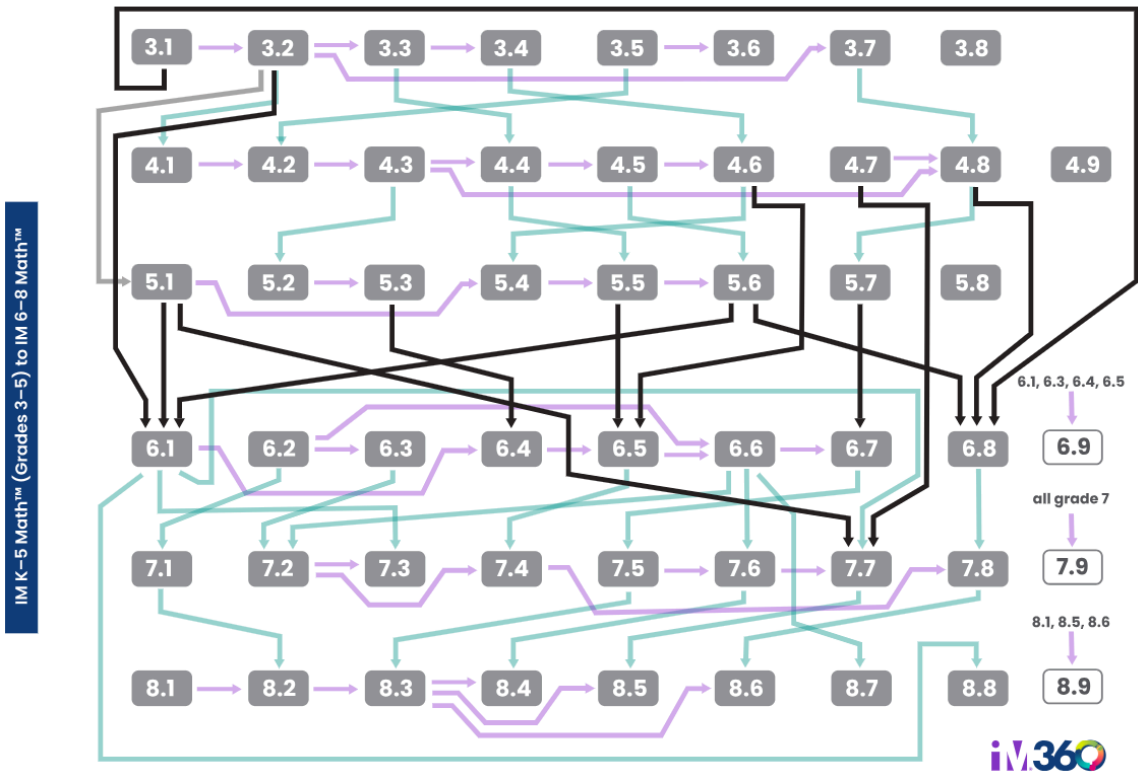


Course Title	Content Area	Grade Level	Credit (if applicable)						
Grade 7 Math	Mathematics	Grade 7	N/A						
Course Description									
<p>IM Grade 7 begins with students studying scale drawings, an engaging geometric topic that sets the stage for the subsequent work on proportional relationships in the following three units. Students also have opportunities to build fluency with IM Grade 6 arithmetic. They work with proportional relationships represented by tables, equations, and graphs. Geometry and proportional relationships are interwoven in the third unit, when the important proportional relationship between a circle's circumference and its diameter is studied. Then students work with percent increase and percent decrease.</p> <p>By the fifth unit, on operations with rational numbers, students have had time to brush up on and solidify their understanding of, and skill in, IM Grade 6 arithmetic. At this point, the emphasis becomes the role of the properties of operations in determining the rules for operating with negative numbers. This is a natural lead-in to the work on solving equations and simplifying expressions in the next unit. Students then put their arithmetical and algebraic skills to work in the last two units: on angles, triangles, and prisms, and on probability and sampling.</p>									
Aligned Core Resources									
Connection to the BPS Vision of the Graduate									
<p>CT Core Standards (aligned to National Common Core Standards)</p> <p>Imagine Learning iM Resources (Imagine 6-8) BPS teacher login through ClassLink required</p> <p>https://accessim.org/6-8/grade-7/course-guide/further-reading?a=teacher</p> <ul style="list-style-type: none"> Empowering All Storytellers: Tips for Engaging Special Populations Using IM® v.360 for Grade 6-12 Tackling Wordy Problems: How the Three Reads Math Language Routine Supports Access for All Learners Think Pair Share Math Language Routines: Discourse with a Purpose Unlocking Learners' Thinking Using the Mathematical Language Routines 	<p>Common Core State Standards: Math Practice (MP) Standards</p> <p>MP 1: Make sense of problems and persevere in solving them. MP 2: Reason abstractly and quantitatively. MP 3: Construct viable arguments and critique the reasoning of others. MP 4: Model with mathematics. MP 5: Use appropriate tools strategically. MP 6: Attend to precision. MP 7: Look for and make use of structure. MP 8: Look for and express regularity in repeated reasoning.</p>								
	<i>Lessons that Showcase Math Practice Standards</i>								
		Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6	Unit 7	Unit 8
	MP 1	1, 8, 13	7, 9, 15	4, 5, 10, 11	2, 3, 6, 7, 10, 12, 14, 15	1, 4, 10, 12	1, 4, 5, 9, 11, 12, 17	2, 4, 6, 11	8, 14, 19, 20
	MP 2	7, 8	3-6, 10-12	4, 5, 9, 10	2, 4, 5, 8, 11, 16	1, 2, 4, 6-9, 12, 14-16	4, 5, 11, 14, 22	15	2, 3, 9, 15
	MP 3	1, 3, 4, 12	7, 10, 14	2, 5, 8, 10	1, 3, 5, 7, 11, 12, 14, 16	1, 5, 7, 13, 15, 16	2, 3, 6, 7, 10, 12, 13, 16, 19-21	1, 2, 4, 5, 8, 11, 12, 15	2, 5, 7, 11, 12, 14, 16, 18, 20
	MP 4	13	1, 11, 14, 15	1, 6, 11	13	17	2, 17, 22	16	6, 7
	MP 5	2, 3, 5, 7	-	2, 7	-	-	-	3	7, 10
	MP 6	1, 4, 11, 12	1, 4, 5, 8, 9, 11, 13	2, 4, 7, 9, 10	5-10, 12, 15	2, 7, 10, 11, 14	3, 6, 8, 9, 13, 15, 17, 18, 20-22	3, 4, 9, 10	4, 6, 10, 16, 18, 19
	MP 7	2-6, 9-12	2, 3, 6, 8, 10, 12, 13	3, 6, 8, 9	2-6, 8-10, 14	3, 5, 10, 11, 13, 15	2, 4-7, 10, 14, 16, 18-21	1, 4, 5, 7, 9, 13, 14	2, 8, 13, 15, 17
MP 8	2, 5, 6, 10	4-6, 12	1, 3, 5, 7, 8	1, 4, 8-10, 13	2, 3, 5, 6, 8, 9, 16	1, 15	2, 3	2-, 5, 17	
<p>Bristol Public Schools Vision of the Graduate</p> <p>Problem Solving</p> <ul style="list-style-type: none"> iM's focus on real-world modeling and problem-solving strategies Multiple solution pathways are encouraged and explored Students develop perseverance through challenging tasks <p>Critical Thinking</p> <ul style="list-style-type: none"> Students analyze mathematical relationships and justify their reasoning Regular opportunities to critique others' reasoning Emphasis on understanding "why" not just "how" <p>Communication and Collaboration</p> <ul style="list-style-type: none"> Structured mathematical discourse is built into lessons Students explain their thinking both verbally and in writing Many activities involve partner and group work 									
Link to Equity Audit	Equity Curriculum Review Audit (Gr. 8 Alg+)								
Additional Course Information:									

Knowledge/Skill Dependent courses/prerequisites



Standard Matrix

Standard	Lesson(s)				
7.EE.A	Unit 7, Lesson 3				
7.EE.A.1	Unit 6, Lesson 18	Unit 6, Lesson 19	Unit 6, Lesson 20	Unit 6, Lesson 21	Unit 9, Lesson 6
7.EE.A.2	Unit 4, Lesson 4	Unit 4, Lesson 8	Unit 4, Lesson 10	Unit 6, Lesson 12	Unit 9, Lesson 6
7.EE.B	Unit 9, Lesson 7				
7.EE.B.3	Unit 3, Lesson 11 Unit 5, Lesson 12 Unit 5, Lesson 15	Unit 5, Lesson 17 Unit 6, Lesson 2	Unit 6, Lesson 3 Unit 6, Lesson 4	Unit 6, Lesson 5 Unit 6, Lesson 6	Unit 6, Lesson 11 Unit 6, Lesson 12
7.EE.B.4	Unit 5, Lesson 15 Unit 5, Lesson 16	Unit 6, Lesson 5 Unit 6, Lesson 9 Unit 6, Lesson 11	Unit 6, Lesson 12 Unit 6, Lesson 13 Unit 6, Lesson 15	Unit 7, Lesson 5	Unit 9, Lesson 2
7.EE.B.4.a	Unit 6, Lesson 4 Unit 6, Lesson 5	Unit 6, Lesson 7 Unit 6, Lesson 8	Unit 6, Lesson 9 Unit 6, Lesson 10	Unit 6, Lesson 11 Unit 6, Lesson 12	Unit 9, Lesson 6
7.EE.B.4.b	Unit 6, Lesson 14	Unit 6, Lesson 16	Unit 6, Lesson 17		
7.G.A	Unit 3, Lesson 2	Unit 3, Lesson 7	Unit 7, Lesson 1	Unit 7, Lesson 4	Unit 7, Lesson 9
7.G.A.1	Unit 1, Lesson 1 Unit 1, Lesson 2 Unit 1, Lesson 3 Unit 1, Lesson 4	Unit 1, Lesson 5 Unit 1, Lesson 6 Unit 1, Lesson 7 Unit 1, Lesson 8	Unit 1, Lesson 9 Unit 1, Lesson 10 Unit 1, Lesson 11 Unit 1, Lesson 12	Unit 1, Lesson 13 Unit 2, Lesson 1 Unit 3, Lesson 6 Unit 3, Lesson 11	Unit 9, Lesson 3 Unit 9, Lesson 8 Unit 9, Lesson 12
7.G.A.2	Unit 3, Lesson 2 Unit 7, Lesson 6	Unit 7, Lesson 7 Unit 7, Lesson 8	Unit 7, Lesson 9	Unit 7, Lesson 10	Unit 7, Lesson 17
7.G.A.3	Unit 7, Lesson 11	Unit 7, Lesson 13			
7.G.B	Unit 3, Lesson 6 Unit 3, Lesson 7	Unit 7, Lesson 1 Unit 7, Lesson 2	Unit 7, Lesson 4 Unit 7, Lesson 15	Unit 7, Lesson 16	Unit 9, Lesson 5
7.G.B.4	Unit 3, Lesson 3 Unit 3, Lesson 4	Unit 3, Lesson 5 Unit 3, Lesson 7	Unit 3, Lesson 8 Unit 3, Lesson 9	Unit 3, Lesson 10 Unit 3, Lesson 11	Unit 9, Lesson 3 Unit 9, Lesson 10 Unit 9, Lesson 11
7.G.B.5	Unit 7, Lesson 2	Unit 7, Lesson 3	Unit 7, Lesson 4	Unit 7, Lesson 5	
7.G.B.6	Unit 1, Lesson 6	Unit 7, Lesson 13	Unit 7, Lesson 16	Unit 9, Lesson 3	Unit 9, Lesson 7

