Course Title:	Content Area:	Grade Level:	Credit (if applicable)
Precalculus ACA	Mathematics	11-12	1.0

Course Description:

This course is the fourth course in the college preparatory mathematics sequence. It is a prerequisite for the analytic geometry and calculus course offered by colleges. Topics studied are algebraic functions, logarithms and exponential functions, trigonometric functions and conics. All topics studied will involve the use of a TI-83+ or TI-84+ graphing calculator. Students enrolled in this class must take the common mid-term and final assessment for Pre-Calculus Academic.

Aligned Core Resources:	Connection to the **BPS Vision of the Graduate**
Precalculus, 7th Edition (2022) by Robert F. Blitzer Published by Pearson	CRITICAL THINKING AND PROBLEM SOLVING • Collect, assess and analyze relevant information • Reason effectively. Use systems thinking • Make sound judgments and decisions. • Identify, define and solve authentic problems and essential questions. • Reflect critically on learning experience, processes and solutions • Transfer knowledge to other situations CONTENT MASTERY • Develop and draw from a baseline understanding of knowledge in academic disciplines from our Bristol curriculum.

Additional Course Information: Knowledge/Skill Dependent courses/prerequisites	Link to **Completed Equity Audit**
Algebra 2 prerequisite course	Equity Curriculum Review- PreCalculus

Standard Matrix

Standard	Unit P	Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6
A.APR.7	X						
A.APR.A.1			X				
A.APR.B.2			X				
A.APR.B.3			X				
A.CED.1	X						
A.CED.4	X						
A.CED.A.2				X			
A.REI.11	X			X			X
A.REI.2	X						
A.REI.3	X						
A.REI.4	X						
A.REI.D.11				X			X
A.SSE.3	X			X			
A.SSE.A.1			X	X			
F.BF.4		X					
F.BF.A.1		X		X			
F.BF.B.3		X					
F.BF.B.4		X					
F.BF.B.5				X			
F.IF.A.1		X					
F.IF.A.2		X					

F.IF.B.4		X					
F.IF.B.5		X					
F.IF.C.7		X	X	X			
F.IF.C.8			X	X			
F.IF.C.9			X				
F.LE.A.1				X			
F.LE.A.2				X			
F.LE.A.3				X			
F.LE.A.4				X			
F.LE.B.5				X			
F.TF.A.1					X		
F.TF.A.2					X		
F.TF.A.3					X		
F.TF.A.4					X		
F.TF.B.6					X		
F.TF.C.8						X	
F.TF.C.9						X	
G.SRT.C.6					X		
G.SRT.C.7					X		
G.SRT.C.8					X		
G.SRT.D.10						X	
G.SRT.D.11						X	
G.SRT.D.9						X	
HSA.CED.A.3							X
HSA.REI.C.6							X
HSA.REI.C.7							X
HSA.REI.D.12							X
N.CN.3			X				
N.CN.A.1			X				
N.CN.A.2			X				
N.CN.C.7			X				
N.CN.C.8			X				
N.CN.C.9			X				
S.ID.7	X						

See each unit for the standard language

Unit Links

Unit P: Fundamental Concepts of Algebra
Unit 1: Functions and Graphs
Unit 2: Polynomials and Rational functions
Unit 3: Exponential and Logarithmic Functions
Unit 4: Trigonometric Functions
Unit 5: Analytic Trigonometry
Unit 6: Systems of Equations and Inequalities

<table>
<tr><th colspan="2">Unit Title</th></tr>
<tr><td colspan="2">Unit P: Fundamental Concepts of Algebra</td></tr>
<tr><th colspan="2">Relevant Standards: Bold indicates priority</th></tr>
<tr><td colspan="2">A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
• A.SSE.3a. Factor a quadratic expression to reveal the zeros of the function it defines.
• A.SSE.3b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
• A.SSE.3c. Use the properties of exponents to transform expressions for exponential functions
A.APR.7 (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple (rational) and exponential functions.
A.REI.2 Solve simple (rational) and radical equations in one variable, and give examples showing how extraneous solutions may arise.
A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.
A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
A.REI.4 Solve quadratic equation in one variable
• A.REI.3a Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)$ that has the same solutions. Derive the quadratic formula from this form. $2 = q$
• A.REI.3b Solve quadratic equations by inspection, taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.
A.REI.11 Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solution of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and /or g(x) are linear, polynomial, (rational), absolute value, exponential, and logarithmic functions.
S.ID.7 Interpret the slope (rate of change) and the intercept of a linear model in the context of data</td></tr>
<tr><th>Essential Question(s)</th><th>Enduring Understanding(s)</th></tr>
<tr><td>• Why do we structure expressions in different ways?
• In what ways can the problem be solved, and why should one method be chosen over another?
• How can the properties of the real number system be useful when working with polynomials and rational expressions?
• Which function is being modeled in specific real world applications and how can I use acquired knowledge to solve the problem?</td>
<td>• Expressions can be written in multiple ways using the rules of algebra; each version of the expression tells something about the problem it represents.
• There is often an optimal method of manipulating equations to solve a mathematical problem; however other methods, which may not be as efficient, can still provide insight to the problem.
• Algebraic expressions such as polynomials and rational expressions, symbolize numerical relationships and can be manipulated in much the same way as numbers.</td></tr>
<tr><th>Demonstration of Learning</th><th>Pacing for Unit</th></tr>
<tr><td>Homework
Class Practice
Readiness Quizzes
Skills Check Quizzes
Mid-Unit Checkpoint
End of Unit Assessment</td><td>13 blocks</td></tr>
<tr><th>Family Overview (link below)</th><th>Integration of Technology:</th></tr>
<tr><td>Precalculus ACA - Family Overview (2024-25)
Precalculus ACA - Family Overview (2024-25) SPANISH</td><td>TI-84 graphing calculator
Desmos calculator</td></tr>
<tr><th>Unit-specific Vocabulary</th><th>Aligned Unit Materials, Resources, and Technology</th></tr>
</table>

<table>
<tr><th></th><th>(beyond core resources)</th></tr>
<tr><td>Linear Equations, Slope, Y-intercept, Radical Equation, Rational Exponent, Rational Expression, Rational Equation, Extraneous Solution, Literal Equation, Quadratic Equation, Polynomial, Factoring, Square root method, Completing the square, Quadratic Formula, Radicand, Complex Number, Imaginary, Solution</td><td>TI-84 graphing calculator
Desmos calculator
Mini white boards
Teacher created activities
Vertical surfaces</td></tr>
<tr><td>Opportunities for Interdisciplinary Connections</td><td>Anticipated misconceptions:</td></tr>
<tr><td>
• Quadratic expressions frequently appear in physics, especially in kinematics, where equations of motion describe how objects move under gravity.
• Quadratic equations model motion under gravity, where the height or position of an object is a function of time.
• Exponential functions model financial growth (compound interest) and depreciation (exponential decay).
• Exponential functions are crucial in biology for modeling population growth, radioactive decay, and disease spread.
</td><td>
• Every quadratic equation can be factored easily.
• Completing the square is useful only to find the vertex of a quadratic function.
• The rules for simplifying polynomials apply to exponential expressions in the same way.
• Rational expressions can always be added, subtracted, multiplied, or divided without considering restrictions.
• Rearranging an equation always preserves all original solutions.
• The quadratic formula should always be used when solving quadratics.
• Every rational or radical equation has a valid solution.
• When solving $f(x)=g(x)$ there is always a single intersection.
• Exponential growth can be analyzed using the concept of slope.
• A line's slope is always positive or negative.
</td></tr>
<tr><td>Connections to Prior Units</td><td>Connections to Future Units</td></tr>
<tr><td>These standards served as a bridge from Algebra 1 to Algebra 2 by establishing essential algebraic skills. Algebra 2 extended these ideas by introducing higher-degree polynomials, logarithms, complex numbers, advanced transformations, and real-world applications of these concepts.</td><td>These Algebra standards lay the foundation for more advanced topics in Precalculus, including polynomials, exponentials/logs, complex numbers, nonlinear systems, and limits. Each concept builds towards a deeper understanding of functions and prepares students for Calculus applications.</td></tr>
<tr><td colspan="2">Differentiation through Universal Design for Learning</td></tr>
<tr><td>UDL Indicator</td><td>Teacher Actions</td></tr>
<tr><td colspan="2">
Multiple Means of Representation (Principle I)
Consideration 2.5 - Illustrate through multiple media:
Use graphing technology (Desmos, GeoGebra) to show how different forms of an expression impact the graph.
• Show how factoring a quadratic highlights the zeros, while vertex form highlights the maximum/minimum.
• Have students manipulate sliders to see real-time changes in quadratic and exponential expressions.
Consideration 3.2 - Highlight patterns, critical features, big ideas, and relationships:
Use step-by-step examples with color-coding to highlight transformations and relationships.
• Color different parts of an expression when factoring, completing the square, or using exponent rules.
• Provide scaffolded notes where students fill in missing steps to highlight key algebraic patterns.
Multiple Means of Action & Expression (Principle II)
Consideration 5.1 - Use multiple media for communication:
Encourage students to verbally explain why they choose a particular algebraic method.
• Have students record voice memos or videos explaining how different forms of an expression provide insights.
• Use math journaling to reflect on how algebraic manipulations reveal different properties of functions.
Multiple Means of Engagement (Principle III)
Consideration 7.1 - Optimize individual choice and autonomy:
Provide students with a choice of problem-solving strategies.
</td></tr>
</table>

- Example: Solve a quadratic equation using three different methods (factoring, completing the square, quadratic formula) and discuss which is most efficient.

