in communicating geometric ideas, but more important is that constructing examples of these concepts, such as drawing angles and triangles that are acute, obtuse, and right, help students form richer concept images connected to verbal definitions.	Polygons can be compared, sorted and classified using attributes and relationships, such as number of sides, types of angles, and parallel and perpendicular sides.	Right angle Acute angle Obtuse angle Two-dimensional figures Perpendicular lines Parallel lines Classify Equilateral triangle Scalene triangle Isosceles triangle Acute triangle Obtuse triangle Right triangle
4.G.A.3 Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line	Through building, drawing, and analyzing shapes, students expand their knowledge of properties of	Symmetry is used to describe and analyze figures and has many applications in the real world.	Line of symmetry Symmetry Two-dimensional figure

into matching parts. Identify line-symmetric figures and draw lines of symmetry.	two-dimensional objects and the use of them to solve problems involving symmetry.		
4.MD.A.3 Apply the area and perimeter formulas for rectangles in real world and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.	Such abstraction and use of formulas underscores the importance of distinguishing between area and perimeter in Grade 3 (3.MD.8 3) and maintaining the distinction in Grade 4 and later grades, where rectangle perimeter and area problems may get more complex and problem solving can benefit from knowing or being able to rapidly remind oneself of how to find an area or perimeter. By repeatedly reasoning about how to calculate areas and perimeters of rectangles, students can come to see area and perimeter formulas as summaries of all such calculations.	The area of a rectangle can be found by multiplying the lengths of adjacent sides (length and width) of the rectangle. Given an area or a perimeter of a rectangle and one side length, the adjacent side length can be determined.	Perimeter Area Formula Length Width Square units

UNIT 8: PROPERTIES OF TWO-DIMENSIONAL SHAPES

How can polygons be compared, sorted and classified?
Why is symmetry important?

Section A: Side Lengths, Angles, and Lines of Symmetry

CCSS Standards #	Learning Targets	Summative Assessment Strategy	Lesson Progression and Connection to Math Practices	Common Learning Experiences and Assessments
4.G.A.2 4.G.A.3	I can classify two-dimensional figures based on their attributes. I can identify and draw lines of symmetry in two-dimensional figures	X Selected Response X Constructed Response Performance X Observation	**Lesson Progression:** Students consider different ways of looking at two-dimensional shapes: by the number of sides, length of sides, size of angles, presence of parallel or perpendicular lines, and symmetry. Students examine these attributes in shapes, classify the shapes by the attributes, and explain their classifications. For example, they identify quadrilaterals as parallelograms if they have two pairs of parallel sides, as squares if they have four equal sides and four right angles, and so on. In studying symmetry, students characterize shapes based on whether they can be folded into two equal halves that match up exactly, draw lines of symmetry, and complete drawings of figures that are halved by a line of symmetry.	**Mandatory Lessons/Activities:** Lessons 1, 2, 3, 4, 5
Pacing:	6 days		**Math Practices:** SMP 3, 5, 6, 7, 8	**Assessments:** Cool Downs: 1,2,4,5 Section A Checkpoint

Section B: Reason about Properties to Solve Problems

CCSS Standards #	Learning Targets	Summative Assessment Strategy	Lesson Progression and Connection to Math Practices	Common Learning Experiences and Assessments
4.G.A.2 4.G.A.3 4.MD.A.2 4.MD.A.3 4.MD.C.7	I can use my understanding of geometry to solve problems.	X Selected Response X Constructed Response	**Lesson Progression:** Students apply their developing knowledge of geometric attributes to reason about measurements in various two-dimensional shapes. They begin by finding the perimeter of shapes where the side lengths are all given. Then, they	**Mandatory Lessons/Activities:** Lessons 7, 8

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<td rowspan="2"><table><tr><td></td><td>Performance</td></tr><tr><td>X</td><td>Observation</td></tr></table></td>
<td>move on to cases where the side lengths are not explicitly given but can be deduced based on information about the shapes. Later, students reason in the other direction: given the perimeter and some information about a shape, they reason about its side lengths. The activities also enable students to practice performing operations on whole numbers and fractions.</td>
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<td>Pacing:</td>
<td>4 days</td>
<td>Math Practices:
SMP 1, 2, 3, 5, 6, 7, 8</td>
<td>Assessments:
Cool Downs: 7
Section B Checkpoint</td>
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ADDITIONAL CONSIDERATIONS			
COMMON MISCONCEPTIONS	**PRIOR KNOWLEDGE NEEDED TO MASTER STANDARDS FOR THIS UNIT**	**ADVANCED STANDARDS FOR STUDENTS WHO HAVE DEMONSTRATED PRIOR MASTERY**	**OPPORTUNITIES FOR STUDENT-DIRECTED LEARNING WITHIN THE UNIT**
Students think that when describing geometric shapes and placing them in subcategories, the last category is the only classification that can be used. Students may misclassify a shape due to its orientation. For example, students may classify a square tipped on its side as a rhombus. Students believe a wide angle with short sides may seem smaller than a narrow angle with long sides. Students can compare two angles by tracing one and placing it over the other. Students will then realize that the length of the sides does not determine whether one angle is larger or smaller than another angle. The measure of the angle does not change. Students are confused as to which number to use when determining the measure of an angle using a protractor because most protractors have a double set of numbers. Students should decide first if the angle appears to be an angle that is less than the measure of a right angle (90°) or greater than the measure of a right angle (90°). If the angle appears to be less than 90°, it is an acute angle and its measure ranges from 0° to	4.G.A.2: 4.G.A.1 4.MD.A.2: 4.MD.A.1, 4.NF.C.5, 4.NF.C.6, 4.NF.B.4 4.MD.A.3: 3.MD.D.8, 3.OA.A.4, 3.MD.C.7 4.MD.C.7: 4.MD.C.5	Choose from iM leveled centers and exploration problems to differentiate for students who are ready.	iM Centers District-approved online resources

89°. If the angle appears to be an angle that is greater than 90°, it is an obtuse angle and its measures range from 91° to 179°. Ask questions about the appearance of the angle to help students in deciding which number to use.			

RESOURCES
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