	Unit 3, Lesson 6 Unit 7, Lesson 12	Unit 7, Lesson 14 Unit 7, Lesson 15	Unit 7, Lesson 17	Unit 9, Lesson 4	Unit 9, Lesson 8
7.NS.A	Unit 5, Lesson 13	Unit 5, Lesson 15			
7.NS.A.1	Unit 5, Lesson 1	Unit 5, Lesson 4	Unit 5, Lesson 7	Unit 6, Lesson 18	
7.NS.A.1.a	Unit 5, Lesson 2	Unit 5, Lesson 3	Unit 5, Lesson 4		
7.NS.A.1.b	Unit 5, Lesson 2	Unit 5, Lesson 3			
7.NS.A.1.c	Unit 5, Lesson 5	Unit 5, Lesson 6	Unit 5, Lesson 7	Unit 6, Lesson 18	
7.NS.A.1.d	Unit 5, Lesson 3	Unit 5, Lesson 6	Unit 5, Lesson 13		
7.NS.A.2.a	Unit 5, Lesson 8	Unit 5, Lesson 9			
7.NS.A.2.b	Unit 5, Lesson 11				
7.NS.A.2.c	Unit 5, Lesson 9	Unit 5, Lesson 10	Unit 5, Lesson 13		
7.NS.A.2.d	Unit 4, Lesson 5	Unit 8, Lesson 16	Unit 9, Lesson 3		
7.NS.A.3	Unit 5, Lesson 7	Unit 5, Lesson 10 Unit 5, Lesson 12	Unit 5, Lesson 14 Unit 5, Lesson 15	Unit 5, Lesson 16 Unit 5, Lesson 17	Unit 9, Lesson 2 Unit 9, Lesson 5
7.RP.A	Unit 2, Lesson 5 Unit 2, Lesson 9 Unit 2, Lesson 11 Unit 2, Lesson 14	Unit 5, Lesson 8 Unit 5, Lesson 12 Unit 7, Lesson 16 Unit 8, Lesson 4	Unit 8, Lesson 7 Unit 8, Lesson 16 Unit 8, Lesson 20	Unit 9, Lesson 5 Unit 9, Lesson 9	Unit 9, Lesson 10 Unit 9, Lesson 11 Unit 9, Lesson 12
7.RP.A.1	Unit 2, Lesson 8	Unit 4, Lesson 2	Unit 4, Lesson 3	Unit 9, Lesson 4	
7.RP.A.2	Unit 2, Lesson 2 Unit 2, Lesson 4 Unit 2, Lesson 5 Unit 2, Lesson 6	Unit 2, Lesson 7 Unit 2, Lesson 8 Unit 2, Lesson 9 Unit 2, Lesson 10	Unit 2, Lesson 12 Unit 2, Lesson 13 Unit 2, Lesson 14 Unit 2, Lesson 15	Unit 3, Lesson 3	Unit 4, Lesson 3 Unit 4, Lesson 4 Unit 4, Lesson 5 Unit 9, Lesson 2 Unit 9, Lesson 4
7.RP.A.2.a	Unit 2, Lesson 3 Unit 2, Lesson 7	Unit 2, Lesson 8 Unit 2, Lesson 10	Unit 3, Lesson 1 Unit 3, Lesson 3	Unit 3, Lesson 5 Unit 3, Lesson 7	
7.RP.A.2.b	Unit 2, Lesson 2	Unit 2, Lesson 3	Unit 2, Lesson 5	Unit 2, Lesson 11	Unit 2, Lesson 13
7.RP.A.2.c	Unit 2, Lesson 4 Unit 2, Lesson 5	Unit 2, Lesson 6 Unit 2, Lesson 7	Unit 2, Lesson 8	Unit 2, Lesson 13	Unit 3, Lesson 5
7.RP.A.2.d	Unit 2, Lesson 11	Unit 2, Lesson 13			
7.RP.A.3	Unit 3, Lesson 5 Unit 4, Lesson 6 Unit 4, Lesson 7 Unit 4, Lesson 8	Unit 4, Lesson 9 Unit 4, Lesson 10 Unit 4, Lesson 11	Unit 4, Lesson 12 Unit 4, Lesson 13 Unit 4, Lesson 14 Unit 4, Lesson 15	Unit 4, Lesson 16 Unit 5, Lesson 17 Unit 9, Lesson 1 Unit 9, Lesson 2	Unit 9, Lesson 3 Unit 9, Lesson 5 Unit 9, Lesson 7 Unit 9, Lesson 12
7.SPA	Unit 8, Lesson 13	Unit 8, Lesson 16	Unit 8, Lesson 17	Unit 8, Lesson 20	
7.SPA.1	Unit 8, Lesson 12	Unit 8, Lesson 13	Unit 8, Lesson 14	Unit 8, Lesson 15	Unit 8, Lesson 20
7.SPA.2	Unit 8, Lesson 13 Unit 8, Lesson 14	Unit 8, Lesson 15	Unit 8, Lesson 16	Unit 8, Lesson 17	Unit 8, Lesson 20
7.SP.B	Unit 8, Lesson 11	Unit 8, Lesson 12			
7.SP.B.3	Unit 8, Lesson 11	Unit 8, Lesson 18			
7.SP.B.4	Unit 8, Lesson 15 Unit 8, Lesson 16	Unit 8, Lesson 18	Unit 8, Lesson 19	Unit 8, Lesson 20	Unit 9, Lesson 2
7.SPC	Unit 8, Lesson 6				
7.SPC.5	Unit 8, Lesson 2	Unit 8, Lesson 3	Unit 8, Lesson 4	Unit 8, Lesson 5	Unit 8, Lesson 6
7.SPC.6	Unit 8, Lesson 1	Unit 8, Lesson 3	Unit 8, Lesson 4	Unit 8, Lesson 5	Unit 8, Lesson 6
7.SPC.7	Unit 8, Lesson 3	Unit 8, Lesson 4	Unit 8, Lesson 5	Unit 8, Lesson 14	
7.SPC.7.a	Unit 8, Lesson 3	Unit 8, Lesson 20			
7.SPC.7.b	Unit 8, Lesson 4	Unit 8, Lesson 5	Unit 8, Lesson 6		
7.SPC.8.a	Unit 8, Lesson 9				
7.SPC.8.b	Unit 8, Lesson 8	Unit 8, Lesson 9			
7.SPC.8.c	Unit 8, Lesson 6	Unit 8, Lesson 7	Unit 8, Lesson 10		

Unit Links

[Grade 7 Math](#)

Use of Instructional Time (181 School Days)
→ 162 iM Content and Assessment Days

Unit 1: Scale Drawings Unit 2: Introducing Proportional Relationships Unit 3: Measuring Circles Unit 4: Proportional Relationships and Percentages Unit 5: Rational Number Arithmetic Unit 6: Expressions, Equations, and Inequalities Unit 7: Angles, Triangles, and Prisms Unit 8: Probability and Sampling Course Assessment Map	<ul style="list-style-type: none">→ 6 Climate and Culture Days: 2 days at start of year, 2 shortened days before breaks, and 2 days at end of year→ 9 IAB Days: 1 day Strategic Review and 2 day IAB in fall, winter, and spring→ 4 SBA Days: 1 day Strategic Review and 3 day SBA
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Unit Title:**Unit 1: Scale Drawings****Relevant Standards: Bold indicates priority**

Lesson	Standards	Lesson	Standards
Lesson 1	7.G.A.1	Lesson 8	7.G.A.1
Lesson 2	7.G.A.1	Lesson 9	7.G.A.1
Lesson 3	7.G.A.1	Lesson 10	7.G.A.1
Lesson 4	7.G.A.1	Lesson 11	7.G.A.1
Lesson 5	7.G.A.1	Lesson 12	7.G.A.1
Lesson 6	7.G.A.1, 7.G.B.6	Lesson 13	7.G.A.1
Lesson 7	7.G.A.1		

Essential Question(s):	Enduring Understanding(s):
<ul style="list-style-type: none"> How can I identify if a copy is "to scale" compared to an original? What is the relationship between the scale factor and the resulting size of a figure? How do scale drawings represent real-world objects and locations? 	<ul style="list-style-type: none"> A scaled copy is a two-dimensional image created by multiplying all original lengths by a consistent scale factor while keeping the corresponding angle measures the same. When a scale factor is greater than 1, the copy will be larger than the original; when it is less than 1, the copy will be smaller. A scale drawing uses a specific scale to relate drawn measurements to actual measurements, allowing for the calculation of real-world distances and areas.

Demonstration of Learning:	Pacing for Unit
CFA 1: Checkpoint A (after Lesson 6) CFA 2: Checkpoint B (after Lesson 12) Unit 1 End of Unit Assessment	14 Days Lessons to Add/Remove: <ul style="list-style-type: none"> If prior knowledge needs to be reviewed, add the following lessons from Grade 6: <ul style="list-style-type: none"> 6.2.5, 6.2.8, 6.2.11, 6.2.14 Combine 6.1.5 and 6.1.6 - focus on finding area of polygons. If you need to condense the unit: <ul style="list-style-type: none"> Combine lessons 7.1.4 and 7.1.5 skipping optional activities. Remove 7.1.6 - optional activity calculating and comparing areas of multiple scaled copies of the same shape. Remove 7.1.8 - optional lesson using a scale drawing to estimate the distance an object traveled Remove 7.1.11 - activity can be moved to at home if additional time is needed Remove 7.8.13 - lesson can be removed as it is additional practice with creating scale drawings BPS Lesson Modifications: <ul style="list-style-type: none"> Lesson 6 - skip activity, supplement to do more with area on a grid then using the blocks (aligns to EOU #3) Lesson 7 (Section B) - more examples/hands on of items that are scaled (minibrands, expandable water toys that "grow, lego sets) Lesson 8 - skip? (could use for CCR in Unit 2) Lesson 11-13 - optional, could make a BPS lesson/project based on scaling a real object by hand

Family Overview	Integration of Technology:
https://accessim.org/6-8/grade-7/unit-1?a=family	<ul style="list-style-type: none"> Desmos Online Graphing Calculator Pear Assessment (Edulastic) iM v.360 Digital Applets (see below)

			every 3 students): Activity 2
6	<ul style="list-style-type: none"> • Pattern blocks: Warm-up, Activity 1 • Geometry toolkits: Activity 2 	<ul style="list-style-type: none"> • Scaled Copies Cards (1 copy for every 3 students): Activity 1 • Scaling A Puzzle Cutouts (1 copy for every 3 students): Activity 2 	
7	<ul style="list-style-type: none"> • Math Community Chart: Warm-up • Geometry toolkits: Activity 1, Activity 2 		
8	<ul style="list-style-type: none"> • Geometry toolkits: Activity 2 		
9	<ul style="list-style-type: none"> • Geometry toolkits: Activity 2 		
10	<ul style="list-style-type: none"> • Math Community Chart: Lesson • Math Community Chart: Warm-up • Geometry toolkits: Activity 2 	Same Plot, Different Drawings Cards (1 copy for every 24 students): Activity 1	
11	<ul style="list-style-type: none"> • Geometry toolkits: Activity 1, Activity 2 		
12	<ul style="list-style-type: none"> • Metric and customary unit conversion charts: Activity 2 • Geometry toolkits: Activity 3 	<ul style="list-style-type: none"> • Scales Cards (1 copy for every 4 students): Activity 1 • Units of Length Reference Sheet (1 copy for every 2 students): Activity 1 and 2 	
13	<ul style="list-style-type: none"> • Blank paper: Activity 1 • Measuring tools: Activity 1 • Graph paper: Activity 2 		

Opportunities for Interdisciplinary Connections:	Anticipated misconceptions:
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<ul style="list-style-type: none"> • Geography: Students use scale drawings and maps to calculate actual real-world distances • Art and Design: Students explore scaling in the context of printing portraits and analyzing the proportions of various artistic figures • Biology: Scaling is explored through the lens of "Movie Monsters," examining how size changes affect physical properties 	<ul style="list-style-type: none"> • Grid Dependency: Students may believe that vertices in a scaled copy must land exactly on the intersections of grid line • Additive Scaling: A common error is thinking that adding a fixed amount to all side lengths, rather than multiplying by a scale factor, creates a scaled copy • Area vs. Length: Students often mistakenly assume that if the side lengths of a figure double, the area also doubles, rather than quadruples <p>See teacher's guide for specific misconceptions aligned to each lesson.</p>
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Connections to Prior Units:	Connections to Future Units:
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<p>Essential prior concepts to engage with this unit:</p> <ul style="list-style-type: none"> • Find the area of polygons. • Generate equivalent ratios and justify why they are equivalent <p>Relevant Unit(s)/Lesson(s) to Review:</p> <ul style="list-style-type: none"> • Grade 6 Unit 2: Introducing Ratios 	<p>This unit lays the foundation for work on proportional relationships later in Grade 7. It also prepares students for Grade 8 topics including dilations, similarity, translations, rotations, and reflections</p>
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Differentiation through <i>Universal Design for Learning</i>

Engagement:

- Display sentence frames during peer collaboration (Lesson 1, Activity 1 Synthesis)
LT1: Determine whether a figure is a scaled copy of another figure, by examining corresponding side lengths and angle measures.
- Leverage choice around perceived challenge in drawing tasks (Lesson 3, Activity 1 Launch)
LT2: Draw a scaled copy of a figure using a given scale factor.