Consideration 8.3 - Foster collaboration and community:
Encourage students to defend different algebraic methods.

- Pose a problem and have students debate the best method for solving it.
- Assign roles where some students advocate for factoring, while others argue for completing the square.

Supporting Multilingual/English Learners

Related CELP standards	**Learning Targets**

A MLL can . . . determine the meaning of words and phrases in oral presentations and literary and informational text.
I can rewrite and manipulate algebraic expressions, including polynomials and rational expressions, using algebraic rules to evaluate multiple solution methods to identify the most efficient strategy, while understanding how alternative approaches provide insight into mathematical relationships.

- **Level 1** – I can use numbers, symbols, and simple words to show how to rewrite basic algebraic expressions, such as combining like terms or factoring simple polynomials, with visual or guided support.
- **Level 2** – I can describe and follow steps to rewrite and simplify algebraic expressions, including polynomials and fractions, using models, word banks, and sentence frames.
- **Level 3** – I can explain how to rewrite and manipulate algebraic expressions, including polynomials and rational expressions, using algebraic rules, and describe how different methods lead to the same solution.
- **Level 4** – I can analyze and compare different methods of rewriting algebraic expressions, explain why one method may be more efficient than another, and describe how each approach provides insight into a mathematical problem.
- **Level 5** – I can justify my choice of algebraic manipulation strategies by evaluating efficiency, comparing alternative methods, and explaining how different forms of an expression reveal key mathematical relationships.

Lesson Sequence	**Learning Target**	**Success Criteria/ Assessment**	**Resources**
P.2 Exponents and Scientific Notation	I can identify and apply the key rules of exponents to simplify expressions with exponents	• I can apply the product rule, quotient rule, power of a power rule, negative exponent rule, and zero exponent rule to simplify expressions with exponents • I can demonstrate a clear understanding of how to manipulate exponents with the same base when multiplying, dividing, or raising to a power.	
P.4 Polynomials	I can perform operations on polynomials	• I can add, subtract, and multiply polynomials	
P.5 Factoring Polynomials	I can factor polynomials	• I can identify which type of factoring is needed. • I can factor by greatest common factor, grouping, trinomials, difference of squares.	
P.6 Rational Expressions	I can use perform operations on rational expressions	• I can simplify rational expressions • I can multiply, add and subtract rational expressions	
P.7 Equations	I can solve polynomial equations using various methods	• I can solve radical and rational equations • I can solve quadratic equations ○ Quadratic formula (no complex numbers) ○ completing the square ○ Factoring ○ Square root property	

Unit Title

Unit 1: Functions and Graphs

Relevant Standards: Bold indicates priority

F.IF.A.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).
F.IF.A.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
F.IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
F.IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.
F.IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

- F.IF.C.7.B Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

F.BF.A.1 Write a function that describes a relationship between two quantities.

- F.BF.A.1.A Determine an explicit expression, a recursive process, or steps for calculation from a context.
- F.BF.A.1.C (+) Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.
- F.BF.B.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
- F.BF.B.4 Find inverse functions.
  - F.BF.4a Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, f(x) =2(x^3) or f(x) = (x+1)/(x-1) for x ≠ 1 (x not equal to 1).
  - F.BF.4b (+) Verify by composition that one function is the inverse of another.
  - F.BF.4c (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.
  - F.BF.4d (+) Produce an invertible function from a non-invertible function by restricting the domain.

Essential Question(s)

- What is a function and what are the different ways they can be represented?
- How are functions used in the real world?
- What are the key characteristics of a graph and why are they important?
- How does composing functions affect the domain?
- What is the relationship between a function and its inverse?

Enduring Understanding(s)

- A function is a specific type of relation where each input is associated with exactly one output. This concept is fundamental in describing how variables interact and is a building block for analyzing more complex mathematical relationships.
- Functions can be transformed through translations, reflections, stretches, and compressions. These transformations affect the graph of the function and help in understanding how changes in the function's formula impact its graph.
- Functions can be combined through addition, subtraction, multiplication, and division, and can be composed with each other. Understanding these operations is essential for building more complex functions and analyzing their behavior.
- An inverse function essentially reverses the effect of the original function. Understanding how to find and use inverse functions is important for solving equations and understanding the relationship

	between functions and their inverses. • Functions are used to model and solve real-world problems in various fields such as physics, economics, biology, and engineering. Applying functions to practical situations helps in interpreting data and making predictions.
Demonstration of Learning	**Pacing for Unit**
Homework Class Practice Readiness Quizzes Skills Check Quizzes Mid-Unit Checkpoint End of Unit Assessment	20 blocks
Family Overview (link below)	**Integration of Technology**
Precalculus ACA - Family Overview (2024-25) Precalculus ACA - Family Overview (2024-25) SPANISH	TI-84 graphing calculator Desmos calculator
Unit-specific Vocabulary	**Aligned Unit Materials, Resources, and Technology (beyond core resources)**
Function, Input, Output, Independent variable, Dependent variable, Domain, Range, Function notation: f(x), Parent function, Transformation, Translations, Vertical stretch, Vertical compression, Vertical reflection, Horizontal reflection, Piecewise function, Vertical line test, One-to-one function, Inverse function, Domain, Restricted domain, Composition	TI-84 graphing calculator Desmos calculator Mini white boards Teacher created activities Vertical surfaces
Opportunities for Interdisciplinary Connections	**Anticipated misconceptions**
• Functions are used to model relationships between variables in science, such as the decay of radioactive substances, population growth, and the motion of objects. • Functions are the foundation of programming and computational thinking, used in defining algorithms, recursive processes, and data transformations. • Functions describe relationships in economics, such as cost, revenue, and profit models, as well as statistical trends in business.	• Every equation represents a function. • A function must always be a linear equation. • If a function has the same output for two different inputs, it is not a function. • The vertical line test determines if a graph is a function based on its shape alone. • Function notation f(x) = y means multiplication between f and x. • The domain of a function is always all real numbers. • Graph transformations (shifts, stretches, reflections) change the function's properties unpredictably. • The inverse of a function always exists. • Composing two functions always produces another function. • The x-values where two graphs intersect are only approximate solutions.
Connections to Prior Units	**Connections to Future Units:**
Algebraic Manipulation & Functions – Factoring, completing the square, and working with rational expressions (A.SSE.3, A.APR.7) help reveal key features of functions like zeros, domain restrictions, and asymptotic behavior. **Function Transformations & Equation Solving** – Graphing transformations (F.BF.B.3) align with solving and rearranging equations (A.REI.3, A.CED.4), while finding inverse functions (F.BF.4) requires algebraic solving	The Interpreting and Building Functions standards connect to future units by... • Preparing students for function transformations, composition, inverses, graphing, and real-world applications. • Understanding domain, range, and key features leads to advanced functions like trigonometric, logarithmic, and exponential functions. • Skills in composition and inverse functions help

(A.REI.2). **Data Interpretation & Functions** – Understanding slopes, intercepts, and key features of graphs (F.IF.B.4, S.ID.7) connects functions to real-world data trends.	with nested functions and transformations. • Graphing concepts extend to polynomials, rational functions, and asymptotic behavior. • Modeling functions supports sequences, series, and calculus readiness.
Differentiation through *Universal Design for Learning*	
UDL Indicator	**Teacher Actions:**
Multiple Means of Representation (Principle I) Consideration 1.2: Support multiple ways to perceive information – Use graphs, tables, equations, and real-world scenarios to show the same function in multiple ways. Consideration 3.2: Highlight patterns, critical features, big ideas, and relationships – Help students recognize transformations and connections between functions through color coding, annotations, and guided questions.	**Multiple Means of Representation (Principle I)** • Interactive Technology: Use Desmos, GeoGebra, or graphing calculators to visually manipulate function transformations. Let students drag sliders to see how coefficients affect graphs dynamically. • Color-Coded Notes: Assign different colors for transformations (e.g., green for shifts, blue for reflections) to help students track changes in equations vs. graphs. • Real-World Analogy Mapping: Compare function transformations to real-world scenarios (e.g., stretching a rubber band = vertical stretch, flipping a pancake = reflection).
Multiple Means of Action & Expression (Principle II) Consideration 2.5: Illustrate through multiple media– Allow students to explain functions using writing, drawings, verbal descriptions, or digital tools. Consideration 6.3: Organize information and resources– Guide students in breaking down multi-step function problems through problem-solving scaffolds, peer discussions, and structured templates.	**Multiple Means of Action & Expression (Principle II)** • Function Storytelling: Have students personify functions (e.g., "I am a quadratic function. My vertex is my home, and I always curve symmetrically!"). • Function Transformations Dance: Assign different movements for function transformations (e.g., stepping forward = horizontal shift, jumping = vertical shift, turning = reflection). • Peer Teaching with Choice: Let students choose between creating a tutorial video, writing a math comic strip, or presenting a mini-lesson on function transformations.
Supporting Multilingual/English Learners	
Related *CELP standards:*	**Learning Targets:**
A MLL can . . . determine the meaning of words and phrases in oral presentations and literary and informational text. I can analyze and interpret functions by identifying key characteristics such as domain, range, intercepts, asymptotes, intervals of increase and decrease, and transformations, and represent them graphically and algebraically. • **Level 1** – I can use words, symbols, or gestures to show basic information about a function, such as points on a graph or increasing and decreasing parts. • **Level 2** – I can name and describe parts of a function, such as domain, range, or intercepts, using simple words, labels, and visuals. • **Level 3** – I can explain key features of a function, such as intercepts, asymptotes, or transformations, using simple sentences and diagrams. • **Level 4** – I can describe and compare functions by explaining their key characteristics, such as domain, range, and transformations, using mathematical vocabulary. • **Level 5** – I can analyze and justify how key characteristics of functions impact their graphs and equations using precise mathematical language in written and verbal explanations.	

