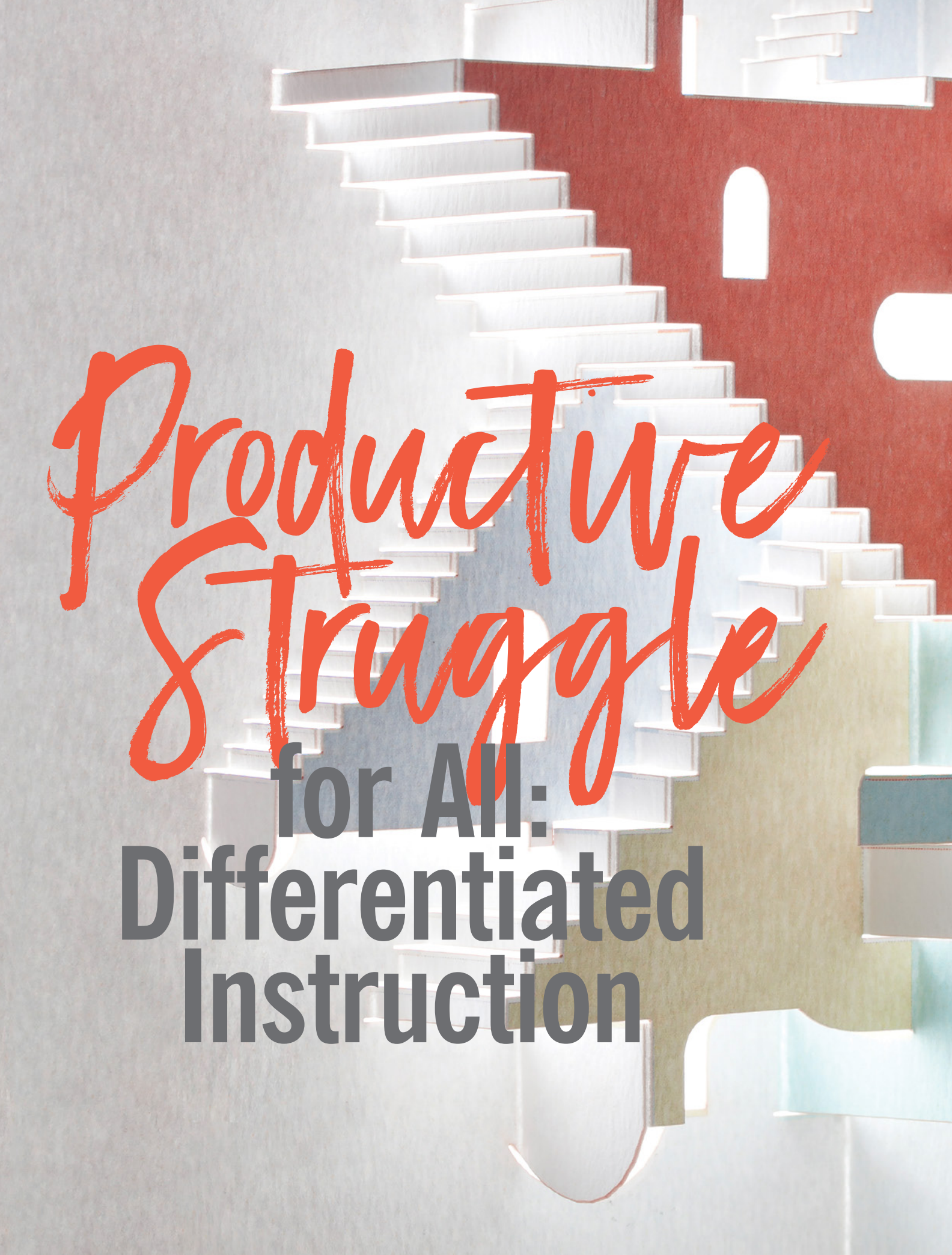

Productive Struggle for All: Differentiated Instruction

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Consider strategies that create access while maintaining the cognitive demand of a mathematics task.

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Ms. Young is determined to engender flexible thinking in all her students. She identified and launched a mathematically rich task that could promote opportunities for productive struggle. As she observed discussions, she found that some students were not engaging in the task and were instead employing surface-level procedures that were not connected to the mathematics concepts. After class, she noted that although the task allowed for several solution pathways, the content was not accessible to all students. "I have tried differentiating tasks in the past by giving students part of the solution or even some sort of procedure, but this only seemed to remove students from reasoning about the mathematics. I know there is a better way." How can Young differentiate instruction while keeping the task mathematically rich for all students?