Representation:

- Use multiple examples and non-examples of scaled copies (Lesson 1, Activity 2 Launch)
LT1: Determine whether a figure is a scaled copy of another figure
- Maintain vocabulary displays for corresponding points, segments, and angles (Lesson 2, Activity 1 Launch)
LT1: Determine whether a figure is a scaled copy of another figure
- Use color coding and annotations to highlight connections between representations (Lesson 4, Activity 1 Synthesis)
LT1: Determine whether a figure is a scaled copy of another figure

Action & Expression:

- Support working memory with access to sticky notes or mini-whiteboards (Lesson 2, Warm-up Launch)
LT1: Determine whether a figure is a scaled copy of another figure
- Chunk task of drawing classroom floor plan into manageable parts (Lesson 13, Activity 2 Launch)
LT5: Generate an appropriate scale to represent an actual distance on a limited drawing size

Supporting Multilingual Learners**Math Language Routines**

The Illustrative Mathematics curriculum incorporates eight Mathematical Language Routines (MLRs) that support English Language Learners:

- MLR1: *Stronger and Clearer Each Time* - Students revise and refine their mathematical language through multiple drafts
- MLR2: *Collect and Display* - Students capture and organize language in visual displays
- MLR3: *Clarify, Critique, Correct* - Students analyze mathematical writing/talk
- MLR4: *Information Gap* - Students share information to solve problems
- MLR5: *Co-Craft Questions* - Students create and improve questions
- MLR6: *Three Reads* - Students analyze complex mathematical text
- MLR7: *Compare and Connect* - Students connect different mathematical representations
- MLR8: *Discussion Supports* - Students participate in mathematical discussions

In this unit:

- MLR1: Stronger and Clearer Each Time (Lessons 4, 12)
- MLR2: Collect and Display (Lessons 2, 7, 10)
- MLR3: Critique, Correct, Clarify (Lessons 9, 12)
- MLR7: Compare and Connect (Lessons 6, 8, 9, 11, 13)
- MLR8: Discussion Supports (Lessons 1, 2, 3, 4, 5, 7, 11, 12, 13)

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes, such as comparing, explaining, and describing. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Compare

- Compare, contrast, and critique (orally) scale drawings of the classroom. (Lesson 7 & 8)
- I can use corresponding distances and corresponding angles to tell whether one figure is a scaled copy of another. (Lesson 4)
- When I see a figure and its scaled copy, I can explain what is true about corresponding angles. (Lesson 4)
- When I see a figure and its scaled copy, I can explain what is true about corresponding distances. (Lesson 4)
- I can tell whether two scales are equivalent (Lesson 12).

Explain

- How to use scale drawings to find actual distances (Lessons 7 and 11).
- How to use scale drawings to find actual distances, speed, and elapsed time (Lesson 8).
- The meaning of scales expressed without units (Lesson 11)
- How to use scale drawings to find actual areas (Lesson 12).

Describe

- I can describe some characteristics of a scaled copy (Lesson 1).
- I can describe what the scale factor has to do with a figure and its scaled copy (Lesson 2).
- I can describe the effect on a scaled copy when I use a scale factor that is greater than 1, less than 1, or equal to 1 (Lesson 5).
- I can describe how the area of a scaled copy is related to the area of the original figure and the scale factor that was used (Lesson 6).

Sentence Frames and Stems

Section A

- I know _____ is/is not a scaled copy of _____ because ...
- Corresponding side lengths in scaled copies _____ . Angle measures in scaled copies _____ .
- The scale factor between _____ and _____ is _____ because ...
- To create a scaled copy of the figure _____ , I multiplied each side length by the scale factor _____ to get side lengths ...
- If the scale factor is greater/less than one, the copy will be _____ because ...

Section B

- The scale shows that _____ on the drawing represents _____ on the actual object.
- Since the scale is _____ to _____ , then _____ on the drawing means _____ on the actual object.
- When a scale drawing has a scale of _____ to _____ with no units, it means ...
- The scale _____ to _____ and _____ to _____ are equivalent because ...
- If the scale of the drawing is _____, the area of the actual object would be _____ because...

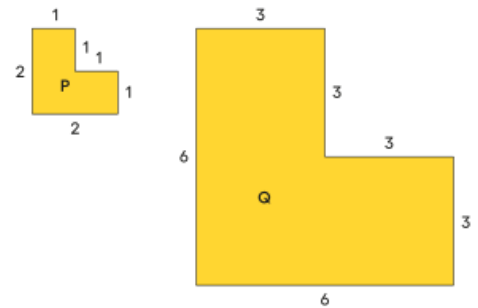
Section C

- I chose to use a scale of _____ to _____ because ...
- Since the length of _____ is _____ and I want to make a drawing with length of _____, my scale will be _____ to _____.
- To make a scale floor plan of _____, first I would ...

Unit Outline

In this unit, students study scaled copies of plane figures and scale drawings of real-world objects. Students learn that all lengths in a scaled copy are the result of multiplying the original lengths by a scale factor. Also, the angle measures in a scaled copy are the same as in the original figure.

This work builds on what students learned in previous grades about measuring lengths, areas, and angles. This unit provides a geometric context to preview the type of reasoning that students will use with proportional relationships later in grade 7. It also lays the foundation for grade 8 work on dilations and similarity.



Students begin the unit by looking at copies of a picture and describing what differentiates scaled and non-scaled copies. They calculate scale factors and draw scaled copies of figures. Note that the study of scaled copies is limited to pairs of figures that have the same orientation — in other words, they are not rotations or reflections of each other. In grade 8, students will extend their knowledge of scaled copies when they study translations, rotations, reflections, and dilations.

Next, students study scale drawings. They see that the principles and strategies that they used to reason about scaled copies of figures can also be used with scale drawings. They use scale drawings to calculate actual lengths and areas, and they create scale drawings.

A note about the geometry toolkit:

In the unit, several lesson plans suggest that each student have access to a geometry toolkit. Each toolkit contains tracing paper, graph paper, colored pencils, scissors, a centimeter ruler, a protractor (clear protractors with no holes that show radial lines are recommended), and an index card to use as a straightedge or to mark right angles. Providing students with these toolkits gives opportunities for students to develop abilities to select appropriate tools and use them strategically to solve problems (MP5).

Note that even students in a digitally enhanced classroom should have access to such tools. Applets and simulations should be considered additions to their toolkits, not replacements for physical tools.

Lesson Sequence	Learning Target(s)	Success Criteria/Assessment
Section A: Scaled Copies (Lessons 1-6)	Learning Target #1 Determine whether a figure is a scaled copy of another figure, by examining corresponding side lengths and angle measures. Learning Target #2 Draw a scaled copy of a figure using a given scale factor.	Lesson 1 What are Scaled Copies? <ul style="list-style-type: none"> • I can describe some characteristics of a scaled copy. • I can tell whether or not a figure is a scaled copy of another figure. Lesson 2 Corresponding Parts and Scale Factors <ul style="list-style-type: none"> • I can describe what the scale factor has to do with a figure and its scaled copy. • In a pair of figures, I can identify corresponding points, corresponding segments, and corresponding angles. Lesson 3 Making Scaled Copies <ul style="list-style-type: none"> • I can draw a scaled copy of a figure using a given scale factor. • I know what operation to use on the side lengths of a figure to produce a scaled copy. Lesson 4 Scaled Relationships <ul style="list-style-type: none"> • I can use corresponding distances and corresponding angles to tell whether one figure is a scaled copy of another.

		<ul style="list-style-type: none"> When I see a figure and its scaled copy, I can explain what is true about corresponding angles. When I see a figure and its scaled copy, I can explain what is true about corresponding distances. <p>Lesson 5 The Size of a Scale Factor</p> <ul style="list-style-type: none"> I can describe the effect on a scaled copy when I use a scale factor that is greater than 1, less than 1, or equal to 1. I can explain how the scale factor that takes Figure A to its copy Figure B is related to the scale factor that takes Figure B to Figure A <p>Lesson 6 Scaling and Area</p> <ul style="list-style-type: none"> I can describe how the area of a scaled copy is related to the area of the original figure and the scale factor that was used.
Checkpoint A	<p>Responding to Student Thinking</p> <ul style="list-style-type: none"> Problem 1: More Chances: Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons. Problem 2: Points to Emphasize: If most students struggle with creating a scaled copy of a figure, revisit the characteristics of scaled copies when opportunities arise over the next several lessons. For example, invite multiple students to share their thinking about how to create a scale drawing in these activities <ul style="list-style-type: none"> Grade 7, Unit 1, Lesson 9, Activity 3 Two Maps of Utah Grade 7, Unit 1, Lesson 10, Activity 3 A New Drawing of the Playground 	
<p>Section B Scaled Drawings (Lessons 7-12)</p>	<p>Learning Target #3 Create a scale drawing given the actual measurements of the object or given another scale drawing at a different scale.</p> <p>Learning Target #4 Explain how to use scales and scale drawings to calculate actual distances and areas.</p>	<p>Lesson 7 Scale Drawings</p> <ul style="list-style-type: none"> I can explain what a scale drawing is, and I can explain what its scale means. I can use actual distances and a scale to find scaled distances. I can use a scale drawing and its scale to find actual distances. <p>Lesson 8 Scale Drawings & Maps</p> <ul style="list-style-type: none"> I can use a map and its scale to solve problems about traveling. <p>Lesson 9 Creating Scale Drawings</p> <ul style="list-style-type: none"> I can determine the scale of a scale drawing when I know lengths on the drawing and corresponding actual lengths. I know how different scales affect the lengths in the scale drawing. When I know the actual measurements, I can create a scale drawing at a given scale. <p>Lesson 10 Changing Scales in Scale Drawings</p> <ul style="list-style-type: none"> Given a scale drawing, I can create another scale drawing that shows the same thing at a different scale. I can use a scale drawing to find actual areas. <p>Lesson 11 Scales without Units</p> <ul style="list-style-type: none"> I can explain the meaning of scales expressed without units. I can use scales without units to find scaled distances or actual distances. <p>Lesson 12 Units in Scale Drawings</p> <ul style="list-style-type: none"> I can tell whether two scales are equivalent. I can write scales with units as scales without units
Checkpoint B	<p>Responding to Student Thinking</p> <ul style="list-style-type: none"> Press Pause By this point in the unit, there should be some student mastery of calculating actual lengths and areas from a scale drawing. If students struggle with this, make time to examine related work in the section referred to here. The Course Guide provides additional ideas for revisiting earlier work. (Grade 7, Unit 1, Section B Scale Drawings) 	
<p>Section C Draw it to Scale (Lessons 13 - optional)</p>	<p>Learning Target #5 Generate an appropriate scale to represent an actual distance on a limited drawing size, and explain (orally) the reasoning.</p>	<p>Lesson 13 Drawing it to Scale</p> <ul style="list-style-type: none"> I can create a scale drawing of my classroom. When given requirements on drawing size, I can choose an appropriate scale to represent an actual object.
End of Unit Assessment		

NOTES:

#7: scale without units, change to have units (if bedroom door is 6 ft tall, and scale is 1 ft = 0.5 cm, how tall is the door?), could look at released SBA questions

Unit Title:

Unit 2: Introducing Proportional Relationships

Relevant Standards: Bold indicates priority

Lesson	Standards	Lesson	Standards
Lesson 1	7.G.A.1	Lesson 9	7.RP.A, 7.RP.A.2
Lesson 2	7.RP.A.2, 7.RP.A.2.b	Lesson 10	7.RP.A.2, 7.RP.A.2.a
Lesson 3	7.RP.A.2.a, 7.RP.A.2.b	Lesson 11	7.RP.A, 7.RP.A.2.b, 7.RP.A.2.d
Lesson 4	7.RP.A.2, 7.RP.A.2.c	Lesson 12	7.RP.A.2
Lesson 5	7.RP.A.2, 7.RP.A.2.b, 7.RP.A.2.c	Lesson 13	7.RP.A.2, 7.RP.A.2.b, 7.RP.A.2.c, 7.RP.A.2.d
Lesson 6	7.RP.A.2, 7.RP.A.2.c	Lesson 14	7.RP.A, 7.RP.A.2
Lesson 7	7.RP.A.2, 7.RP.A.2.a, 7.RP.A.2.c	Lesson 15	7.RP.A.2
Lesson 8	7.RP.A.1, 7.RP.A.2, 7.RP.A.2.a, 7.RP.A.2.c		

Essential Question(s):

- What defines a proportional relationship between two quantities?
- How can a proportional relationship be identified across different representations?
- Why are there two different constants of proportionality for any proportional relationship?