Lesson Sequence	**Learning Target**	**Success Criteria/ Assessment**	**Resources**
1.2 Basics of functions and	• I can determine whether a relation is a function or not	• I can determine if a relation is a function using ○ A table of values	

their graphs	• I can determine the domain and range of a function	○ Vertical line test • I can determine the domain and range given a graph
1.3 More on functions and their graphs	• I can identify intervals on which a function increases or decreases • I can locate relative extrema	• I can identify intervals on which a function increases or decreases given a graph • I can locate relative minima and maxima given a graph
1.4 Linear functions and slope	I can analyze linear functions	• I can calculate slope • I can write point slope form of equation • I can write and graph slope-intercept form of an equation • I can graph horizontal and vertical lines • I can recognize and use standard form of an equation • I can use intercepts to graph an equation
1.6 Transformations of Functions	I can identify transformations of Functions	• I can use use vertical shifts to graphs functions • I can use horizontal shifts to graph functions. • I can use reflections to graph functions. • I can use vertical stretching and shrinking to graph functions • I can use horizontal stretching and shrinking to graph functions.
1.7 Combinations of functions, composite functions	I can combine Functions	• I can combine functions using the algebra of functions. • I can form composite functions
1.8 Inverse functions	I can analyze the inverse of a function	• I can verify that functions are inverses. • I can find the inverse of a function. • I can use the horizontal line test to determine if a function has an inverse function.
1.10 Modeling with functions	I can use functions to model real world situations	• I can construct functions from verbal descriptions. • I can construct functions from formulas.

Unit Title:	
Unit 2: Polynomials and Rational Functions	
Relevant Standards: Bold indicates priority	
N.CN.A 1 Know there is a complex number i such that $i^2 = -1$, and every complex number has the form a + bi with a and b real. **N.CN.A 2** Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. **N.CN.3** Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. **N.CN.C 7** Solve quadratic equations with real coefficients that have complex solutions. **N.CN.C 8** Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as (x + 2i)(x – 2i). **N.CN.C 9** Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. **A.SSE.A.1** Interpret expressions that represent a quantity in terms of its context. **A.SSE.B.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. **A.APR.A.1** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. **A.APR.B.2** Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x). **A.APR.B.3** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. **F.IF.C.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ● F.IF.C.7.C Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. **F.IF.C.8** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. **F.IF.C.9** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.	
Essential Question(s):	**Enduring Understanding(s):**
● What are the defining characteristics of polynomial functions and how do their degrees and coefficients affect their behavior? ● Why are there different methods of finding the zeros of a polynomial and how do we decide which method is appropriate? ● How can polynomial functions represent real-life situations? ● What is the Fundamental Theorem of Algebra, and how does it relate to the existence of complex roots? ● How can you use the Factor Theorem and synthetic division to factorize polynomials? ● How do different types of variation apply to real-world problems, and how can understanding these relationships help solve those problems?	● Polynomial functions can be classified by their degree, which determines their general shape and the maximum number of roots they can have. ● The degree and leading coefficient determine the end behavior of a polynomial function, which helps in sketching and analyzing graphs. ● The zeros/roots of a polynomial function correspond to the x-intercepts of its graph. The multiplicity of a zero affects the behavior of the graph at that intercept, such as whether it touches or crosses the x-axis. ● Polynomial functions model various real-world phenomena, from projectile motion to economics. ● The complex number set is composed and the real numbers and the imaginary numbers. ● Complex solutions will not be x-intercepts. ● The intermediate value theorem can be used to determine the zeros of a function. ● Recognizing symmetry in polynomial functions (even, odd, or neither) helps in graphing and analyzing these functions. ● The remainder theorem tells us that if a polynomial f(x) is divided by x-c, then the remainder is f(c). ● The factor theorem tells us that if f(c)=0, then x - c is a

	factor of f(x). • Graphs of rational functions have asymptotes and have many forms. • Many real-life quantities can be modeled by types of variation including: direct, inverse, and joint.
Demonstration of Learning:	**Pacing for Unit**
Homework Class Practice Readiness Quizzes Skills Check Quizzes Mid-Unit Checkpoint End of Unit Assessment	12 blocks
Family Overview (link below)	**Integration of Technology:**
[Precalculus ACA - Family Overview (2024-25)] [Precalculus ACA - Family Overview (2024-25) SPANISH]	TI-84 graphing calculator Desmos calculator
Unit-specific Vocabulary:	**Aligned Unit Materials, Resources, and Technology (beyond core resources):**
Complex Number, Complex Conjugate, Polynomial Function, Leading coefficient test, End Behavior, Multiplicities, Intermediate Value Theorem, Turning Points, Synthetic Division, Remainder Theorem, Factor Theorem, Rational Zero Theorem, Vertical Asymptote, Horizontal Asymptote, Slant Asymptote, Direct Variation, Inverse Variation, Joint Variation	TI-84 graphing calculator Desmos calculator Mini white boards Teacher created activities Vertical surfaces
Opportunities for Interdisciplinary Connections:	**Anticipated misconceptions:**
Science (Physics and Biology) • **Quadratic Expressions in Motion:** Completing the square helps analyze projectile motion by revealing the maximum height (vertex) of a thrown object. • **Exponential Functions in Growth & Decay:** Transforming exponential expressions models radioactive decay, bacterial growth, and population dynamics. **Finance & Economics** • **Interest & Investment Models:** Exponential transformations explain compound interest and financial forecasting. • **Quadratic Cost & Revenue Functions:** Factoring quadratics can help find break-even points in business models. **Computer Science & Engineering** • **Algorithm Efficiency (Big-O Notation):** Exponential and polynomial expressions describe algorithm complexity. • **Structural Engineering:** Quadratic equations help model forces and stress in materials.	• Factoring always works for quadratics. • Completing the square is only for finding the vertex. • Exponential expressions can be transformed like polynomials. • Rational expressions can always be added, subtracted, multiplied, or divided without considering restrictions. • Rearranging an equation always preserves all original solutions. • The quadratic formula should always be used when solving quadratics. • Every rational or radical equation has a valid solution. • When solving f(x)=g(x), there is always a single intersection. • Exponential growth can be analyzed using the concept of slope. • A line's slope is always positive or negative.
Connections to Prior Units:	**Connections to Future Units:**
• Factoring quadratic expressions helps determine the x-intercepts (zeros) of a function. This is essential for graphing parabolas and understanding solutions to equations. • Rewriting a quadratic function in vertex form by	• Using exponent properties (e.g., rewriting $2^x = 8$as $x = log_2(8)$) allows for solving exponential equations, essential in financial modeling, population growth, and radioactive decay.

completing the square reveals the maximum or minimum point, aiding in graphing and real-world applications like projectile motion. • Using exponent properties to transform expressions reveals key features of exponential functions, helping in modeling financial growth, population changes, and radioactive decay. • Adding, subtracting, multiplying, and dividing rational expressions mirrors operations with rational numbers. These functions often have asymptotes and discontinuities, crucial for graphing. • Understanding where two function graphs intersect helps solve systems of equations, interpret business models (e.g., break-even analysis), and analyze competing trends in data.	• Rearranging formulas (e.g., converting $A = Pe^{rt}$ into $t = \frac{ln(A/P)}{r}$) helps analyze interest rates, half-life problems, and inverse exponential relationships in scientific contexts. • Applying factoring, completing the square, or algebraic methods to trigonometric functions (e.g., solving $2sin^2(x) - sin(x) - 1 = 0$ by factoring) supports work in physics, engineering, and wave motion. • Factoring and transforming expressions simplify trigonometric identities, such as rewriting $cos^2(x) - sin^2(x)$ using polynomial techniques to reveal properties of sine and cosine graphs. • Finding points of intersection between exponential and logarithmic functions or trigonometric and polynomial functions is crucial in modeling business trends, sound waves, and electrical circuits.

Differentiation through *Universal Design for Learning*	
UDL Indicator	**Teacher Actions**
Multiple Means of Representation (Principle I) Consideration 1.3 Offer alternatives for visual information – Using dynamic graphing tools, interactive simulations, or color-coded diagrams can help students visualize polynomial functions, their end behavior, and how factors affect the graph. Consideration 3.2: Highlight patterns, critical features, big ideas, and relationships – Emphasizing key connections, such as the relationship between degree, roots, and graph shape, helps students generalize these mathematical concepts.	**Visual and Interactive Tools** – Use graphing calculators, Desmos, or GeoGebra to explore polynomial and rational function behaviors dynamically. **Real-World Applications** – Show how polynomial functions model real-life phenomena like physics (projectile motion) and economics (profit models) to improve relevance. **Scaffolded Problem-Solving** – Use guided questioning and step-by-step problem-solving to help students analyze polynomials and apply theorems (Intermediate Value Theorem, Factor Theorem). **Multiple Forms of Representation** – Present concepts using verbal explanations, symbolic notation, graphical representations, and real-world contexts to reach diverse learners.