Mathematics teachers are challenged to daily engage all students, regardless of prior instructional experiences or current mathematical understandings, in rich mathematical content and practice. Teachers are expected to possess not only the knowledge for facilitating development of all students' mathematical content knowledge but also the pedagogical skill to support the unique learning needs of all students. Even though differentiation is clearly needed to ensure that all students have access, it is not always clear how to differentiate in ways that maintain rigor. Often, attempts to increase accessibility can decrease the mathematical richness of the task.

DIFFERENTIATED INSTRUCTION

Most teachers are familiar with the concept of differentiated instruction (DI) (Tomlinson 1999; Tomlinson and Imbeau 2010)—a process through which teachers can increase access to content by considering unique characteristics of students as they plan instructional experiences. DI identifies three ways to differentiate instruction, which includes modifications to—

1. the *content* (i.e., what the student is learning);
2. the *process* (i.e., how the student accesses the information or the way in which the student engages with the content); and
3. the *product* (i.e., how students demonstrate their knowledge or mastery of the content).

In mathematics, when inconsistent patterns are found in students' prior knowledge, teachers can differentiate the content to increase access.

PRODUCTIVE STRUGGLE

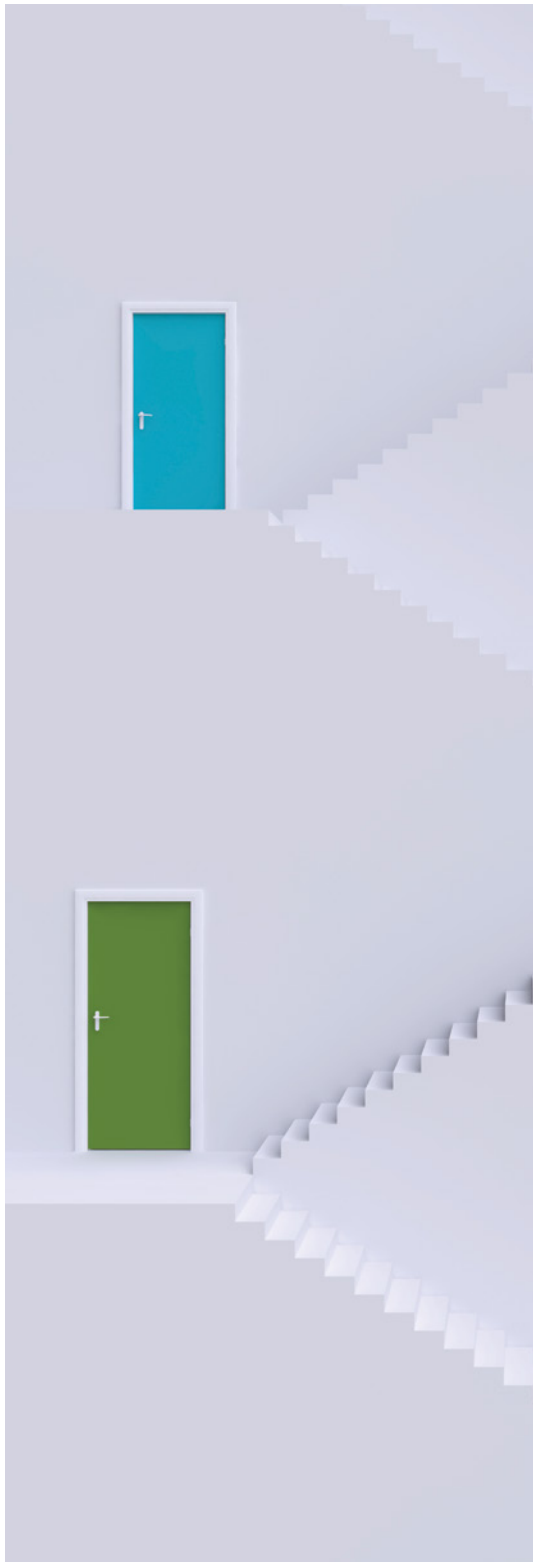
Maintaining the cognitive demand and rigor of the mathematics task is essential for creating classrooms that are more equitable. One essential component of equitable classrooms is providing all students with opportunities to engage in productive struggle. Although productive struggle is not a new concept in mathematics instruction, *Principles to Actions: Mathematical Success for All* (NCTM 2014) re-emphasized its importance in learning mathematics. For students to learn mathematics meaningfully, it is necessary for them to struggle with concepts, procedures, and ideas (Hiebert and Grouws 2007). When *all* students are given opportunities to think critically about important mathematics content and concepts, they have higher achievement than their peers who are not afforded such opportunities (Marzano 2003). Unfortunately, when teaching students who have difficulties learning mathematics, especially those receiving special education services, the struggle is often removed (Karp 2013).