Enduring Understanding(s):

- A proportional relationship exists when two quantities always share the same ratio, meaning one can be found by multiplying the other by a constant multiplier called the constant of proportionality.
- Proportional relationships appear as a set of equivalent ratios in a table, an equation in the form $y=kx$, and a straight line passing through the origin (0, 0) on a graph.
- Because a proportional relationship can be viewed from two perspectives (e.g., y proportional to x or x proportional to y), there are two constants of proportionality that are always reciprocals of each other.

Demonstration of Learning:

CFA 1: Checkpoint A (after Lesson 3)
 CFA 2: Checkpoint B (after Lesson 6)
 CFA 3: Checkpoint C (after Lesson 9)
 CFA 4: Checkpoint D (after Lesson 13)
 Unit 2 End of Unit Assessment

Pacing for Unit

21 Days
 Lessons to Add if prior knowledge needs to be reviewed:

- 6.3.5 - Prioritize activities 2 and 3
- 6.3.6
- 6.3.7 - Prioritize activities 2 and 3
- 6.6.15
- Combine 6.6.16 and 6.6.17

Lessons to condense if needed: :

- Remove 7.2.6 - additional practice that could be done outside of class
- Combine 7.2.7 and 7.2.8 - both lessons focus on comparing relationships (7.2.7 - activities 2 and 3, 7.2.8 - activities 2 and 3)
- Remove 7.2.15 - optional activity

BPS Modifications

- Deemphasize Lesson 5
- Can combine Lessons 14/15 (Lesson 14 is optional)

Family Overview

<https://accessim.org/6-8/grade-7/unit-2?a=family>

Integration of Technology:

- Desmos Online Graphing Calculator
- Pear Assessment (Edulastic)
- iM v.360 Digital Applets (see below)

Unit-specific Vocabulary:

Lesson	New Terminology	
	receptive	productive
7.2.1	equivalent ratios	
7.2.2	constant of	equivalent ratios

Aligned Unit Materials, Resources, and Technology

- Digital Applets
- 2.1 Crescent Moons
 - 2.10 Digital Applet: T-shirts for Sale
 - 2.11 Digital Applet: Seagulls Eat What?
 - 2.12 Race to the Bumper Cars, Space Rocks
 - 2.13 Tables, Graphs, and Equations; Balloon Animal

	proportionality proportional relationship value	row column
7.2.3	___ is proportional to ___ relate constant	reciprocal per
7.2.4	equation quotient	___ is proportional to ___
7.2.5	steady situation	
7.2.6		equation quotient
7.2.7		constant of proportionality proportional relationship
7.2.8		constant
7.2.10	origin coordinate plane plot	
7.2.11	quantity axes coordinates	
7.2.13	x-coordinate y-coordinate	origin
7.2.14		axes
7.2.15	reasonable	

Contest

Provide access as needed throughout the unit:

- Chart paper
- Colored pencils
- Drink mix
- A powder that is mixed with water to create a fruit-flavored or chocolate-flavored drink. Using a sugar-free drink mix is recommended, but not a mix that calls for adding a separate sweetener when mixing up the drink.
- Four-function calculators
- Geometry toolkits (ongoing)
- Internet-enabled device
- Math Community Chart
- Measuring cup
- Measuring spoons
- Mixing containers
- Pre-printed slips, cut from copies of the blackline master
- Rulers
- Small disposable cups
- Snap cubes
- Tools for creating a visual display
Any way for students to create work that can be easily displayed to the class. Examples: Chart paper and markers, whiteboard space and markers, shared online drawing tool, access to a document camera.
- Water

Lesson	Materials to Gather	Materials to Copy
1	<ul style="list-style-type: none"> • Math Community Chart: Activity 1 • Colored pencils: Activity 2 • Drink mix: Activity 2 • Measuring cup: Activity 2 • Measuring spoons: Activity 2 • Mixing containers: Activity 2 • Small disposable cups: Activity 2 • Water: Activity 2 • Geometry toolkits: Activity 3 	
2	<ul style="list-style-type: none"> • Measuring cup: Activity 3 • Measuring spoons: Activity 3 	
3	<ul style="list-style-type: none"> • Chart paper: Activity 1 • Math Community Chart: Activity 1 	
6	Math Community Chart: Activity 1	
7	<ul style="list-style-type: none"> • Four-function calculators: Lesson • Math Community Chart: Activity 1 	
8	Snap cubes: Activity 3	
9		Biking and Rain Cards (1 copy for every 4 students): Activity 2

	10	<ul style="list-style-type: none"> Rulers: Activity 2 Math Community Chart: Activity 3 Pre-printed slips, cut from copies of the blackline master: Activity 3 	Matching Tables and Graphs Cards (1 copy for every 2 students): Activity 3
	11	Rulers: Activity 3	
	12	<ul style="list-style-type: none"> Colored pencils: Activity 2 Rulers: Activity 2 	
	13	Rulers: Activity 2	Tables, Graphs, and Equations Handout (1 copy for every 3 students): Activity 2
	14	Tools for creating a visual display: Activity 2	Creating and Representing Situations Handout (1 copy for every 1 student): Activity 2
	15	<ul style="list-style-type: none"> Internet-enabled device: Activity 2 Tools for creating a visual display: Activity 3 	

Opportunities for Interdisciplinary Connections:	Anticipated misconceptions:
<ul style="list-style-type: none"> Culinary Arts: Students apply proportional reasoning to scale food and drink recipes Environmental Science: The unit explores water usage efficiency in households and the economic value of recycling aluminum cans 	<ul style="list-style-type: none"> Non-origin Graphs: Students may assume any straight-line graph is proportional, neglecting the requirement that it must pass through the origin (0, 0) Pace vs. Speed: Students often confuse "moving faster" with having a steeper line on a graph, which may actually represent a slower pace if the axes are reversed <p>See teacher's guide for specific misconceptions aligned to each lesson.</p>

Connections to Prior Units:	Connections to Future Units:
<p>Essential prior concepts to engage with this unit:</p> <ul style="list-style-type: none"> equivalent ratios derived units: miles per hour; meters per second; dollars per pound; or cents per minute <p>Relevant Unit(s)/Lesson(s) to Review:</p> <ul style="list-style-type: none"> Grade 6 Unit 3 Unit Rates and Percentages Grade 6 Unit 6 Expressions and Equations 	Builds toward the study of linear functions in Grade 8. Reasoning will be applied to multi-step problems and complex rates in later units.

Differentiation through *Universal Design for Learning*

<p>Engagement:</p> <ul style="list-style-type: none"> Recruit interest by inviting students to generate personal examples of proportional relationships (Lesson 14, Activity 1 Launch) LT7: Determine whether a given graph represents a proportional relationship Provide tools like calculators to facilitate information processing (Lesson 3, Activity 2 Launch) LT1: Determine the constant of proportionality for a proportional relationship represented in a table <p>Representation:</p> <ul style="list-style-type: none"> Use physical cubes to connect symbols to concrete objects like "side length" (Lesson 8, Activity 2 Launch) LT5: Determine whether the values in a table could represent a proportional relationship <p>Action & Expression:</p> <ul style="list-style-type: none"> Support executive function with checklists for using digital tools (Lesson 15, Activity 2 Synthesis) LT7: Determine whether a given graph represents a proportional relationship

Supporting Multilingual Learners
<p>Math Language Routines The Illustrative Mathematics curriculum incorporates eight Mathematical Language Routines (MLRs) that support English Language Learners:</p> <p>MLR1: <i>Stronger and Clearer Each Time</i> - Students revise and refine their mathematical language through multiple drafts</p>

MLR2: *Collect and Display* - Students capture and organize language in visual displays
MLR3: *Clarify, Critique, Correct* - Students analyze mathematical writing/talk
MLR4: *Information Gap* - Students share information to solve problems
MLR5: *Co-Craft Questions* - Students create and improve questions
MLR6: *Three Reads* - Students analyze complex mathematical text
MLR7: *Compare and Connect* - Students connect different mathematical representations
MLR8: *Discussion Supports* - Students participate in mathematical discussions

In this unit:

- MLR1: Stronger and Clearer Each Time (Lessons 5, 8, 11, 15)
- MLR2: Collect and Display (Lessons 1, 3, 8, 10, 14)
- MLR3: Critique, Correct, Clarify (Lessons 4, 8, 9, 13)
- MLR4: Information Gap (Lesson 9)
- MLR5: Co-Craft Questions (Lessons 3, 5, 7, 10)
- MLR6: Three Reads (Lessons 2, 6, 9, 12, 15)
- MLR7: Compare and Connect (Lessons 4, 6, 8, 12, 13, 15)
- MLR8: Discussion Supports (Lessons 1, 2, 3, 4, 5, 7, 10, 11, 12, 13, 14, 15)

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes, such as comparing, interpreting, and generalizing. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Compare

- Drink mixtures and figures (Lesson 1).
- Approaches to solving problems involving proportional relationships (Lesson 6).
- Proportional relationships with nonproportional relationships (Lesson 8).
- Tables, descriptions, and graphs representing the same situations (Lesson 10).
- Graphs of proportional relationships (Lesson 12).

Interpret

- Representations showing equivalent ratios (Lesson 1).
- Tables showing equivalent ratios (Lesson 2).
- Situations involving proportional relationships (Lesson 6 and 9).
- How a graph represents features of a situation (Lesson 11).

Generalize

- About proportional relationships (Lesson 4).
- About equations that represent proportional relationships (Lesson 5).
- About how a constant of proportionality is represented by graphs and tables (Lesson 13).

In addition, students are expected to describe proportional relationships and constants of proportionality, explain how to determine whether or not a relationship is proportional and how to compare and represent situations with different constants of proportionality, justify whether or not a relationship is proportional, and represent proportional and nonproportional relationships in multiple ways.

Sentence Frames and Stems

Section A

- I found the missing values in the table by ...
- The constant of proportionality between _____ and _____ is _____ because ...
- A second constant of proportionality between _____ and _____ is _____ because ...
- The relationship between _____ and _____ is proportional because ...

Section B

- The equation _____ represents this proportional relationship because ...
- Two equations that represent the same proportional relationship are _____ and _____.
- I used _____ to represent the relationship and find the unknown values to be ...
- I used the equation _____ to find the number of _____ when the number of _____ is _____.

Section C

- This situation is/is not a proportional relationship because ...
- The values in this table do/do not represent a proportional relationship because ...
- The equation _____ represents a proportional relationship because ...

Section D

- The graph does/does not represent a proportional relationship because ...
- Using the graph, I found the constant of proportionality to be _____ because ...
- The constant of proportionality of this graph describes the relationship between _____ and _____.
- The coordinates _____ represent ...
- The constant of proportionality of _____ is greater/less than the constant of proportionality of _____ because ...
- The equation _____ represents the proportional relationship shown in the graph because ...

Section E

- I created a proportional relationship between _____ and _____. The constant of proportionality is _____ and an equation to represent the relationship is _____.
- I am most comfortable representing a proportional relationship with _____ because ...

Unit Outline

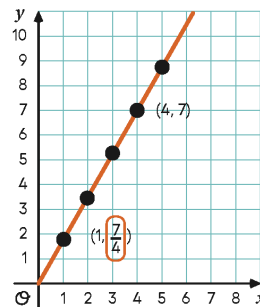
In this unit, students develop the idea of a proportional relationship. They work with proportional relationships that are represented in tables, as equations, and on graphs. This builds on grade 6 work with equivalent ratios and helps prepare students for the study of linear functions in grade 8.

Students begin by looking at tables. In a table of equivalent ratios, a multiplicative relationship between a pair of rows is given by a scale factor, while the multiplicative relationship between the columns is given by a unit rate. Students learn that the relationship between pairs of values in the two columns is called a "proportional relationship," and the unit rate that describes this relationship is called a "constant of proportionality."

Next, students use equations of the form $y = kx$ to represent proportional relationships and solve problems. They determine whether given tables and equations could represent a proportional relationship.

Then students investigate graphs of proportional relationships. They recognize that the graph of a proportional relationship is a straight line through the origin. They interpret points on the graph, including the point $(1, \frac{7}{4})$. Here is an example of a graph, an equation, and a table that all represent the same proportional relationship.