Supporting Multilingual/English Learners	
Related *CELP standards:*	**Learning Targets**
A MLL can . . . determine the meaning of words and phrases in oral presentations and literary and informational text. I can rewrite and manipulate algebraic expressions, including polynomials and rational expressions, using algebraic rules to evaluate multiple solution methods to identify the most efficient strategy, while understanding how alternative approaches provide insight into mathematical relationships. • **Level 1** – I can use numbers, symbols, and simple words to show how to rewrite basic algebraic expressions, such as combining like terms or factoring simple polynomials, with visual or guided support. • **Level 2** – I can describe and follow steps to rewrite and simplify algebraic expressions, including polynomials and fractions, using models, word banks, and sentence frames. • **Level 3** – I can explain how to rewrite and manipulate algebraic expressions, including polynomials and rational expressions, using algebraic rules, and describe how different methods lead to the same solution. • **Level 4** – I can analyze and compare different methods of rewriting algebraic expressions, explain why one method may be more efficient than another, and describe how each approach provides insight into a mathematical problem. • **Level 5** – I can justify my choice of algebraic manipulation strategies by evaluating efficiency, comparing alternative methods, and explaining how different forms of an expression reveal key mathematical relationships.	

Lesson Sequence	**Learning Target**	**Success Criteria/ Assessment**	**Resources**
2.1 Complex numbers	I can identify complex numbers and use them to solve problems	• I can add and subtract complex numbers. • I can multiply complex numbers. • I can perform operations with square roots of negative numbers. • I can solve quadratic equations with complex solutions.	
2.2 Quadratic functions	I can analyze quadratic functions	• I can Recognize characteristics of parabolas. • I can graph parabolas. • I can determine a quadratic functions' max or min value. • I can solve problems involving a quadratic function's min or max value.	
2.3 Polynomial Functions and Their Graphs	I can analyze polynomial functions	• I can identify polynomial functions • I can recognize characteristics of graphs of polynomial functions. • I can determine end behavior. • I can Use factoring to find zeros of polynomial functions • I can identify zeros and their multiplicities • I can graph polynomial functions.	
2.4 Dividing Polynomials Remainder and Factor Theorems.	I can divide polynomials	• I can use long division to divide polynomials • I can use synthetic division to divide polynomials. • I can evaluate a polynomial using the remainder theorem. • I can use the factor theorem to solve a polynomial equation.	
2.5 Zeros of polynomial functions	I can find zeros of a polynomial function.	• I can solve polynomial equations. • I can use the linear factorization theorem to find polynomials with given zeros	
2.6 Rational Functions and Their Graphs	I can analyze rational functions	• I can find the domains of rational functions. • I can use arrow notation. • I can identify vertical asymptotes. • I can identify horizontal asymptotes • I can use transformations to graph rational functions. • I can graph rational functions.	

Unit Title
Unit 3: Exponential and Logarithmic Functions
Relevant Standards: Bold indicates priority

A.SSE.A.1 Interpret expressions that represent a quantity in terms of its context.

- A.SSE.A.1.B Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1 + r)^n$ as the product of P and a factor not depending on P.

A.SSE.B.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

- A.SSE.B.3.C Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15^t can be rewritten as $1.15^{1/12} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.

A.CED.A.2 Create equations that describe numbers or relationships. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

A.REI.D.11 Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

F.IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

- F.IF.C.7.E Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

F.IF.C.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

- F.IF.C.8.B Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = 1.02^t$, $y = 0.97^t$, $y = 1.01^{12t}$, $y = 1.2^{t/10}$, and classify them as representing exponential growth and decay.

F.BF.A.1 Write a function that describes a relationship between two quantities.

F.BF.B.5 Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

F.LE.A.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.

- F.LE.A.1.C Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

F.LE.A.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

F.LE.A.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

F.LE.A.4 For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

F.LE.B.5 Interpret the parameters in a linear or exponential function in terms of a context.

S.ID.B.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

- S.ID.B.6.A Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.

Essential Question(s)	Enduring Understanding(s):
• How can exponential functions be used to model growth and decay? • What is the purpose of logarithms and exponential functions and how are they related to each other? • How do you graph exponential and logarithmic functions by analyzing intercepts and end behavior? • How are exponential and logarithmic functions used to model real-world phenomena?	• Exponential functions model situations which grow or decline at a constant percent rate. • Graphing exponential functions and logarithmic functions will result in asymptotic behavior. Transformations (shifts, stretches, reflections) will affect the graphs. • Logarithms can be used to solve exponential functions; and conversely, exponents are used to solve logarithmic equations.

<table>
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- Logarithms can be used to solve equations for which no other algebraic method exists.
- Exponential functions model situations involving growth or decay, such as population growth, radioactive decay, and interest calculations, while logarithmic functions are used to model phenomena like the Richter scale for earthquakes and pH in chemistry.
- Exponential and logarithmic functions are inverse functions.

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<tr><th>Demonstration of Learning</th><th>Pacing for Unit</th></tr>
<tr><td>Homework
Class Practice
Readiness Quizzes
Skills Check Quizzes
Mid-Unit Checkpoint
End of Unit Assessment</td><td>10 blocks</td></tr>
<tr><th>Family Overview (link below)</th><th>Integration of Technology</th></tr>
<tr><td>Precalculus ACA - Family Overview (2024-25)
Precalculus ACA - Family Overview (2024-25) SPANISH</td><td>TI-84 graphing calculator
Desmos calculator</td></tr>
<tr><th>Unit-specific Vocabulary</th><th>Aligned Unit Materials, Resources, and Technology (beyond core resources)</th></tr>
<tr><td>Exponential Growth, Exponential Decay, Exponential Function, Exponential Regression, Growth Factor, Decay Factor, Compounded monthly, quarterly, etc. Asymptote, Base, Natural base e, Compounded continuously, Logarithm, Logarithmic Regression, Common Logarithm, Natural Logarithm, Power Function, Logarithmic Equation, Extraneous Solution, Inverse functions</td><td>TI-84 graphing calculator
Desmos calculator
Mini white boards
Teacher created activities
Vertical surfaces</td></tr>
<tr><th>Opportunities for Interdisciplinary Connections</th><th>Anticipated misconceptions</th></tr>
<tr><td>

Science (Physics, Biology, and Chemistry)

- **Radioactive Decay & Half-Life (Physics & Chemistry):** Exponential decay models how radioactive substances break down over time. The half-life formula uses logarithms to determine how long it takes for a substance to decay to a certain amount.
- **pH Scale (Chemistry):** Logarithmic functions are used to measure acidity and alkalinity, where pH is the negative logarithm of the hydrogen ion concentration.
- **Population Growth (Biology & Ecology):** Exponential functions model how populations grow under ideal conditions, and logistic growth models (which incorporate carrying capacity) adjust the exponential model for limited resources.

Economics & Finance

- **Compound Interest:** Exponential functions describe how money grows in savings accounts, investments, and loans. The compound interest formula uses exponents, and logarithms are used to solve for unknown time periods or rates.
- **Inflation & Depreciation:** Economic trends like

</td><td>

- Misunderstanding the structure of expressions- seeing each term in an algebraic expression separately rather than recognizing the expression's overall structure.
- Believing that equivalent expressions are always identical in appearance.
- incorrectly apply exponent rules, such as thinking $(a^b)^c = a^{b+c}$ instead of a^{bc}.
- Assuming exponential growth means a constant increase in the function's output, like linear growth.
- Treating logarithms as unrelated to exponents instead of understanding them as inverses.
- Believing that every function graph must cross the x-axis (e.g., assuming all polynomials have real roots).
- Automatically applying linear models, even when an exponential or quadratic model fits better.
- Thinking that an asymptote is a physical boundary the graph cannot cross.
- Thinking the b in $y = ab^x$ represents the total amount rather than the growth/decay factor per unit interval.
- Believing transformations only affect shape but not key features like intercepts, end behavior, and asymptotes.

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</table>

inflation (increasing prices over time) and depreciation (value loss of assets) are modeled using exponential functions. **Earth Science & Natural Disasters** • **Richter Scale for Earthquakes:** The Richter scale is logarithmic, meaning that an earthquake of magnitude 6 is 10 times stronger than a magnitude 5. • **Sound Intensity (Decibels):** Logarithmic functions are used in measuring sound levels in decibels, where an increase of 10 dB represents a tenfold increase in intensity. • **Carbon Dating:** Scientists use exponential decay equations to estimate the age of fossils and ancient artifacts by analyzing the remaining Carbon-14 in the sample.	
Connections to Prior Units	**Connections to Future Units**
• Exponential functions model constant percent growth or decay, distinguishing them from linear and polynomial functions that grow at a constant rate or degree-based rate. • Just like polynomial and rational functions, exponential and logarithmic functions can undergo transformations such as shifts, reflections, and stretches. • Logarithms and exponentials are inverse functions, similar to how square and square root functions relate. • Rational functions and logarithmic functions both model asymptotic behavior, meaning they never cross a certain boundary.	• Solving Trigonometric Equations Involving Exponents and Logarithms: In physics and engineering, exponential functions often appear in damping or wave equations. • Logarithms and Periodic Growth: Logarithms are used in measuring decibels (sound intensity), earthquake magnitudes (Richter scale), and pH levels (acidity), all of which relate to oscillatory or wave-based phenomena. • The Fourier Transform, a key tool in signal processing, represents periodic wave functions (trigonometry) as sums of exponentials (Euler's formula). • The logarithmic spiral, which appears in nature (e.g., shells, hurricanes), is modeled using both exponential and trigonometric functions $r = ae^{b\Theta}$. This function combines exponential growth (size increases) with circular motion (angle-based progression).
Differentiation through *Universal Design for Learning*	
UDL Indicator	**Teacher Actions**
Multiple Means of Representation (Principle 1) Consideration 1.1 Perception: Provide options for perception by offering multiple formats (e.g., visual, auditory, kinesthetic) for understanding exponential growth, decay, and transformations in graphs. Consideration 1.2 Language & Symbols: Clarify vocabulary and symbols by providing definitions, explanations, and examples of terms like growth/decay, logarithms, and transformations.	**Multiple Means of Representation (Principle 1)** • Present the exponential function $y = P(1 + r)^t$ using multiple formats, such as a table, graph, and verbal explanation. Use graphing software (like Desmos or GeoGebra) to show the curve's shape and asymptotic behavior. • Offer a visual representation alongside a text-based explanation for students with different learning preferences. • Provide animated videos showing the behavior of exponential growth (such as population growth) and decay (such as radioactive decay), so students can see how these concepts evolve over time. • Simplify complex expressions by breaking them down into smaller, more digestible pieces.
Supporting Multilingual/English Learners	