Warshauer (2015) describes the following strategies that teachers can use to support productive struggle:

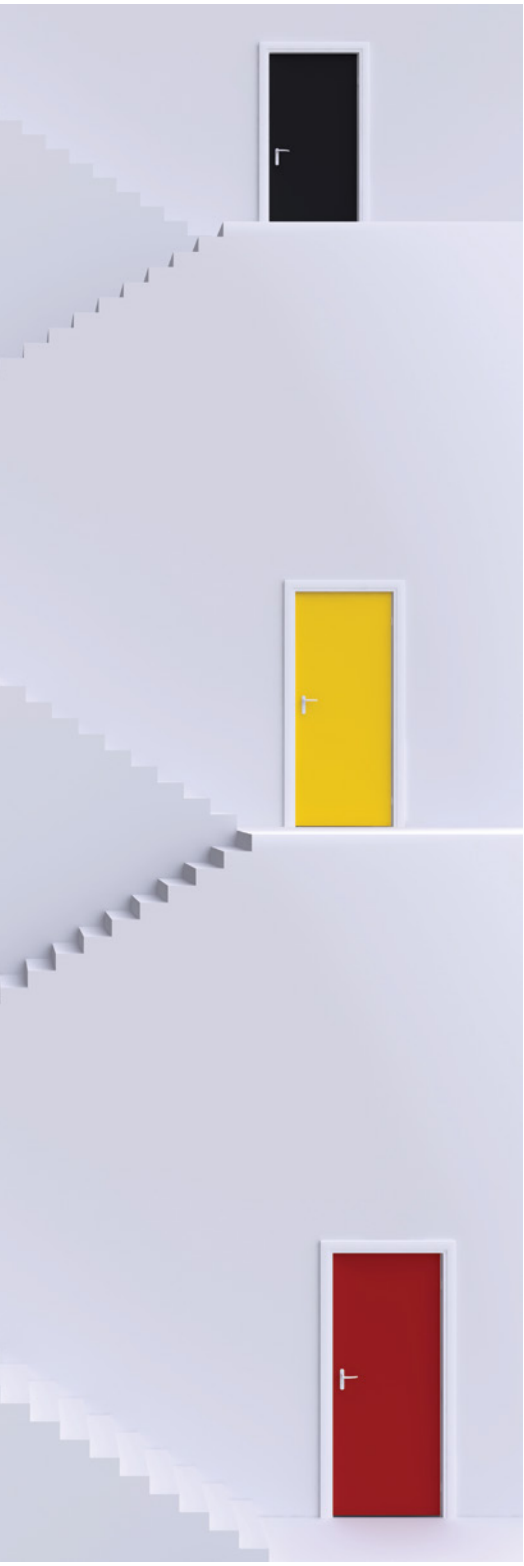
1. Ask purposeful questions to help students reflect on the source of their struggle and focus their thinking.
2. Encourage students to engage in the process of critically thinking about the problem and concepts as opposed to focusing only on obtaining a correct solution.
3. Give students time to manage their struggles; do not step in to help too soon.
4. Create a classroom norm that shows students that struggle is an important part of learning.

Although essential to supporting productive struggle in classrooms, these strategies alone may not be sufficient to support productive struggle for all learners.

In this article, we focus on how to differentiate mathematical tasks for a wide range of learners while maintaining rigor. First, we provide an example of how well-meaning differentiation can sometimes decrease students'



opportunities to engage in productive struggle. Next, we suggest and illustrate accessible practices that teachers can use to promote productive struggle through differentiation. Finally, we present scenarios involving hypothetical cases of students in inclusive classrooms who engaged in productive struggle within the differentiated task. We specifically focus on how appropriate differentiation can provide *increased* opportunities for all students, including those with disabilities.