By the end of the unit, students should be comfortable working with common contexts associated with proportional relationships (such as constant speed, unit pricing, and measurement conversions) and be able to determine whether or not a relationship is proportional. In a later unit, students will apply proportional reasoning to solve multi-step problems and to calculate more complex rates.



$$y = \frac{7}{4}x$$

x	y
0	0
1	$\frac{7}{4}$
2	$\frac{7}{2}$
3	$\frac{21}{4}$
4	7

A note on using the terms "ratio," "proportional relationship," and "unit rate":

In these materials, the term "ratio" is used to mean a type of association between two or more quantities. A quantity is a measurement that can be specified by a number and a unit, for example 4 oranges, 4 centimeters, or "my height in feet." A proportional relationship is a collection of equivalent ratios.

A

Lesson Sequence	Learning Target(s)	Success Criteria/Assessment
Section A Representing Proportional Relationships with Tables (Lessons 1-3)	Learning Target #1 Determine whether a figure is a scaled copy of another figure, by examining corresponding side lengths and angle measures. Learning Target #2 Draw a scaled copy of a figure using a given scale factor.	Lesson 1 One of These Things Is Not Like the Others <ul style="list-style-type: none"> • I can use equivalent ratios to describe scaled copies of shapes. • I know that two recipes will taste the same if the ingredients are in equivalent ratios. Lesson 2 Introducing Proportional Relationships with Tables <ul style="list-style-type: none"> • I can use a table to reason about two quantities that are in a proportional relationship. • I understand the terms proportional relationship and constant of proportionality Lesson 3 More about Constant of Proportionality <ul style="list-style-type: none"> • I can find missing information in a proportional relationship using a table. • I can find the constant of proportionality from information given in a table.
Checkpoint A	Responding to Student Thinking Press Pause: By this point in the unit, there should be some student mastery of working with a table that represents a proportional relationship and identifying the constant of proportionality. If students struggle with this, make time to examine related work in the lessons referred to here. The Course Guide provides additional ideas for revisiting earlier work. <ul style="list-style-type: none"> • Grade 7, Unit 2, Lesson 2 Introducing Proportional Relationships with Tables • Grade 7, Unit 2, Lesson 3 More about Constant of Proportionality 	
Section B Representing Proportional Relationships with	Learning Target #3 Use an equation to solve problems involving a proportional relationship.	Lesson 4 Proportional Relationships and Equations <ul style="list-style-type: none"> • I can write an equation of the form $y = kx$ to represent a proportional relationship shown in a table or described in a story.

<p>Equations (Lessons 4-6)</p>	<p>Learning Target #4 Write an equation of the form $y=kx$ to represent a proportional relationship, given a table or a description of the situation</p>	<ul style="list-style-type: none"> I can write the constant of proportionality as an entry in a table. <p>Lesson 5 Two Equations for Each Relationship</p> <ul style="list-style-type: none"> I can find two constants of proportionality for a proportional relationship. I can write two equations representing a proportional relationship described by a table or story <p>Lesson 6 Writing Equations to Represent Relationships</p> <ul style="list-style-type: none"> I can find missing information in a proportional relationship using the constant of proportionality. I can relate all parts of an equation to the situation it represents.
<p>Checkpoint B</p>	<p>Responding to Student Thinking</p> <ul style="list-style-type: none"> Problem 1: Points to Emphasize: If most students struggle with writing an equation that represents a proportional relationship, as opportunities arise over the next several lessons, revisit the structure and meaning of an equation of the form $y=kx$. For example, in the activity referred to here, invite multiple students to share their thinking about the structure of the equations. <ul style="list-style-type: none"> Grade 7, Unit 2, Lesson 8, Activity 3 Total Edge Length, Surface Area, and Volume Problem 2: Points to Emphasize: If most students struggle with using an equation to find unknown values, revisit this concept as opportunities arise over the next several lessons. For example, in the activity referred to here, invite multiple students to share their thinking about substituting values and evaluating expressions. <ul style="list-style-type: none"> Grade 7, Unit 2, Lesson 8, Activity 2 More Conversions 	
<p>Section C Comparing Proportional and Nonproportional Relationships (Lessons 7-9)</p>	<p>Learning Target #5 Determine whether the values in a table could represent a proportional relationship.</p> <p>Learning Target #6 Use a table to determine whether an equation represents a proportional relationship</p>	<p>Lesson 7 Comparing Relationships with Tables</p> <ul style="list-style-type: none"> I can decide if a relationship represented by a table could be proportional and when it is definitely not proportional. <p>Lesson 8 Comparing Relationships with Equations</p> <ul style="list-style-type: none"> I can decide if a relationship represented by an equation is proportional or not <p>Lesson 9 Solving Problems about Proportional Relationships</p> <ul style="list-style-type: none"> I can ask questions about a situation to determine whether two quantities are in a proportional relationship. I can solve all kinds of problems involving proportional relationships
<p>Checkpoint C</p>	<p>Responding to Student Thinking</p> <ul style="list-style-type: none"> Problem 1: Press Pause: By this point in the unit, there should be some student mastery of determining whether or not a table could represent a proportional relationship. If students struggle with this, make time to examine related work in the section referred to here. The Course Guide provides additional ideas for revisiting earlier work. <ul style="list-style-type: none"> Grade 7, Unit 2, Section C Comparing Proportional and Nonproportional Relationships Problem 2: More Chances: Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons. 	
<p>Section D Representing Proportional Relationships with Graphs (Lessons 10-13)</p>	<p>Learning Target #7 Determine whether a given graph represents a proportional relationship.</p> <p>Learning Target #8 Identify the constant of proportionality from the graph of a proportional relationship.</p> <p>Learning Target #9 Interpret points on the graph of a proportional relationship.</p>	<p>Lesson 10 Introducing Graphs of Proportional Relationships</p> <ul style="list-style-type: none"> I know that the graph of a proportional relationship lies on a line through $(0, 0)$. <p>Lesson 11 Interpreting Graphs of Proportional Relationships</p> <ul style="list-style-type: none"> I can draw the graph of a proportional relationship given a single point on the graph (other than the origin). I can find the constant of proportionality from a graph. I understand the information given by graphs of proportional relationships that are made up of points or a line. <p>Lesson 12 Using Graphs to Compare Relationships</p> <ul style="list-style-type: none"> I can compare two, related proportional relationships based on their graphs. I know that the steeper graph of two proportional relationships has a larger constant of proportionality <p>Lesson 13 Two Graphs for Each Relationship</p>

		<ul style="list-style-type: none"> I can interpret a graph of a proportional relationship using the situation. I can write an equation representing a proportional relationship from a graph. I understand the relationship between a polyhedron and its net.
Checkpoint D	Responding to Student Thinking	<ul style="list-style-type: none"> Problem 1: Points to Emphasize: If most students struggle with determining whether or not a graph represents a proportional relationship, revisit this concept as opportunities arise over the next several lessons. For example, invite multiple students to share their thinking about this using the graphs in the lessons referred to here. <ul style="list-style-type: none"> Grade 7, Unit 3, Lesson 1 How Well Can You Measure? Grade 7, Unit 3, Lesson 3 Exploring Circumference Problem 2: Press Pause: By this point in the unit, there should be some student mastery of interpreting points on the graph of a proportional relationship. If students struggle with this, make time to examine related work in the lessons referred to here. The Course Guide provides additional ideas for revisiting earlier work. <ul style="list-style-type: none"> Grade 7, Unit 2, Lesson 11 Interpreting Graphs of Proportional Relationships Grade 7, Unit 2, Lesson 13 Two Graphs for Each Relationship
Section E Let's Put it To Work (Lessons 14-15)	No new learning targets.	<p>Lesson 14 Four Representations</p> <ul style="list-style-type: none"> I can make connections between the graphs, tables, and equations of a proportional relationship. I can use units to help me understand information about proportional relationships <p>Lesson 15 Using Water Efficiently</p> <ul style="list-style-type: none"> I can answer a question by representing a situation using proportional relationships

End of Unit Assessment

NOTES

Replace question 3 on EOU, completed table asking them to state the constant of proportionality

Reword question 7:

Original A recipe for salad dressing calls for 3 tablespoons of oil for every 2 tablespoons of vinegar

New A recipe for salad dressing calls for 2 tablespoons of vinegar for every 3 tablespoons of oil

m	N
3	9
5	15
6	18
9	27

Constant of proportionality:

Equation: $N =$

Unit Title:

Unit 3: Measuring Circles

Relevant Standards: Bold indicates priority

Lesson	Standards	Lesson	Standards
Lesson 1	7.RP.A.2.a	Lesson 7	7.G.A, 7.G.B, 7.G.B.4, 7.RP.A.2.a
Lesson 2	7.G.A, 7.G.A.2	Lesson 8	7.G.B.4
Lesson 3	7.G.B.4, 7.RP.A.2, 7.RP.A.2.a	Lesson 9	7.G.B.4
Lesson 4	7.G.B.4	Lesson 10	7.G.B.4
Lesson 5	7.G.B.4, 7.RP.A.2.a, 7.RP.A.2.c, 7.RP.A.3	Lesson 11	7.EE.B.3, 7.G.A.1, 7.G.B.4
Lesson 6	7.G.A.1, 7.G.B, 7.G.B.6		

Essential Question(s):

- What is the relationship between a circle’s diameter and its circumference?
- How can we determine the area of a circular region?
- How do I decide whether a situation involves circumference or area?

Enduring Understanding(s):

- The circumference of any circle is proportional to its diameter, and the exact value of this constant of proportionality is pi (π).
- The area of a circle can be calculated using the formula $A=\pi r^2$, which can be justified by decomposing and rearranging circular parts into shapes that approximate polygons like parallelograms.
- Circumference is a one-dimensional measure of the distance around a circle’s boundary, while area is a two-dimensional measure of the surface space inside that boundary.

Demonstration of Learning:

CFA 1: Checkpoint A (after Lesson 5)
 CFA 2: Checkpoint B (after Lesson 9)
 Checkpoint C (after Lesson 11) is an opportunity for feedback
 Unit 3 End of Unit Assessment

Pacing for Unit

13 Days
 Lesson to Review/Add:

- 6.1.6 - Focus on area of parallelograms
- 6.1.9 - Focus on area of triangles

 Lesson Modifications to Condense the Unit:

- Deemphasize lesson 5 and 11
- 7.3.5 - lesson is optional
- 7.3.11 - lesson can be removed or moved to outside of class if time does not allow

Family Overview

<https://accessim.org/6-8/grade-7/unit-3?a=family>

Integration of Technology:

- Desmos Online Graphing Calculator
- Pear Assessment (Edulastic)
- iM v.360 Digital Applets (see below)

Unit-specific Vocabulary:

Lesson	New Terminology	
	receptive	productive
7.3.1	relationship perimeter	
7.3.2	radius diameter circumference center (of a circle)	circle
7.3.3	pi (π)	
7.3.4	half-circle rotation approximation	
7.3.5		diameter circumference pi (π)