<table>
<tr><th colspan="2">Related <u>CELP standards:</u></th><th colspan="2">Learning Targets</th></tr>
<tr><td colspan="4">A MLL can . . . determine the meaning of words and phrases in oral presentations and literary and informational text.
I can analyze and graph exponential and logarithmic functions by identifying key characteristics such as growth and decay, intercepts, asymptotes, and transformations, and I can solve related equations and inequalities to model and solve real-world problems.
Level 1 - I can point to and recognize pictures and labels of exponential and logarithmic function graphs, noticing simple parts like where the graph goes up, down, or gets close to a line.
Level 2 - I can match basic terms (growth, decay, intercept, asymptote, transformation) with examples on exponential and logarithmic graphs using word banks and visuals.
Level 3 - I can describe and label parts of exponential and logarithmic function graphs using key vocabulary (growth, decay, intercepts, asymptotes, and transformations) and solve simple related equations.
Level 4 - I can analyze and graph exponential and logarithmic functions by identifying key features—such as growth, decay, intercepts, asymptotes, and transformations—and solve equations and inequalities that model real-world problems.
Level 5 - I can independently analyze, graph, and solve complex equations and inequalities involving exponential and logarithmic functions by accurately identifying key characteristics (growth, decay, intercepts, asymptotes, and transformations) and applying these concepts to model and address real-world scenarios.</td></tr>
<tr><th>Lesson Sequence</th><th>Learning Target</th><th>Success Criteria/ Assessment</th><th>Resources</th></tr>
<tr><td>3.1 Exponential Functions</td><td>I can analyze exponential functions</td><td colspan="2">I can evaluate exponential functions
I can graph exponential functions
I can evaluate functions with base e.
I can use compound interest formula.</td></tr>
<tr><td>3.2 Logarithmic Functions</td><td>I can analyze logarithmic functions</td><td colspan="2">I can change from log to exponential form.
I can change from exponential to log form.
I can evaluate logarithms.
I can use basic logarithmic properties.
I can use common and natural logarithms</td></tr>
<tr><td>3.3 Properties of Logarithms</td><td>I can use properties of logarithms to find equivalent expressions</td><td colspan="2">I can use the product rule.
I can use the quotient rule.
I can use the power rule.
I can expand logarithmic expressions.
I can condense logarithmic expressions.
I can use the the change of base property</td></tr>
<tr><td>3.4 Exponential and Logarithmic Equations</td><td>I can solve exponential and logarithmic equations.</td><td colspan="2">I can use like bases to solve exponential equations.
I can use logarithmic to solve exponential equations
I can use the definition of a logarithm to solve logarithmic equations.</td></tr>
<tr><td>3.5 Exponential Growth and Decay</td><td>I can solve real world problems using logarithms.</td><td colspan="2">I can model exponential growth and decay.
I can use logistic growth models.
I can express and exponential model in base e.</td></tr>
</table>

Unit Title	
Unit 4: Trigonometric Functions	
Relevant Standards: Bold indicates priority	
F.TF.A.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. **F.TF.A.2** Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. **F.TF.A.3** Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosine, and tangent for π - x, π + x, and 2π - x in terms of their values for x, where x is any real number. **F.TF.A.4** Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. **F.TF.B.6** Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. **G.SRT.C.6** Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. **G.SRT.C.7** Explain and use the relationship between the sine and cosine of complementary angles. **G.SRT.C.8** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.	
Essential Question(s)	**Enduring Understanding(s)**
• How are sine, cosine, and tangent defined using the sides of a right triangle? • What are the reciprocal trigonometric ratios (cosecant, secant, cotangent)? • How do the coordinates of points on the unit circle relate to trigonometric functions? • What are the radian and degree measures, and how do they relate to each other? • How do amplitude, period, phase shift, and vertical shift affect the graphs of trigonometric functions? • What are the key features of the graphs of sine, cosine, tangent, cosecant, secant, and cotangent functions? • What are the definitions and domains of the inverse sine, cosine, and tangent functions? • How are the inverse functions used to find angles given a trigonometric ratio?	• The trigonometric functions (sine, cosine, tangent, and their reciprocals) represent specific relationships between the angles and side lengths of a right triangle. Understanding these functions as ratios helps students connect geometric intuition with algebraic representation. • The unit circle is a foundational concept in trigonometry that connects the angle measures in radians with the coordinates of points on the circle. This understanding leads to the concept of periodicity in trigonometric functions, where patterns repeat over regular intervals. • The graphs of sine, cosine, and tangent functions exhibit unique characteristics such as amplitude, period, phase shift, and vertical shift. Recognizing these properties allows students to model real-world phenomena and understand oscillatory behavior. • Inverse trigonometric functions, such as arcsine, arccosine, and arctangent, are essential for finding angles when given a trigonometric ratio. Understanding their domains, ranges, and interpretations is crucial for solving trigonometric equations and modeling. • Trigonometric functions can be represented analytically (using equations) and graphically (using graphs). Understanding both forms and how they interrelate is important for solving problems and interpreting data. • Understanding different units of angle measurement, such as degrees and radians, and how to convert between them is crucial for working with trigonometric functions and applying them in various contexts.
Demonstration of Learning	**Pacing for Unit**
Homework Class Practice Readiness Quizzes	16 blocks

Skills Check Quizzes Mid-Unit Checkpoint End of Unit Assessment	
Family Overview (link below)	**Integration of Technology**
Precalculus ACA - Family Overview (2024-25) Precalculus ACA - Family Overview (2024-25) SPANISH	TI-84 graphing calculator Desmos calculator
Unit-specific Vocabulary	**Aligned Unit Materials, Resources, and Technology (beyond core resources)**
Radian, Degree, Coterminal Angles, Complement, Supplement, Sine, Cosine, Tangent, Secant, Cosecant, Cotangent, Reference Angle, Unit Circle, Amplitude, period	TI-84 graphing calculator Desmos calculator Mini white boards Teacher created activities Vertical surfaces
Opportunities for Interdisciplinary Connections:	**Anticipated misconceptions**
Physics: Wave Motion and Harmonics Trigonometric functions such as sine and cosine are commonly used to model wave motion, which is a fundamental concept in physics. The oscillations of waves, such as sound waves, light waves, and mechanical vibrations, can be described using trigonometric functions. The concepts of amplitude, period, and phase shift are used to understand wave behavior and harmonic motion. **Engineering: Signal Processing** In electrical engineering, trigonometric functions are integral to analyzing and designing circuits, particularly in signal processing. The concept of periodicity in trigonometric functions is crucial for understanding alternating current (AC) signals and their behaviors over time. **Astronomy: Orbital Mechanics** Trigonometry plays a vital role in astronomy and space science, especially in understanding the orbits of planets and satellites. The unit circle and trigonometric functions are used to calculate the positions of celestial bodies and to model orbital motion, such as the movement of planets around the sun or artificial satellites orbiting Earth. **Computer Science: Graphics and Animation** Trigonometric functions are foundational in computer graphics, particularly in the rendering of 3D graphics and animations. Sine and cosine functions help model rotations and scaling transformations of objects in a virtual space. **Economics: Cyclical Behavior and Seasonal Trends** Trigonometric functions can be used to model cyclical or periodic behaviors in economics, such as seasonal demand, temperature fluctuations, or economic cycles. Understanding how periodic functions like sine and cosine apply to real-world data can help economists predict trends and make forecasts.	• Students often confuse radians and degrees as two equivalent systems of measuring angles, without fully understanding the concept of radians as the ratio of the arc length to the radius on the unit circle. • Thinking that the unit circle and trigonometric functions only apply to angles within one full rotation (0 to 360 degrees or 0 to 2π radians). • Students may not fully grasp that trigonometric functions (like sine, cosine, and tangent) repeat at regular intervals (i.e., they are periodic functions). • Thinking that all angles on the unit circle are positive (in a counterclockwise direction), neglecting the fact that angles can also be negative, which correspond to clockwise motion. • Incorrectly applying the unit circle by misunderstanding how to find the sine, cosine, and tangent values based on the coordinates on the unit circle. • Mistaking special triangle values for all angles
Connections to Prior Units:	**Connections to Future Units:**