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DIFFERENTIATED INSTRUCTION AND OPPORTUNITIES FOR PRODUCTIVE STRUGGLE

Consider Aunt Martha's Cupcake task, presented by Young:

> Aunt Martha has 5 trays of cupcakes. She describes how they were arranged: There are 100 total cupcakes on the trays. The first and second trays have 52 cupcakes, the second and third trays have 43 cupcakes, the third and fourth trays have 34 cupcakes, and the fourth and fifth trays have 30 cupcakes. How many cupcakes are on each tray? (Bair and Mooney 2013, p. 326)

Some students—particularly those with disabilities—may lack some of the requisite prior knowledge to effectively engage with this problem. In considering how to differentiate this task, it may be tempting to provide these students with the total number of cupcakes that are on one of the trays. At face value, students would still participate in the task assigned to the class and would arrive at the same answer as their classmates. However, this fundamentally changes the task demands: By providing the number of cupcakes on one tray, the problem has become a series of subtraction problems. Because the student is no longer required to reason flexibly about composing and decomposing numbers, the student has fewer opportunities to engage in productive struggle.

Several common issues can occur when differentiating a mathematical task. Watch for the following pitfalls when planning DI:

- Providing hints that remove the cognitive demand of the task
- Focusing exclusively on the procedure, without a contextual emphasis, when helping the student generate the answer or solution
- Providing students with formulas (process) to solve a task without enabling them the opportunity to engage in the mathematics content
- Considering only the characteristics of students who struggle and not those of students working at above-grade-level expectations.

Although avoiding these pitfalls is important, it does not suggest how one might approach meaningfully differentiating a task. To provide support for differentiating instruction in ways that increase opportunities for productive struggle, we have created an Accessible Practices Planning framework.

ACCESSIBLE PRACTICES

In our collective work, we have found some questions to be useful as a planning framework in promoting productive struggle in inclusive settings:

Whole-Class Considerations

- What is the underlying mathematical concept the activity is designed to build? How can the differentiated task address those same underlying concepts?
- What are the prerequisite skills needed to complete this task, and how can I incorporate those into the lesson launch?
- How much time is sufficient to enable all students to engage with the mathematical substance of this task?

Individual Student Considerations

- What are the barriers the student

might experience in engaging with the problem?
- How can the student engage with the mathematical goal in a way that builds on and extends prior knowledge?
- What feedback should I provide?
- What supporting or extending questions should I ask?
- What structures or discussions should I facilitate to help support students' understanding?

These questions are appropriate for all students and need to be addressed at a whole class and individual level. Because a variety of student characteristics are found in inclusive classrooms, individual student considerations could differ greatly for the same task. For example, when identifying the barriers to engaging with the problem, the task's context might be problematic for an ELL or a student with developmental delays, whereas a student with a physical disability may have challenges using physical manipulatives to represent the task. In contrast, students working above grade level may solve the problem quickly and need an extending question to truly engage with a cognitively demanding task that promotes productive struggle.

Practices Applied to a Mathematically Rich Task

To illustrate how differentiated instruction facilitates productive struggle with rich mathematics content for all students, we provide a differentiated example of the Aunt Martha's Cupcake task. **Table 1** shows options for differentiating the content and evaluating whether these modifications adhere to the practices that promote productive struggle or are pitfalls that would remove productive struggle in inclusive settings.

For students working significantly below grade level who may not have the requisite skills needed to make sense of the problem, consider variations of the problem that address underlying conceptual foundations. Differentiation possibilities that target the same underlying concepts include the following:

- Have the student imagine that he or she is writing this problem, deciding on the number of cupcakes on each tray, and then figuring out how to write a similar problem.
- Have the student think about 2 trays with a given number of cupcakes and determine all the arrangements that could produce that total number. (Then do the same thing for 3 trays.)

When identifying the barriers to engaging with the problem, the task's context might be problematic for an ELL or a student with developmental delays, whereas a student with a physical disability may have challenges using physical manipulatives to represent the task.

For those students in need of a task extension, consider adjusting the number of trays or cupcakes to extend students' understanding of the underlying conceptual foundations of the task; or ask the students to represent their solution algebraically.

CASE STUDY EXAMPLES

The following student descriptions from Young's class depict typical student characteristics found in inclusive classrooms. We provide suggestions for what to note when planning for each student using our differentiation considerations, which specifically build on student strengths while addressing individual and whole-class consideration questions from our planning framework.