Aligned Unit Materials, Resources, and Technology

- Digital Applets
- 3.1 Perimeter of a Square
 - 3.2 Drawing Circles
 - 3.3 Measuring Circumference and Measuring Circumference Graph
 - 3.4 Using pi
 - 3.7 Estimating the Area of a Circle
 - 3.8 Making Another Polygon out of a Circle
- Provide access as needed throughout the unit:
- Blank paper
 - Compasses
 - Cylindrical household items
 - Empty toilet paper roll
 - Four-function calculators
 - Geometry toolkits (ongoing)
 - Glue or glue sticks

		travel	<ul style="list-style-type: none"> • Markers • Math Community Chart • Measuring tapes • Receipt tape • Rulers • Rulers marked with centimeters • Scissors 																																	
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7.3.7	area of a circle																																			
7.3.8	squared formula	radius area of a circle																																		
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7.3.10		squared center (of a circle) formula																																		
7.3.11	design																																			
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<p>Opportunities for Interdisciplinary Connections:</p> <ul style="list-style-type: none"> ● History and Culture: Students examine the geometry involved in Hopi Basket Weaving ● Art: Students design stained-glass windows using circumference and area calculations ● Agriculture: The unit uses the context of field irrigation to estimate circular areas 	<p>Anticipated misconceptions:</p> <ul style="list-style-type: none"> ● Measurement Error: Students may discount mathematical patterns because their physical measurements of circumference and diameter are not perfectly proportional due to human error. ● Pi Placement: When solving for diameter ($d=C/\pi$), students might multiply the circumference by π instead of dividing. ● Formula Confusion: Students often apply the circumference formula ($2\pi r$) when the problem asks for area (πr^2), and vice versa <p>See teacher's guide for additional misconceptions by lesson.</p>
<p>Connections to Prior Units:</p> <p>Essential prior concepts to engage with this unit:</p> <ul style="list-style-type: none"> ● Find the perimeter of polygons. ● Find the constant of proportionality. <p>Relevant Unit(s)/Lesson(s) to Review:</p> <ul style="list-style-type: none"> ● Grade 6 Unit 1: Area and Surface Area 	<p>Connections to Future Units:</p> <p>Serves as a bridge to advanced proportional applications and prepares students for Grade 8 work on the volume of spheres, cylinders, and cones.</p>
<p>Differentiation through <i>Universal Design for Learning</i></p>	
<p>Engagement:</p> <ul style="list-style-type: none"> ● Develop effort and persistence by connecting new concepts like scaling circles to prior success with pattern blocks (Lesson 8, Activity 2 Launch) <p>LT3: Justify that the area of a circle can be calculated with the formula</p> <p>Representation:</p> <ul style="list-style-type: none"> ● Provide reference sheets for polygon area formulas (Lesson 6, Activity 1 Launch) ● Use digital applets to visualize cutting and "unrolling" circles (Lesson 8, Activity 2 Launch) <p>LT5: Solve problems involving the area of a circle LT3: Justify that the area of a circle can be calculated with the formula</p> <p>Action & Expression:</p> <ul style="list-style-type: none"> ● Support communication by providing sentence frames for critiquing claims (Lesson 10, Activity 3 Student Task) <p>LT6: Critique claims about the radius, diameter, circumference, or area of a circle</p>	
<p>Supporting Multilingual Learners</p>	
<p>Math Language Routines</p> <p>The Illustrative Mathematics curriculum incorporates eight Mathematical Language Routines (MLRs) that support English Language Learners:</p> <p>MLR1: <i>Stronger and Clearer Each Time</i> - Students revise and refine their mathematical language through multiple drafts</p> <p>MLR2: <i>Collect and Display</i> - Students capture and organize language in visual displays</p> <p>MLR3: <i>Clarify, Critique, Correct</i> - Students analyze mathematical writing/talk</p> <p>MLR4: <i>Information Gap</i> - Students share information to solve problems</p> <p>MLR5: <i>Co-Craft Questions</i> - Students create and improve questions</p> <p>MLR6: <i>Three Reads</i> - Students analyze complex mathematical text</p> <p>MLR7: <i>Compare and Connect</i> - Students connect different mathematical representations</p> <p>MLR8: <i>Discussion Supports</i> - Students participate in mathematical discussions</p> <p>In this unit:</p> <ul style="list-style-type: none"> ● MLR1: Stronger and Clearer Each Time (Lessons 1, 11) ● MLR2: Collect and Display (Lessons 2, 4, 7, 8, 10) ● MLR3: Critique, Correct, Clarify (Lessons 2, 8, 10, 11) ● MLR4: Information Gap (Lesson 10) ● MLR5: Co-Craft Questions (Lessons 4, 5, 11, 13) ● MLR6: Three Reads (Lessons 6, 8, 11, 14) ● MLR7: Compare and Connect (Lessons 4, 9, 10, 13) ● MLR8: Discussion Supports (Lessons 1, 2, 3, 5, 9, 10, 11) <p>Progression of Disciplinary Language</p> <p>In this unit, teachers can anticipate students using language for mathematical purposes, such as generalizing, justifying, and interpreting. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:</p> <p>Generalize</p> <ul style="list-style-type: none"> ● About categories for sorting circles (Lesson 2). 	

- About the relationship between circumference and diameter (Lesson 3).
- About circumference and rotation (Lesson 5).
- About the relationship between the radius and the area of a circle (Lesson 8).

Justify

- Reasoning about circumference and perimeter (Lesson 4).
- Estimates for the areas of circles (Lesson 7).
- Reasoning about areas of curved figures (Lesson 9).
- Reasoning about the cost of stained-glass windows (Lesson 11).

Interpret

- Situations involving circles (Lessons 5 and 8).
- Floor plans and maps (Lesson 6).
- Situations involving circumference and area (Lesson 10).

Sentence Frames and Stems

Section A

- The diameter of the circle is _____.
- I know the radius of the circle is _____ because the diameter is _____ and ...
- The equation _____ can be used to find the circumference of a circle with a diameter/radius of _____.
- The diameter of the circle is _____ with a circumference of _____. I know these values have a proportional relationship because ...
- If I know the _____ of a circle, I can find the _____ by ...
- The circumference of the circle is _____ because ...

Section B

- The difference between circumference and area of a circle is ...
- I calculated the area of the circle by ...
- The area of the circle is _____ because ...
- The equation _____ can be used to find the area of a circle with a diameter/radius of _____.
- To find the area of a shaded region, first I _____, then I _____ ...

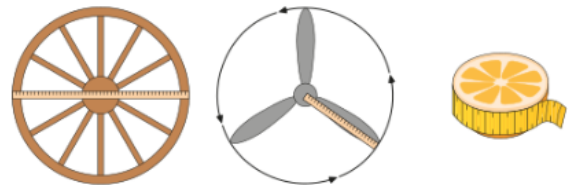
Section C

- For this question, I need to find the _____ of a circle because...
- I know the _____ of the circle, which I can use to find the _____. This will help me solve this problem because ...
- I used the equation _____ to solve this problem because ...

Unit Outline

In this unit, students apply their knowledge of proportional relationships to the context of measuring circles. They learn the relationships between radius, diameter, circumference, and area of circles and use these relationships to solve problems. This builds on students' work from previous grades with perimeter and area of polygons. Students will build on this work in grade 8 when they study the volume of spheres, cylinders, and cones.

The unit begins with activities designed to help build up students' vocabulary for describing circles more precisely. The terms "center," "radius," "diameter," and "circumference" are introduced. Then students investigate the relationship between circumference and diameter and see that it is a proportional relationship. They apply this relationship to solve problems.



Next, students explore the area of circular regions. They see an informal derivation that shows where the formula $A = \pi r^2$

comes from and then use this formula to solve problems. Finally, students solve problems that require deciding whether the situation relates to the circumference or area of a circle.

The first section of this unit, in which students recognize and apply proportional relationships involving circumference, serves as a bridge between the foundational work with proportional relationships in the previous unit and the more advanced applications in the following unit. The remaining sections of this unit, which deal with the area of circles, are preparation for the continued geometry work students will do later in this course.

A note on using the term "circle":

Strictly speaking, a circle is one-dimensional. It is the boundary of a two-dimensional region, rather than the region itself. The circular region is called a "disk." Because students are not yet expected to make this distinction, these materials refer to both disks and the boundaries of disks as "circles," using illustrations to eliminate ambiguity.

Lesson Sequence	Learning Target(s)	Success Criteria/Assessment
Section A Circumference of a	Learning Target #1	Lesson 1 How Well Can You Measure?

<p>Circle (Lessons 1-5)</p>	<p>Recognize that there are proportional relationships between the circumference, diameter, and radius of circles and express these relationships using equations.</p> <p>Learning Target #2 Solve problems involving circumference.</p>	<ul style="list-style-type: none"> I can examine quotients and use a graph to decide whether two associated quantities are in a proportional relationship. I understand that it can be difficult to measure the quantities in a proportional relationship accurately. <p>Lesson 2 Exploring Circles</p> <ul style="list-style-type: none"> I can describe the characteristics that make a shape a circle. I can identify the diameter, center, radius, and circumference of a circle. <p>Lesson 3 Exploring Circumference</p> <ul style="list-style-type: none"> I can describe the relationship between circumference and diameter of any circle I can explain what pi means <p>Lesson 4 Applying Circumference</p> <ul style="list-style-type: none"> I can choose an approximation for pi based on the situation or problem. If I know the radius, diameter, or circumference of a circle, I can find the other two <p>Lesson 5 Circumference & Wheels</p> <ul style="list-style-type: none"> If I know the radius or diameter of a wheel, I can find the distance the wheel travels in some number of revolutions.
<p>Checkpoint A</p>	<p>Responding to Student Thinking More Chances Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons</p>	
<p>Section B Area of a Circle (Lessons 6-9)</p>	<p>Learning Target #3 Justify that the area of a circle can be calculated with the formula.</p> <p>Learning Target #4 Recognize that the area of a circle is not proportional to its diameter or radius.</p> <p>Learning Target #5 Solve problems involving the area of a circle.</p>	<p>Lesson 6 Estimating Areas</p> <ul style="list-style-type: none"> I can calculate the area of a complicated shape by breaking it into shapes whose area I know how to calculate. <p>Lesson 7 Exploring the Area of a Circle</p> <ul style="list-style-type: none"> If I know a circle's radius or diameter, I can find an approximation for its area. I know whether or not the relationship between the diameter and area of a circle is proportional and can explain how I know. <p>Lesson 8 Relating Area to Circumference</p> <ul style="list-style-type: none"> I can explain how the area of a circle and its circumference are related to each other I know the formula for the area of a circle. <p>Lesson 9 Applying Area of Circles</p> <ul style="list-style-type: none"> I can calculate the area of more complicated shapes that include fractions of circles I can write exact answers in terms of pi
<p>Checkpoint B</p>	<p>Responding to Student Thinking Points to Emphasize If most students struggle with finding the area of a circle and expressing it in terms of π, revisit this concept when opportunities arise over the next several lessons.</p>	
<p>Section C Lets put it to work (Lessons 10-11)</p>	<p>Learning Target #6 Critique (orally and in writing) claims about the radius, diameter, circumference, or area of a circle in a real-world situation.</p> <p>Learning Target #7 Decide whether to calculate the circumference or area of a circle to solve a problem in a real-world situation, and justify (orally) the decision..</p> <p>Learning Target #8 Estimate measurements of a circle in a real-world situation, and explain (orally and in writing) the estimation strategy.</p>	<p>Lesson 10 Distinguishing Circumference and Area</p> <ul style="list-style-type: none"> I can decide whether a situation about a circle has to do with area or circumference. I can use formulas for circumference and area of a circle to solve problems. <p>Lesson 11 Stained-Glass Windows</p> <ul style="list-style-type: none"> I can apply my understanding of area and circumference of circles to solve more complicated problems.

End of Unit Assessment

NOTES

Replace question 3 on the EOU test version A with question 3 on the EOU test version B

Possibility for a question like this added to the test? Name 2 radii and name 2 diameters?

Select all true statements.

Circle A



Circle B

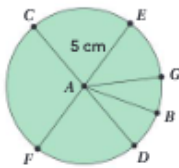


Circle C



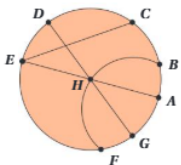
- A Circle A has a circumference of π .
- B Circle B has a circumference of π .
- C Circle B has an area of π .
- D Circle C has an area of π .
- E π is the constant of proportionality relating the radius of a circle to its circumference.
- F π is the constant of proportionality relating the diameter of a circle to its circumference.

A is the center of the circle, and the length of AE is 5 centimeters.



- a. What is the length of segment CD ? _____
- b. What is the length of segment AB ? _____
- c. Name a segment that has the same length as segment AB . _____

2 Here is a circle with center H and some line segments and curves joining points on the circle.



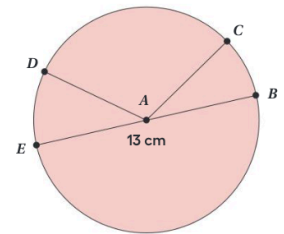
Identify examples of the following. Explain your reasoning.

- a. Diameter _____

- b. Radius _____

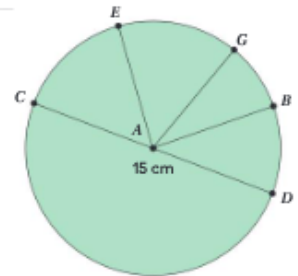
4 from Unit 3, Lesson 3
Point A is the center of the circle, and the length of EB is 13 centimeters.

- a. What is the radius of this circle? _____
- b. What is the circumference of this circle? _____



A is the center of the circle, and the length of CD is 15 centimeters.

- a. Name a segment that is a radius. _____
How long is it? _____
- b. Name a segment that is a diameter. _____
How long is it? _____



Rewrite question 7:

Original: A store sells 50-pound bags of grass seed. One pound of grass seed covers about 400 square feet of field.

New: A store sells bags of grass seed. One bag of grass seed covers 20,000 sq feet.