• Just as algebraic functions (polynomials, rational functions) undergo transformations such as shifts, reflections, and stretches, so do trigonometric functions. These transformations (e.g., amplitude changes, phase shifts, vertical shifts) help students understand how functions behave in a coordinate plane, similar to the transformations applied to polynomial or rational graphs. • Exponential functions and trigonometric functions are deeply connected through Euler's formula, $e^{ix} = cos(x) + isin(x)$. This formula connects exponential growth or decay to trigonometric functions, demonstrating how trigonometric functions can model oscillatory behavior (e.g., in waveforms) alongside exponential growth or decay. • Trigonometric identities (such as the Pythagorean identity $sin^2(x) + cos^2(x) = 1$ allow trigonometric expressions to be simplified into polynomial forms. This is useful for solving equations that may involve both trigonometric and polynomial terms.	• Analytic trigonometry involves the use of identities (like the Pythagorean identity, sum and difference formulas, etc.) to simplify trigonometric expressions and solve equations. For example, solving trigonometric equations often requires applying identities to reduce complex expressions into simpler forms, making the equations easier to handle analytically. • Systems of equations that involve trigonometric functions can often be solved using algebraic techniques, such as substitution or elimination. For example, if you have a system that involves both sine and cosine, you might use the Pythagorean identity $sin^2(x) + cos^2(x) = 1$ to convert one equation into a simpler form. • In analytic trigonometry, inverse functions (like arcsin, arccos, and arctan) are used to solve for angles in trigonometric equations. These functions allow us to find specific angle measures when given a trigonometric ratio, which is essential when solving systems of equations involving trigonometric relationships. • In systems of equations, graphing trigonometric functions can visually show the solutions where different trigonometric functions intersect. For instance, solving a system of two sine and cosine equations can be done graphically by plotting the functions and identifying where their graphs intersect, providing a visual understanding of the solution.
Differentiation through [Universal Design for Learning]	
UDL Indicator	**Teacher Actions:**
Representation Clarify vocabulary, symbols, and language structures for trigonometric functions (2.1)	• Have students create index cards for key trigonometric terms (e.g., amplitude, period, phase shift, vertical shift, unit circle, radian). They can write the term on one side and a definition with a sketch or example on the other. Use these cards in a memory match or Jeopardy-style game to reinforce definitions and symbols. These same cards can be used as a fun "heads up" game. • Create a class set of index cards where one card has the vocabulary term and the other has the definition and students need to find their match. • Set up a digital or physical vocabulary wall where each key term is displayed with its definition, symbol, and a visual representation (e.g., a graph highlighting the amplitude or a unit circle with marked angles). Allow students to add sticky notes or digital comments with examples or questions, creating a collaborative resource they can reference throughout the unit. • Challenge students to create a short video, song, or digital poster that explains a set of trigonometric vocabulary and symbols. For instance, they might create a catchy mnemonic song for "SOH CAH TOA" while illustrating what each term means with

	examples from graphs or the unit circle. Sharing these projects in class reinforces language structures and provides multiple representations of the same information.

Supporting Multilingual/English Learners	
Related *CELP standards:*	**Learning Targets:**

A MLL can . . . determine the meaning of words and phrases in oral presentations and literary and informational text.
I can analyze and graph trigonometric functions by identifying key characteristics such as amplitude, period, phase shift, and vertical shift, and apply these concepts to model periodic real-world scenarios.

- **Level 1 -** I can recognize and point to parts of a sine or cosine graph using pictures and simple labels.
- **Level 2 -** I can match basic terms like “maximum,” “minimum,” “wave,” and “interval” with features on a trigonometric graph using visual supports and word banks.
- **Level 3 -** I can label the key features of trigonometric graphs—amplitude, period, phase shift, and vertical shift—using provided sentence frames and visuals.
- **Level 4 -** I can analyze and graph trigonometric functions by identifying and explaining how amplitude, period, phase shift, and vertical shift affect the graph’s shape.
- **Level 5 -** I can independently analyze and graph trigonometric functions by accurately interpreting amplitude, period, phase shift, and vertical shift and applying these concepts to model periodic real-world scenarios.

Lesson Sequence	**Learning Target**	**Success Criteria/ Assessment**	**Resources**
4.1 Angles and Radian Measure	I can recognize and use the vocabulary of angles	• I can use degree measure. • I can use radian measure. • I can convert between degrees and radians. • I can draw angles in standard position. • I can find coterminal angles. • I can find the length of a circular arc. • I can use linear and angular speed to describe motion on a circular arc.	
4.2 Trigonometric Functions: The unit Circle	I can use a unit circle to define trigonometric functions of real numbers	• I can recognize the domain and range of sine and cosine functions. • I can find exact values of trigonometric functions • I can evaluate trigonometric functions.	
4.3 Right Triangle Trigonometry	I can use right triangles to evaluate trigonometric functions.	• I can find function values for 30 degrees and 60 degrees. • I can use right triangle trig to solve applied problems.	
6.1 The Law of Sines	I can use the Law of Sines to solve triangles.	• I can use the law of sines to solve oblique triangles. • I can use the law of sines to solve, if possible, the triangle or triangles in the ambiguous case. • I can find the area of an oblique triangle. • I can solve applied problems using the law of sines.	
6.2 Law of Cosines	I can use the Law of Cosines to solve triangles	• I can use the law of cosines to solve oblique triangles. • I can solve applied problems using the law of cosines. • I can use heron's formula to find the area of a triangle	
4.4 Trigonometric Functions of Any Angle	I can use the definitions of trigonometric functions of any angle.	• I can use the signs of the trigonometric functions. • I can find reference angles. • I can use reference angles to evaluate trigonometric functions.	
4.5 Graphs of Sine and Cosine Functions	I can understand the graphs of Sin and Cos Functions	• I can graph variations of a sine function. • I can graph variations of a cosine function. • I can use vertical shifts of sine and cosine curves. • I can model periodic behavior.	

4.7 Inverse Trigonometric Functions	I can understand and use the inverse sin and cos functions.	• I can find exact values of inverse sin and cos functions.
4.8 Applications of Trigonometric Functions	I can use Trigonometry to solve applied problems.	• I can solve a right triangle. • I can solve problems involving bearing.

<table>
<tr><th colspan="2">Unit Title</th></tr>
<tr><td colspan="2">Unit 5: Analytic Trigonometry</td></tr>
<tr><th colspan="2">Relevant Standards: Bold indicates priority</th></tr>
<tr><td colspan="2">F.TF.C.8 Prove the Pythagorean identity $sin^2(\theta) + cos^2(\theta) = 1$ and use it to find $sin(\theta)$, $cos(\theta)$, or $tan(\theta)$ given $sin(\theta)$, $cos(\theta)$, $tan(\theta)$ and the quadrant of the angle.
F.TF.C.9 Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems
A.SSE.A.2 Use the structure of an expression to identify ways to rewrite it.
G.SRT.D.9 Apply trigonometry to general triangles. Derive the formula A = (1/2)ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
G.SRT.D.10 Apply trigonometry to general triangles. Prove the Laws of Sines and Cosines and use them to solve problems.
G.SRT.D.11 Apply trigonometry to general triangles. Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).</td></tr>
<tr><th>Essential Question(s)</th><th>Enduring Understanding(s)</th></tr>
<tr><td>• Why do we structure trig functions in different ways?
• What is the difference between solving an equation and verifying a trigonometric identity?
• How do you decide which strategies to use when verifying a trigonometric identity?
• How are trigonometric identities used in the process of solving trig equations?
• How are trigonometric ratios used in solving real-world problems?
• How do you determine whether the law of sines or law of cosines is most appropriate for solving a particular problem?
• In ambiguous cases, how many possible triangles can the Law of Sines produce?
• Is it possible to solve for the area of a triangle if the altitude is not known?
• What kind of real-life situations can be modeled by sinusoidal equations?
• How can sinusoidal equations be used to predict future values?
• How can the area of a triangle be found using the law of sines and cosines?</td><td>• Trigonometric functions are useful for modeling periodic phenomena.
• Expressions can be written in multiple ways using the rules of Trigonometry and Algebra and used to solve trigonometric functions.
• The law of sines and cosines are fundamental trigonometry laws that can be used to solve problems involving the sides and angles of non-right triangles.
• In a non-right triangle the given information determines which law is most appropriate to solve a problem.
• When you know two sides and a non-included angle (SSA), there can be zero, one, or two possible triangles, known as the "ambiguous case."
• If the altitude of a non-right triangle is not given, it can be expressed using trigonometry and used to calculate its area.</td></tr>
<tr><th>Demonstration of Learning:</th><th>Pacing for Unit</th></tr>
<tr><td>Homework
Class Practice
Readiness Quizzes
Skills Check Quizzes
Mid-Unit Checkpoint
End of Unit Assessment</td><td>5 blocks</td></tr>
<tr><th>Family Overview (link below)</th><th>Integration of Technology</th></tr>
<tr><td>Precalculus ACA - Family Overview (2024-25)
Precalculus ACA - Family Overview (2024-25) SPANISH</td><td>TI-84 graphing calculator
Desmos calculator</td></tr>
<tr><th>Unit-specific Vocabulary</th><th>Aligned Unit Materials, Resources, and Technology (beyond core resources)</th></tr>
<tr><td>Sine, Cosine, Tangent, Secant, Cosecant, Cotangent, Sum Formula, Difference Formula, Double Angle Formula, Half Angle Formulas, Pythagorean identity, Pythagorean theorem, Unit circle, Quotient identity, Law of Sines, Law</td><td>TI-84 graphing calculator
Desmos calculator
Mini white boards
Teacher created activities</td></tr>
</table>