Carly

Carly has an identified learning disability (LD), and as noted in her Individual Education Program (IEP), she works below grade level in both reading and mathematics. She is a motivated student who works hard to solve problems when working in groups. She tends to understand the context of grade-level problems when they are read to her, but her solution accuracy is far below grade-level expectations. She has difficulty recalling basic arithmetic facts and has just started using a hundred chart to help her calculate addition problems. When visual supports are provided during mathematics instruction, Carly tends to be more engaged in the task. In planning for the Cupcake problem, Young notes that Carly may have difficulty accessing the content because of the complexity of the word problem presentation and the range of the numbers.

In planning for differentiated instruction, Young's first thought when considering this task for Carly is to facilitate it using some type of a cooperative-group strategy because Carly is the

Table 1 Differentiation examples can help teachers determine whether task modifications promote or impede productive struggle.

Possible Differentiation	Example	Does It Maintain the Underlying Mathematical Demands of the Task?	Practice (to Maintain Productive Struggle) or Pitfall?
Give the student a strategy to solve the problem.	Use guess and check by subtracting.	(No; *decreased*) Subtraction problems using a teacher-given strategy	Pitfall— Providing the process
Use a smaller number of cupcakes.	Aunt Martha has **5** trays of cupcakes. She describes how they are arranged: There are **15** total cupcakes on the trays. The first and second trays have **8** cupcakes, the second and third trays have **4** cupcakes, the third and fourth trays have **3** cupcakes, and the fourth and fifth trays have **6** cupcakes. How many cupcakes are on each tray?	(Yes; *maintained*) Solving the same problem but can now use manipulatives or guess-and-check solution methods and approaches	Practice— Considering barriers to access
Use a larger number of cupcakes.	Aunt Martha has **5** trays of cupcakes. She describes how they were arranged: There are **200** total cupcakes on the trays. The first and second trays have **102** cupcakes, the second and third trays have **86** cupcakes, the third and fourth trays have **68** cupcakes, and the fourth and fifth trays have **60** cupcakes. How many cupcakes are on each tray?	(No; *increased*) Solving same problem type yet larger numeric values	Practice— Considering extensions for engagement
Ask the student to rewrite the problem.	Ask the student to rewrite the problem with a different number of trays and see if more than one answer exists.	(No; *increased*) Encourages justification	Practice—Considering extensions for engagement
Give the student the incorrect number of cupcakes on the first tray and have him or her evaluate the plausibility of that answer.	What if someone said that there were 16 cupcakes on the first tray; would you agree or disagree? Show how you figured it out. If there are not 16 cupcakes on the first tray, what would a better guess be and why?	(Yes; *maintained*) Suggests one particular solution strategy—guess and check—but it does allow the student to start evaluating the differences between various guesses. (How one guesses shapes whether or not the total number of cupcakes is 100?)	Practice— • considering feedback • supporting questions • extending questions

most motivated when working with others to solve a task. From a holistic standpoint, Young notes that Carly is able to solve problems more easily when the information in the problem is presented in a visual format, and she is generally more successful if the word problem is read to her. However, she also knows that Carly often struggles with knowing where to start. She concludes that Carly would benefit from participating in an opening discussion about the problem context as well as having a visual representation of the problem statement. To help Carly engage with this problem, it may be beneficial to have a smaller numbers of cupcakes and use a think-pair-share format to motivate her engagement with the task.

Demetrios

Demetrios is a competitive and social student whose family moved to the United States from Greece. When solving contextual problems, Demetrios will work hard to find a solution if a personal connection exists, but when a disconnect is present between the task and his life experiences, he is less likely to put forth the needed

effort to truly engage in the task and will do just enough to get by. He works above grade-level expectations when performing mathematical calculations. Demetrios is an ELL who tends to be more engaged in tasks when he works with others to solve them. Young notes that barriers that Demetrios may experience include the ability to contextualize the task and frustration if he is asked to complete it independently. Given these characteristics, Demetrios might be most engaged when the context of the task is connected to Demetrios's daily life experiences and he is asked to work with others in solving this task.

In planning for differentiated instruction, Young's first thought when considering this task for Demetrios is to use his life experiences as leverage to help him access the task content and connect it to his daily experiences. Specifically, Young plans to introduce the task in a manner that can connect to an upcoming sports banquet (Demetrios is a multisport athlete). She also plans to facilitate this task using a cooperative group strategy because Demetrios displays self-motivation when working with others to solve a task.