Unit Title:

Unit 4: Proportional Relationships and Percentages

Relevant Standards: Bold indicates priority

Lesson	Standards	Lesson	Standards
Lesson 1		Lesson 9	7.RP.A.3
Lesson 2	7.RP.A.1	Lesson 10	7.EE.A.2, 7.RP.A.3
Lesson 3	7.RP.A.1, 7.RP.A.2	Lesson 11	7.RP.A.3
Lesson 4	7.EE.A.2, 7.RP.A.2	Lesson 12	7.RP.A.3
Lesson 5	7.NS.A.2.d, 7.RP.A.2	Lesson 13	7.RP.A.3
Lesson 6	7.RP.A.1	Lesson 14	7.RP.A.3
Lesson 7	7.RP.A.3	Lesson 15	7.RP.A.3
Lesson 8	7.EE.A.2, 7.RP.A.3	Lesson 16	7.RP.A.3

Essential Question(s):	Enduring Understanding(s):
<ul style="list-style-type: none"> How can I calculate a new amount after a percentage increase or decrease? What does the "original amount" represent in percentage problems? How are real-world costs like sales tax, tips, and discounts modeled? 	<ul style="list-style-type: none"> A percent change can be found by adding or subtracting a percentage of the original amount to/from the original, or by multiplying the original amount by a single scale factor like $(1+0.25)$ for an increase. In situations involving percent increase or decrease, the initial or original value always corresponds to 100%. Taxes and tips are modeled as percent increases on a subtotal, while discounts and markdowns are modeled as percent decreases.

Demonstration of Learning:	Pacing for Unit
<p>CFA 1: Checkpoint A (after Lesson 5) CFA 2: Checkpoint B (after Lesson 9) CFA 3: Checkpoint C (after Lesson 14) Checkpoint D (after Lesson 16) is an opportunity for feedback) Unit 4 End of Unit Assessment</p>	<p>20 Days Lessons to Add/Review:</p> <ul style="list-style-type: none"> 6.3.14 - focus on strategies to solve problems involving percentages. Problem and Data Card discussion is highly suggested. 6.5.8 - focus on strategies for solving decimal multiplication problems. 6.5.11 - focus on strategies for solving decimal division problems. <p>Lesson Modifications to Condense Unit if Needed:</p> <ul style="list-style-type: none"> Remove lessons 2 and 3 (Not relevant to overarching percent learning targets). Combine 7.4.8 and 7.4.9 - remove optional activities to help compact lessons. Optional lessons could be used as additional practice outside of class. Deemphasize 7.4.15 - an optional lesson finding and analyzing intervals of possible error based on maximum possible errors. Move 7.4.16 to outside of class - culminating lesson on percentages, students work at home to collect news clippings that mention percentages and sort them according to whether they are about percent increase or percent decrease, and formulate questions about them. Discussion could take place in class. Combine lesson 8 and 9 and remove optional activities if needed

Family Overview	Integration of Technology:
<p>https://accessim.org/9-12-aga/algebra-1/unit-5?a=family</p>	<ul style="list-style-type: none"> Desmos Online Graphing Calculator Pear Assessment (EduLastic) iM v.360 Digital Applets (see below)

Unit-specific Vocabulary:	Aligned Unit Materials, Resources, and Technology

Lesson	New Terminology	
	receptive	productive
4.1	percentage	
7.4.2		unit rate
7.4.4	(a fraction) more than (a fraction) less than initial / original amount final / new amount	tape diagram distributive property
7.4.5	repeating decimal long division decimal representation	
7.4.6	percent increase percent decrease	(a fraction) more than (a fraction) less than
7.4.7	discount	initial / original amount final / new amount
7.4.10	sales tax tax rate tip	percent increase
7.4.11	interest commission markup markdown	percent decrease
7.4.12		percentage discount
7.4.13	measurement error	
7.4.14	percent error	

Lesson	Materials to Gather	Materials to Copy
4		<ul style="list-style-type: none"> Fractional Relationships Cards (1 copy for every 2 students): Activity 2
5		<ul style="list-style-type: none"> More Representations Cards (1 copy for every 2 students): Activity 3
8	<ul style="list-style-type: none"> Four-function calculators: Activity 1 	
10	<ul style="list-style-type: none"> Four-function calculators: Activity 1, Activity 2, Activity 3 	
11	<ul style="list-style-type: none"> Math Community Chart: Activity 1 Four-function calculators: Activity 2 	<ul style="list-style-type: none"> Percentage Situations Cards (1 copy for every 2 students): Activity 1
12	<ul style="list-style-type: none"> Four-function calculators: Activity 1 	<ul style="list-style-type: none"> Sporting Goods Cards (1 copy for every 2 students): Activity 1
13		<ul style="list-style-type: none"> Measurement to the Nearest Cutouts (1 copy for every 2 students): Warm-up
14	<ul style="list-style-type: none"> Four-function calculators: Activity 2 	
15		
16	<ul style="list-style-type: none"> Grocery store circulars: Warm-up, Activity 1 Sticky notes: Activity 2 Tools for creating a visual display: Activity 2 	

Opportunities for Interdisciplinary Connections:	Anticipated misconceptions:
<ul style="list-style-type: none"> Social Studies: Students investigate the history and specific dimensions required for the U.S. Flag Economics: Real-world financial applications include calculating sales tax, tips, and store discounts Earth Science: Percentages are used to analyze environmental changes, such as the drying of the Aral Sea 	<ul style="list-style-type: none"> Long Division Setup: Students may reverse the divisor and dividend when converting fractions to decimals "Moving the Decimal": Students may overgeneralize decimal rules, thinking 3.5% is 0.35 instead of 0.035 Percent Error Reference: In percent error problems, students might divide the error by the measured value instead of the actual value <p>See teacher's guide for specific misconceptions aligned to each lesson.</p>

Connections to Prior Units:	Connections to Future Units:
<p>Essential prior concepts to engage with this unit:</p> <ul style="list-style-type: none"> Ratio, rates, and unit rates Understanding of rates to include percentages as rates per 100 and reasoning about situations involving whole-number percentages <p>Relevant Unit(s)/Lesson(s) to Review:</p> <ul style="list-style-type: none"> Grade 6 Unit 3: Unit Rates and Percentages Grade 6 Unit 5: Arithmetic and Base Ten 	<p>Provides the groundwork for high school work with exponential functions and compounded percent changes.</p>

Differentiation through *Universal Design for Learning*

<p>Engagement:</p> <ul style="list-style-type: none"> Leverage choice around perceived challenge in sorting or matching activities (Lesson 1, Activity 3 Synthesis) LT1: Create algebraic expressions that represent a situation involving adding or subtracting a fraction of the initial value <p>Representation:</p> <ul style="list-style-type: none"> Activate background knowledge with documents containing mixed number arithmetic strategies (Lesson 1, Activity 1 Launch) LT1: Create algebraic expressions that represent a situation involving adding or subtracting a fraction of the initial value <p>Action & Expression:</p> <ul style="list-style-type: none"> Internalize executive functions by inviting students to rephrase directions in their own words (Lesson 9, Activity 1 Launch) LT3: Create algebraic expressions or equations that represent a situation involving percent increase or decrease

Supporting Multilingual Learners

Math Language Routines

The Illustrative Mathematics curriculum incorporates eight Mathematical Language Routines (MLRs) that support English Language Learners:

- MLR1: *Stronger and Clearer Each Time* - Students revise and refine their mathematical language through multiple drafts
- MLR2: *Collect and Display* - Students capture and organize language in visual displays
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- MLR6: *Three Reads* - Students analyze complex mathematical text
- MLR7: *Compare and Connect* - Students connect different mathematical representations
- MLR8: *Discussion Supports* - Students participate in mathematical discussions

In this unit:

- MLR1: Stronger and Clearer Each Time (Lessons 3, 5, 8, 9, 15)
- MLR2: Collect and Display (Lessons 3, 4, 5, 8, 10, 13)
- MLR3: Critique, Correct, Clarify (Lessons 3, 5, 7, 12, 14)
- MLR4: Information Gap (Lessons 9, 12)
- MLR5: Co-Craft Questions (Lessons 2, 8, 10, 15, 16)
- MLR6: Three Reads (Lessons 2, 6, 7, 14, 15)
- MLR7: Compare and Connect (Lessons 1, 2, 4, 6, 10, 13, 16)
- MLR8: Discussion Supports (Lessons 1, 2, 4, 6, 10, 11, 14)

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes, such as interpreting, explaining, and representing. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

<p>Interpret</p> <ul style="list-style-type: none"> Situations involving constant speed (Lesson 2). Concrete problems involving percent increase and decrease (Lesson 7). Problems involving sales tax and tip (Lesson 10). Concrete situations involving percent error (Lesson 14).
<p>Explain</p> <ul style="list-style-type: none"> How to solve concrete and abstract problems involving an amount plus (or minus) a fraction of that amount (Lesson 4). How to solve percent change problems (Lesson 6). Strategies for solving percent problems with fractional percentages (Lesson 9). How to measure lengths and interpret measurement error (Lesson 13). Strategies for solving percent error problems (Lesson 14).

Represent

- Situations involving percent increase and decrease (Lessons 8 and 15).
- Situations from the news involving percent change (Lesson 16).

Sentence Frames and Stems

Section A

- I know _____ is/is not a scaled copy of _____ because ...
- The scale factor from _____ to _____ is _____ because ...
- The constant of proportionality is _____, and I can use it to help me find _____ because ...
- I can use the distributive property to represent _____ more than _____ with the equation _____.
- I used long division to generate the decimal _____ from the fraction _____.

Section B

- The amount increased/decreased by _____ which is _____ percent of the original amount _____.
- I used _____ to represent _____ percent increase/decrease because ...
- I found the new amount to be _____ after the original amount _____ increased/decreased by _____ percent.
- I wrote the equation _____ to model this situation because ...

Section C

- In this situation, the _____ can be represented with a percent increase/decrease and the equation _____.
- I used the decimal value _____ to represent the percent _____ in the expression _____ to find the _____.
- If the regular price is _____, a discount of _____ percent would make the sale price _____.
- The percent error in this situation is _____ because ...

Section D

- There is a percent increase/decrease of _____ from _____ to _____.
- I know this situation is asking about a percent increase/decrease because ...
- I used _____ to model the situation because ...
- I used the equation _____ to solve this problem because ...

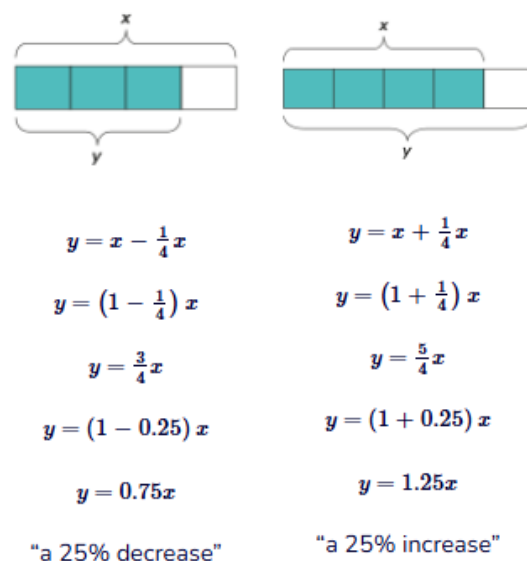
Unit Outline

In this unit, students deepen their understanding of proportional relationships and percentages. They solve multi-step problems and work with situations that involve fractional amounts. This builds on the work students did in grade 6 with ratios, rates, and percentages as well as previous units in grade 7 with proportional relationships. Students will build on this work in high school with exponential functions representing compounded percent increase and decrease.

Students begin the unit by revisiting constant rates, but this time the given values are fractional amounts. To determine the unit rate for the situation, students must compute the quotient of two fractions. Students also make sense of situations where an increase or decrease is expressed as a fraction of the initial amount. They create diagrams and apply the distributive property to generate expressions that represent these situations. They also use long division to write fractions as decimals, including their first introduction to repeating decimals.

Next, students make sense of situations where an increase or decrease is expressed as a percentage of the initial amount. They continue creating diagrams and writing equations to represent the situations. They solve for any one of the three quantities—the initial amount, the final amount, or the percentage of the change—given the other two quantities. They also reason about fractional percentages.

Then students apply percent increase and decrease to solve problems in a variety of real-world situations, such as tax, tip, interest, markup, discount, depreciation, and commission. Lastly, students make sense of situations where the difference between a correct measurement and an incorrect measurement is expressed as a percentage of the correct amount.