of Cosines, Angle of Elevation, Angle of Depression, Bearings, Heron's Formula, Ambiguous Case, Included Angle, Solving Triangles, Acute Triangle, Right Triangle, Obtuse Triangle, Oblique Triangle	Vertical surfaces
Opportunities for Interdisciplinary Connections:	**Anticipated misconceptions:**
Physics - Wave Motion and Oscillations: Trigonometric functions (especially sine and cosine) model periodic phenomena such as sound waves, light waves, and mechanical vibrations. Understanding the relationships between the amplitude, frequency, and phase of wave functions helps students in physics to analyze oscillatory systems. **Engineering - Signal Processing and Electrical Engineering:** Analytical trigonometry plays a key role in signal processing, particularly in analyzing alternating current (AC) circuits and Fourier transforms. Engineers use sine and cosine functions to model signals and study the frequency and phase shifts in electrical systems. **Astronomy - Orbital Mechanics:** Trigonometric functions are used to calculate the positions and motions of celestial bodies. The concepts of angle measurements, periodicity, and phase shifts help describe the orbits of planets, satellites, and the apparent motion of stars. **Geography and Navigation:** Bearings, Angle of Elevation, and Angle of Depression are essential concepts used in geography and navigation. In these fields, trigonometric principles, like the Law of Sines and Law of Cosines, are applied to mapmaking, surveying, and GPS navigation.	• Students often confuse the unit circle with a simple circle, not realizing that its radius is always 1 and the coordinates on the circle represent the cosine and sine values of the corresponding angles. • Incorrectly assuming that trigonometric functions, like sine or cosine, have a range of all real numbers. • Students may incorrectly apply trigonometric identities, such as assuming $sin(\theta) = cos(\theta)$ for all angles or misapplying the Pythagorean identity $sin^2\theta + cos^2\theta = 1$. • Overlooking the periodic nature of trigonometric functions and assuming that functions like sine, cosine, and tangent behave as if they are linear and continue to increase or decrease indefinitely. • Students may mix up angle measurements in degrees and radians, especially when using trigonometric functions. • Confusing angle of elevation and angle of depression. • Incorrect application of the Law of Sines and Law of Cosines.
Connections to Prior Units:	**Connections to Future Units:**
• Trigonometric functions (sine, cosine, tangent, etc.) can be analyzed as functions with specific domains and ranges. These functions can be graphed on the coordinate plane, providing insights into the relationship between angles and side lengths in triangles, similar to how algebraic functions are graphed and interpreted. This connection reinforces the understanding of functions, their graphs, and the properties of periodicity, amplitude, and symmetry. • Just as algebraic functions undergo transformations (shifts, stretches, reflections), trigonometric functions can also be transformed. For example, the graph of $y = sin(x)$ can be shifted, reflected, and stretched. These transformations are central to understanding how the periodic nature of trigonometric functions is affected by changes in the input. • The concept of inverse functions extends from algebra to trigonometry. Just as logarithmic functions are inverses of exponential functions, inverse trigonometric functions (such as arcsine, arccosine, and arctangent) allow for the solving of equations involving trigonometric functions. Understanding their domains and ranges is similar to understanding how logarithmic functions are used to solve exponential equations.	• Many trigonometric equations, such as $sin(x) = 1/2$ or $tan(x) = 3$, can be solved as part of a system of equations. By applying inverse trigonometric functions, students can solve for angles and use the solutions to form systems of equations with multiple variables. • Trigonometry plays a significant role in polar coordinates, where equations involving trigonometric functions can describe curves or points. Students may need to solve systems of equations involving sine and cosine functions to find points of intersection or curves represented in polar form. • Trigonometric identities can be used to simplify and solve systems of trigonometric equations. For example, by using the sum and difference identities, students can simplify equations in terms of sine and cosine and solve systems of equations to find values of angles or unknowns. • Sometimes, trigonometric functions are involved in linear systems, such as solving systems of equations where one equation is trigonometric (e.g., $sin(x) + 2 = 3$) and the other is linear or polynomial. These systems can be solved using algebraic techniques, substitution, or numerical methods. • Analytical trigonometry connects to real-world

• Trigonometric identities, such as the Pythagorean identities, sum and difference identities, and double-angle formulas, often require algebraic manipulation for simplification and solving equations. For example, simplifying expressions involving trigonometric functions can be approached in a manner similar to simplifying polynomial or rational expressions in algebra.	systems, such as modeling wave motion, oscillations, or sound waves. In these cases, students may solve systems of equations involving trigonometric functions to model physical phenomena like harmonic motion or electrical circuits, where multiple trigonometric relationships interact.
Differentiation through *Universal Design for Learning*	
UDL Indicator	**Teacher Actions**
Representation Connect prior knowledge to new learning (3.1)	• Organize a small-group workshop where each group receives a mixed set of problems that require both factoring techniques and trigonometric manipulation (e.g., simplifying expressions like $\frac{sin^2x - cos^2x}{sinx - cosx}$). Begin by having them recall how they factored quadratic or rational expressions, then challenge them to apply similar techniques to the trigonometric problem. Afterward, groups share their strategies and solutions with the class, emphasizing how their prior knowledge helped them tackle new analytical trigonometry challenges. • Present a real-world problem (such as modeling sound waves or periodic phenomena) that requires both rational functions and trigonometric identities to solve. Begin with a review of how they factored expressions and solved rational equations. Then, guide them in applying these techniques to break down the real-world problem into manageable parts. For instance, ask, “How can we factor this trigonometric expression to simplify the model?” This approach helps students see how their past learning directly informs and supports new analytical challenges. • Have students create a short digital story or comic strip that explains a “day in the life” of a math problem. Their story should begin with a familiar process like factoring or solving a rational expression and then transition into a scenario where these techniques are used to simplify a complex trigonometric identity. Students can illustrate how the basic unit circle, sine, cosine, and tangent relationships serve as the foundation for solving higher-level analytical trigonometry problems.
Supporting Multilingual/English Learners	
Related *CELP standards:*	**Learning Targets**
A MLL can . . . determine the meaning of words and phrases in oral presentations and literary and informational text. I can simplify trigonometric expressions, prove identities, and solve trigonometric equations using algebraic and analytical techniques to deepen my understanding of trigonometric relationships. • **Level 1 -** I can recognize and point to trigonometric expressions and symbols with the help of visual aids. • **Level 2 -** I can match simple trigonometric expressions and identities with their corresponding forms using word banks and pictures. • **Level 3 -** I can simplify basic trigonometric expressions and solve simple equations with guided support and partial sentence frames. • **Level 4 -** I can simplify trigonometric expressions, prove basic identities, and solve equations using algebraic	

and analytical methods while explaining my steps in clear academic language.

- **Level 5 -** I can work independently through challenging trigonometric problems—simplifying expressions, proving identities, and solving equations—to show my deep understanding of their relationships.

Lesson Sequence	Learning Target	Success Criteria/ Assessment	Resources
5.1 Verifying Trigonometric Identities	I can use the fundamental trigonometric identities to verify identities.	• I can show equivalence of expressions using trigonometric identities	
5.5 Trigonometric Equations	I can solve trigonometric equations	• I can find all solutions of a trigonometric equation. • I can solve equations with multiple angels. • I can solve trigonometric equations in linear form. • I can solve trigonometric equations in quadratic form. • I can use factoring to separate different functions in trigonometric equations. • I can use identities to solve equations. • I can use a calculator to solve trigonometric equations.	

<table>
<tr><th colspan="2">Unit Title</th></tr>
<tr><td colspan="2">Unit 6: Systems of Equations and Inequalities</td></tr>
<tr><th colspan="2">Relevant Standards: Bold indicates priority</th></tr>
<tr><td colspan="2">HSA.CED.A.3: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.
HSA.REI.C.6: Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
HSA.REI.C.7: Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.
HSA.REI.D.11: Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations.
HSA.REI.D.12: Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.
N.VM.C.6 Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
N.VM.C.7 Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
N.VM.C.8 Add, subtract, and multiply matrices of appropriate dimensions.
N.VM.C.9 Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.</td></tr>
<tr><th>Essential Question(s)</th><th>Enduring Understanding(s)</th></tr>
<tr><td>● How can we model real-world situations using systems of equations or inequalities?
● How do we interpret solutions to systems of equations and inequalities in the context of a problem?
● How do constraints affect the solutions to systems of equations or inequalities?
● What methods can we use to solve systems of linear equations exactly (algebraically and graphically)?
● How do the solutions to systems of linear equations relate to their graphical representations?
● How do the solutions to a system consisting of a linear and a quadratic equation differ from those in a purely linear system?
● How do we solve systems involving both linear and quadratic equations algebraically and graphically?
● Why do the points of intersection of two graphs represent the solutions to the system of equations?
● How do the solutions to systems of equations change based on the nature of the functions?
● What is a matrix, and how does it represent data or systems of equations in a structured way?
● How can matrices be used to solve systems of linear equations, and why is this method sometimes more efficient than traditional methods?
● Under what conditions does a matrix have an inverse, and why is the inverse of a matrix important in solving systems of equations and other applications?</td><td>● Systems of equations and inequalities model real-world scenarios, and their solutions represent viable and nonviable options depending on the constraints of the problem.
● Representing constraints through equations and inequalities provides insight into relationships between different variables in a given context.
● Systems of equations can be used to predict outcomes, make decisions, and optimize processes in various fields.
● A system of linear equations can be solved using various methods, including substitution, elimination, and graphing.
● Solutions to systems of linear equations can be interpreted graphically as the points of intersection of the corresponding lines.
● The process of solving systems of linear equations builds a foundation for more complex systems, including those involving nonlinear equations.
● Solutions to linear inequalities are represented as half-planes on the coordinate plane, and the solution set of a system of inequalities is the intersection of these half-planes.
● The boundary of the solution set in a system of inequalities is defined by the equation of the inequality, and whether the boundary is included or excluded depends on whether the inequality is strict or non-strict.
● Graphing systems of inequalities provides a visual representation of the range of possible solutions, helping to understand the feasibility of different</td></tr>
</table>