During the launch Young plans to ask questions that she can use to support productive struggle and extend thinking. The questions and prompts include, "What types of food have you seen arranged on trays for parties?" and "Please think about, and tell me, how the food might be arranged on the trays."

Jessica

Jessica was diagnosed with Attention Deficit Hyperactivity Disorder (ADHD). She was considered learning disabled (LD) at the age of nine and is currently in the seventh grade. Recent assessments show that she is two grade levels behind in reading and one grade level behind in mathematics. Jessica experiences difficulty with accessing word problems presented in a verbal format only because of auditory processing difficulties and problems focusing and maintaining her attention. These barriers often result in Jessica getting up and wandering around the room during mathematics class. When she is engaged in the lesson, Jessica often blurts out numerous "answers" to questions posed by the teacher and other classmates. For example, when Young launched the Cupcake problem the first time, Jessica shouted out, "One hundred! Wait, . . . 100 divided by 5. Subtract the 52!" When she began to work on the problem, Jessica jotted a few numbers on her paper before she began to look around the room. When her classmate asked about her reasoning and the numbers she had written, Jessica threw her pencil on the floor and mumbled, "This is stupid."

Differentiated instruction for Jessica might include asking supporting questions, supplying representations, allowing structured time for discussions, or posing questions that allow Jessica time to think and organize her thoughts before responding. For example, Young notes that Jessica would benefit from having verbal information presented side-by-side with a visual representation of the words or the mathematical situation. Moreover, a verbal launch of the situation may cause difficulty for Jessica in engaging in the discussion where the unpacking of the problem situation occurs. So, Young knows that, if verbal discussion is a part of the launch, Jessica might benefit from having "think time" included after each question is posed so that she can have time to process the verbal information being presented. Finally, presenting a "conceptual preview" of the problem situation with a similar problem (using visually presented, paired problem-solving situations) may help maintain and focus Jessica's attention during the larger problem.

In considering all three of these students' diverse access needs, an opening discussion will clearly be the best way to start students off on this task. In the opening discussion, Young writes the problem on the board and covers the numbers in the Cupcake problem. To motivate Demetrios and help Carly (as well as other students) access the problem context, she draws 5 large trays and asks the class to think about what the problem is asking them, while encouraging them to connect the problem to their everyday life experiences. She makes sure to provide "think time" and use think-pair-share strategies to help Jessica engage in the discussion and get the class talking about their understanding of the problem context. As she concludes the whole-class discussion, she hands out the problem. Because Carly would benefit from having a smaller number of cupcakes, Young has created a differentiated task that has a total of 15 cupcakes.

SUPPORTING MULTIPLE STUDENTS IN AN INCLUSIVE CLASSROOM

The case studies of Carly, Demetrios, and Jessica exemplify how teachers can support multiple students in an inclusive classroom in productive struggle with cognitively demanding mathematics tasks when planning includes DI. In addition to creating a classroom norm of students engaging in the mathematics (Warshauer 2015), we have proposed specific lesson-planning ideas, that are grounded in the tenets of DI, for maintaining productive struggle in an inclusive classroom: They include the following:

- Consider the underlying mathematical principles that are entailed in the problem;

- Think about how each student can gain access to the mathematical context of the task;
- Consider how each student can engage with the mathematics in a way that builds on and extends his or her prior knowledge;
- Provide feedback in the form of supporting or extending questions;
- Design structures or discussions to help support student understanding; and
- Provide ample time for each student to engage in the mathematical content of the task.

This article has demonstrated how to consider differentiating instruction for diverse learners while maintaining the cognitive demand of a mathematics task. The strategies that create an accessible mathematical task for these students can benefit not only students with disabilities or ELL but all students in the class.

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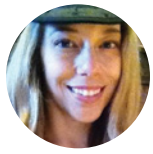

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Let's Chat about Differentiated Instruction

On Wednesday, January 17, 2018, at 9:00 p.m. ET, we will expand on "Productive Struggle for All: Differentiated Instruction" (pp. 194–201), by Sararose D. Lynch, Jessica H. Hunt, and Katherine E. Lewis. Join us at #MTMSchat.

We will also Storify the conversation for those who cannot join us live. The *MTMS* monthly chats fall on the third Wednesday of the month.