	options.
Demonstration of Learning	**Pacing for Unit**
Homework. Class Practice, Readiness Quizzes, Skills Check Quizzes, Mid-Unit Assessment, End-of-Unit Assessment	14 blocks
Family Overview (link below)	**Integration of Technology**
Precalculus ACA - Family Overview (2024-25) Precalculus ACA - Family Overview (2024-25) SPANISH	TI-84 graphing calculator Desmos calculator
Unit-specific Vocabulary	**Aligned Unit Materials, Resources, and Technology (beyond core resources)**
System of Equations, Solution to a System, Linear System, Linear Inequality, Substitution Method, Elimination Method, Graphical Method, Intersection, Ordered Pair, Ordered Triple, Feasible Region, Constraint, Dependent System, Independent System, Inconsistent System, Partial Solution, Graphing Linear Inequalities, Bounded Region, Unbounded Region, Dual System, Substituting into Inequalities, Critical Points, Convex Set, Basic Feasible Solution, Linear Programming, Objective Function, Slopes of Lines, Parallel Lines, Coincident Lines, Linear Transformation	TI-84 graphing calculator Desmos calculator Mini white boards Teacher created activities Vertical surfaces
Opportunities for Interdisciplinary Connections	**Anticipated misconceptions**
Economics • Optimization and Resource Allocation: Systems of equations and inequalities are commonly used in economics to model scenarios such as resource allocation, budget constraints, and supply and demand. **Engineering:** • Structural Analysis and Design: Systems of equations in three variables and inequalities are often used in engineering to analyze and design structures, such as bridges, buildings, and mechanical systems. For example, equations can represent forces, moments, and equilibrium conditions, while inequalities can describe safety factors or material strength constraints. **Environmental Science** • Modeling Population Growth and Resource Management: In environmental science, systems of equations and inequalities are used to model population dynamics, resource consumption, and environmental factors.	• Misunderstanding the meaning of solutions- Students often think that the solution to a system of equations represents a single point on a graph, rather than the set of all points that satisfy the system. • Believing that the solution is only where the graphs of the equations physically intersect on the graph, ignoring other possible solution sets such as parallel lines (no solutions) or coincident lines (infinitely many solutions). • Forgetting to consider the domain and range of the inequalities, assuming all values of x and y are solutions within a system of inequalities. • Graphing systems of inequalities by shading regions incorrectly, confusing the boundary lines (whether the inequality is strict or non-strict). • Confusing systems of linear equations with nonlinear equations, leading to incorrect graphing or solution methods. • Assuming that solving systems of three variables is always more complex and that it is unnecessary to check for special cases like infinitely many solutions or no solution. • Mixing up the approach to solving systems of equations with systems of inequalities, believing that the solutions are always specific points rather than regions of possible solutions.
Connections to Prior Units	**Connections to Future Units**
• Systems of equations in two or more variables involve algebraic manipulation to isolate variables, solve for unknowns, and simplify expressions. These processes connect directly to basic algebraic	• In introductory college math, students often learn about matrices and determinants as tools for solving systems of linear equations. The concepts of row reduction, matrix multiplication, and matrix inverses

concepts such as substitution, elimination, and solving for unknowns. • A system of equations often represents two or more functions (e.g., linear, quadratic) and their intersections. Understanding how functions relate to each other graphically is crucial for solving these systems, whether through graphical or algebraic methods. • Systems of equations in two variables are often solved graphically by plotting the equations on the coordinate plane and identifying the intersection points. This graphical representation connects to visualizing functions and understanding the relationships between them. • Systems may involve polynomials or rational functions, especially when dealing with non-linear systems (e.g., quadratic systems or rational equations). Solving these systems can involve factoring polynomials, solving quadratic equations, or working with rational expressions.	are introduced as methods for solving systems efficiently, especially for larger systems. • Systems of inequalities are foundational to linear programming and optimization problems, often introduced in introductory college courses. These systems model constraints and objective functions in maximizing or minimizing quantities, such as cost or profit. • In introductory calculus, systems of equations often appear in optimization problems where students analyze functions to find maximum and minimum values. Systems may also be used in solving problems that involve rates of change, like related rates or optimization under constraints. • A matrix is a rectangular array of numbers arranged in rows and columns, representing a set of data or a system of linear equations. • The size or dimension of a matrix (rows × columns) is a fundamental concept that affects how matrices can be manipulated and used in calculations. • Students should understand that matrices of the same dimensions can be added or subtracted element-wise, with each element in the result being the sum or difference of corresponding elements. • Matrix multiplication is not commutative, meaning the order of multiplication matters. Students should understand the conditions under which matrix multiplication is defined and how to perform it. • Matrices provide a powerful way to represent and solve systems of linear equations, with each system corresponding to a matrix equation. • Matrices can be used to model and solve problems in fields such as economics, engineering, computer graphics, and more. • As matrices grow in size, operations involving them can become computationally intensive. Understanding the complexity of these operations is important for practical applications. • While manual computation with matrices is important for learning, recognizing the role of technology in handling large and complex matrices is also essential.
Differentiation through *Universal Design for Learning*	
UDL Indicator	**Teacher Actions**
Representation Support decoding of text, mathematical notation, and symbols. (2.2)	• Create large posters that display a system of equations or inequalities with different components color-coded. For instance, use one color for variables, another for coefficients, and another for operation symbols. Display these in the classroom and refer to them during lessons so students can see a visual breakdown of each part of the expression. • In small groups, give students a complex system of equations or inequalities and have them work together to "decode" it. Ask each group to write a simplified explanation of what each symbol and term means in plain language, then share with the class.

	For instance, one group might explain, “The ‘+’ between the equations means we’re combining the results to find the common solution,” and so on. • Organize a classroom scavenger hunt where students search for different symbols and notations on handouts or on an interactive whiteboard. Have half of the posters as “need to solve” and half of the posters be error analysis challenges. Provide a checklist of skills and “look for’s.” This game reinforces familiarity with mathematical notation and strategy analysis in a fun, active way. • Create a matching activity where students need to match each part of the process—for example, “Start here with the given system,” “Identify the coefficients and constants,” “Decide whether to use substitution or elimination,” “set up the system,” “multiply to get coefficients that will cancel,” etc. with the parallel math step.

Supporting Multilingual/English Learners

Related *CELP standards:*	**Learning Targets**

A MLL can . . . determine the meaning of words and phrases in oral presentations and literary and informational text.
I can analyze systems of equations and inequalities to model real-world scenarios by representing constraints, graphing solutions as points or regions, and using methods like substitution, elimination, and graphing to predict outcomes, assess viable options, and optimize solving processes.

- **Level 1 -** I can use pictures, labels, and simple diagrams to identify basic parts of equations and inequalities that show how real-world limits work.
- **Level 2 -** I can recognize key words like "constraint," "solution," and "graph" and match them with examples of simple systems of equations and inequalities in everyday situations using visual aids.
- **Level 3 -** I can solve basic systems of equations and inequalities using methods like substitution or graphing, with support, and explain how these systems represent simple real-world constraints and options.
- **Level 4 -** I can analyze real-world problems by representing constraints as systems of equations and inequalities, and then solve them using substitution, elimination, or graphing, explaining how the solutions show viable and nonviable options.
- **Level 5 -** I can independently model complex real-world scenarios by translating constraints into systems of equations and inequalities, use a variety of strategies (substitution, elimination, and graphing) to solve them, and interpret the results—whether points or regions—to predict outcomes and optimize decision-making processes.

Lesson Sequence	**Learning Target**	**Success Criteria/ Assessment**	**Resources**
6.1 System of equations in two variable.	I can solve a system of linear equations with 2 variables.	• I can determine whether an ordered pair is a solution of a linear system • I can solve linear systems by substitution • I can solve linear systems by elimination • I can identify systems that do not have exactly one ordered pair solutions • Solve applied problems using a system of linear equations.	
6.2 Systems of linear equations in 3 variables	I can solve a system of equations with 3 variables.	• I can verify the solution of a system of linear equations in 3 variables. • I can solve a system of linear equations in 3 variables. • I can solve a system of linear equations using matrices.	
6.3 systems of nonlinear	I can solve a nonlinear system of equations	• I can recognize systems of nonlinear equations in 2 variables.	

equations in 2 variables		• I can solve nonlinear systems by substitution. • I can solve nonlinear systems by addition. • I can solve applied problems using a system of linear equations.
6.4 System of inequalities	I can graph a system of linear inequalities	• I can graph a system of linear inequalities in 2 variables. • I can graph a nonlinear inequality • I can graph a system of inequalities
6.5 Linear programming	I can use linear programming to solve problems.	• I can write an objective function describing a quantity that must be maximized or minimized. • I can use inequalities to describe limitations in a situation.
6.6 Matrix Solutions to Linear Systems	I can represent a system of linear equations as a matrix, use matrix operations to find solutions, and interpret the results in the context of the system.	• I can write the augmented matrix for a linear system. • I can perform matrix row operations. • I can use matrices and Gaussian elimination to solve systems. • I can use matrices and Gauss-Jordan elimination to solve systems.
6.7 Matrix Operations and their Applications	I can perform matrix operations and apply them to solve mathematical and real-world problems.	• I can use matrix notation. • I can explain what is meant by equal matrices. • I can add and subtract matrices. • I can perform scalar multiplication. • I can solve matrix equations. • I can multiply matrices. • I can model applied situations with matrix operations.