Lesson Sequence	Learning Target(s)	Success Criteria/Assessment
Section A Proportional Relationship with	Learning Target #1 Create algebraic expressions that represent a situation involving	Lesson 1 Lots of Flags <ul style="list-style-type: none"> • I can find dimensions on scaled copies of a rectangle • I can remember how to compute percentages

<p>Fractions (Lessons 1-5)</p>	<p>adding or subtracting a fraction of the initial value. Solve problems about proportional relationships with fractional quantities.</p> <p>Learning Target #2 Use long division to generate a decimal representation of a fraction.</p>	<p>Lesson 2 Ratios and Rates with Fractions</p> <ul style="list-style-type: none"> I can solve problems about ratios of fractions and decimals <p>Lesson 3 Revisiting Proportional Relationships</p> <ul style="list-style-type: none"> I can use a table with 2 rows and 2 columns to find an unknown value in a proportional relationship When there is a constant rate, I can identify the two quantities that are in a proportional relationship <p>Lesson 4 More than That, Less than That</p> <ul style="list-style-type: none"> I can use the distributive property to rewrite an expression like $x+1/2x$ as $(1+1/2)x$ I understand that “half as much again” and “multiply by $3/2$” mean the same thing <p>Lesson 5 Say it with Decimals</p> <ul style="list-style-type: none"> I can use the distributive property to rewrite an equation like $x+0.5x=1.5x$ I can write fractions as decimals I understand that “half as much again” and “multiply by 1.5” mean the same thing
<p>Checkpoint A</p>	<p>Responding to Student Thinking</p> <ul style="list-style-type: none"> Problem 1: Students will have more opportunities to develop this understanding in later units. There is no need to slow down or add additional work to review this concept at this time. Problem 2: By this point in the unit, there should be some student mastery of using the distributive property to generate expressions that represent a fractional increase. If students struggle, make time to revisit related work in the lesson referred to here. Problem 3: If students struggle with converting a fraction to a decimal, revisit this concept as opportunities arise over the next several lessons. 	
<p>Section B Percent Increase and Decrease (Lessons 6-9)</p>	<p>Learning Target #3 Create algebraic expressions or equations that represent a situation involving percent increase or decrease.</p> <p>Learning Target #4 Use diagrams to solve problems involving percent increase or decrease.</p>	<p>Lesson 6 Increasing and Decreasing</p> <ul style="list-style-type: none"> I can draw a tape diagram that represents a percent increase or decrease. When I know a starting amount and the percent increase or decrease, I can find the new amount. <p>Lesson 7 One Hundred Percent</p> <ul style="list-style-type: none"> I can use a double number line diagram to help me solve percent increase and decrease problems. I understand that if I know how much a quantity has grown, then the original amount represents 100%. When I know the new amount and the percentage of increase or decrease, I can find the original amount. <p>Lesson 8 Percent Increase and Decrease with Equations</p> <ul style="list-style-type: none"> I can solve percent increase and decrease problems by writing an equation to represent the situation and solving it. <p>Lesson 9 Part of a Percent</p> <ul style="list-style-type: none"> I can find fractional percentages (like 12.5% or 0.4%) of quantities. I understand that to find 0.1% of a quantity, I have to multiply by 0.001.
<p>Checkpoint B</p>	<p>Responding to Student Thinking</p> <ul style="list-style-type: none"> Problem 1: If students struggle with finding the percentage of an increase or decrease, revisit this concept as opportunities arise over the next several lessons. Problem 2: By this point in the unit, there should be some student mastery of representing situations involving percent increase or decrease. If students struggle, make time to revisit related work in the activities referred to here. 	
<p>Section C Applying Percentages (Lessons 10-14)</p>	<p>Learning Target #5 Calculate measurement error, and express it as a percentage of the actual value.</p> <p>Learning Target #6 Solve problems involving tax, tip, simple interest, markup, markdown, or commission.</p>	<p>Lesson 10 Tax and Tip</p> <ul style="list-style-type: none"> I understand and can solve problems about sales tax and tip <p>Lesson 11 Percent Contexts</p> <ul style="list-style-type: none"> I understand and can solve problems about commission, interest, markups and discounts <p>Lesson 12 Solving Multi-step Percentage Problems</p> <ul style="list-style-type: none"> I can solve problems that involve multiple percentages <p>Lesson 13 Measurement error</p> <ul style="list-style-type: none"> I can represent measurement error as a percentage of the correct measurement I understand that all measurements include some error

		Lesson 14 Percent Error <ul style="list-style-type: none"> I can solve problems that involve percent error
Checkpoint C	Responding to Student Thinking <ul style="list-style-type: none"> Problem 1: Press Pause By this point in the unit, there should be some student mastery of finding the value after tax, tip, markup, or interest is added. If students struggle, make time to revisit related work in this section. Problem 2: More Chances Students will have more opportunities to develop this understanding in later units. There is no need to slow down or add additional work to review this concept at this time. 	
Section D Let's put it to work. (Lessons 15-16)	No new learning targets	Lesson 15 Changes on Earth <ul style="list-style-type: none"> I can use percentages to describe changes in real-world situations. Lesson 16 Posing Percentage Problems <ul style="list-style-type: none"> I can write and solve problems about real-world situations that involve percent increase and decrease.

End of Unit Assessment

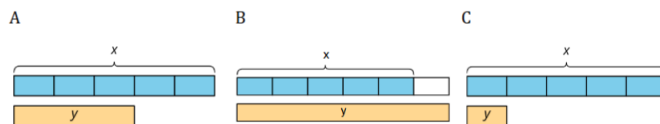
NOTES

Replace question 1 with a possible tape diagram problem from additional practice problems.

Unit 4 Lesson 4 Additional Practice Problems

1. Match each tape diagram to the situation that it models.

- Priya ate x ounces of broccoli. Noah ate $\frac{1}{5}$ of that.
- Noah bought x pounds of chicken. Priya bought $\frac{1}{5}$ more than that.
- Priya walked x miles. Noah walked $\frac{2}{5}$ less than that.



Possibly remove question 4? Since removing lessons 2 and 3.

Break apart other questions into more parts - Question 7 could be broken into 3 different questions.

Change order of problems (remove part C from question 7)

Question 4 removed and replaced with #7 (part A)

Question 7 (part B) used for #5

Question 6 broken up into 2 parts

Part A: Lin's father is paying for a \$20 meal. He has a 15%-off coupon for the meal. What is the price of the meal after the coupon

Part B: Use your answer from part A to help with Part B... After the discount a 7% sales tax is applied. What does Lin's father pay for the meal?

New Question #7 is old question #5 from test

Unit Title:

Unit 5: Rational Number Arithmetic

Relevant Standards: Bold indicates priority

Lesson	Standards	Lesson	Standards
Lesson 1	None Listed	Lesson 9	8.EE.A.3
Lesson 2	8.EE.A.1	Lesson 10	8.EE.A.3,8.EE.A.4
Lesson 3	8.EE.A.1	Lesson 11	8.EE.A.1, 8.EE.A.3, 8.EE.A.4
Lesson 4	8.EE.A.1	Lesson 12	8.EE.A.3, 8.EE.A.4
Lesson 5	8.EE.A.1	Lesson 13	8.EE.A.4
Lesson 6	8.EE.A.1	Lesson 14	8.EE.A.1, 8.EE.A.3, 8.EE.A.4
Lesson 7	8.EE.A.1	Lesson 15	8.EE.A.4
Lesson 8	8.EE.A.1	Lesson 16	8.EE.A.3, 8.EE.A.4

Essential Question(s):

- What do negative numbers represent in real-world contexts?
- How do the rules for arithmetic with negative numbers relate to positive numbers?
- How can position and movement be modeled using signed numbers?

Enduring Understanding(s):

- Negative numbers represent values relative to a reference point of zero, such as elevation below sea level, temperatures below freezing, or financial debt.
- Subtracting a number is equivalent to adding its additive inverse, and the sign of a product or quotient is determined by whether the two numbers have the same sign (positive result) or different signs (negative result).
- Velocity is a signed number indicating both speed and direction, allowing products to be interpreted as changes in position over time.

Demonstration of Learning:

CFA 1: Checkpoint A (after Lesson 7)
 CFA 2: Checkpoint B (after Lesson 12)
 CFA 3: Checkpoint C (after Lesson 16)
 Unit 5 End of Unit Assessment

Pacing for Unit

21 Days
 Lessons to Add/Review:

- Combine Positive and Negative Numbers (6.7.1) / Points on the Number Line (6.7.2) / Interpreting Negative Numbers (7.5.1)
- Comparing Positive and Negative Numbers (6.7.3)
- Combine Ordering Rational Numbers (6.7.4) / Using Negative Numbers to Make Sense of Contexts (6.7.5)
- 6.7.6 Absolute Value of Numbers
- 6.7.7 Comparing Numbers and Distance from Zero
- Combine Points on the Coordinate Plane (6.7.11) / Constructing the Coordinate Plane (6.7.12)
- Interpreting Points on a Coordinate Plane (6.7.13)
- Distances on a Coordinate Plane (6.7.14) / Shapes on the Coordinate Plane (6.7.15)

Lesson Modifications:

- Combine lessons 6 and 7 if needed. (Deemphasize activities 2 and 3 in lesson 7 when combining)
- Combine lessons 8 and 9 if needed. (Deemphasize activity 2 from lesson 8, deemphasize activities 2 and 3 from lesson 9)
- Deemphasize lesson 10
- Combine lessons 12 and 14 if needed. (Deemphasize activity 2 on both lesson 12 and lesson 14)
- Move 7.5.17 to outside of class. In this culminating lesson on percentages, students work at home to collect news clippings that mention percentages and sort them according to whether they are about percent increase or percent decrease, and formulate questions about them. A discussion could take place in class.

Family Overview

<https://accessim.org/6-8/grade-7/unit-5?a=family>

Integration of Technology:

- Desmos Online Graphing Calculator

- Pear Assessment (Edulastic)
- iM v.360 Digital Applets (see below)

Unit-specific Vocabulary:

Aligned Unit Materials, Resources, and Technology

Lesson	New Terminology	
	receptive	productive
7.5.1	absolute value degrees Celsius vertical elevation sea level	positive number negative number
7.5.2	signed numbers	temperature number line
7.5.3	sum opposite expression	
7.5.4	deposit withdrawal account balance debt	
7.5.6	difference	distance
7.5.7		absolute value x-coordinate y-coordinate
7.5.8	velocity	
7.5.11	solution (to an equation) factor	
7.5.13	rational number	sum difference
7.5.15	variable additive inverse multiplicative inverse	opposite solution (to an equation)
7.5.16	operation	
7.5.17		increase decrease

- Digital Applets
- 5.2 Winter Temperatures
 - 5.3 Cliffs and Caves
 - 5.7 Differences and Distances
 - 5.9 Cruising, Rational Numbers Multiplication Grid
 - 5.11 Drilling Down
 - 5.13 Seagulls and Sharks Again
 - 5.15 Match Solutions
 - 5.16 Warmer or Colder than Before?

Provide access as needed throughout the unit:

- Four-function calculators
- Math Community Chart
- Receipt tape
- Tools for creating a visual display

Lesson	Materials to Gather	Materials to Copy
1		Rational Numbers Cards (1 copy for every 3 students): Activity 3
3	Receipt tape: Activity 3	
6	Math Community Chart: Activity 2	
9		Rational Numbers Multiplication Grid Handout (1 copy for every student): Activity 3
10		<ul style="list-style-type: none"> • Temperature and Art Funds Cards (1 copy for every 4 students): Activity 1 • Matching Expressions Cards (1 copy for every 2 students): Activity 2
13		<ul style="list-style-type: none"> • The Same But Different Cards (1 copy for every 2 students): Activity 1
15		<ul style="list-style-type: none"> • Matching Inverses Cards (1 copy for every 2 students): Activity 3
16	Tools for creating a visual display: Activity 3	
17	Four-function calculators: Activity 1, Activity 2, Activity 3	

Opportunities for Interdisciplinary Connections:

Anticipated misconceptions:

- **Meteorology:** Students use signed numbers to model and calculate changes in daily temperatures
- **Oceanography/Earth Science:** Negative numbers represent positions relative to sea level and changes in elevation
- **Finance:** Rational numbers model bank account balances, including deposits, withdrawals, and debt

- **Magnitude vs. Value:** Students often struggle with the idea that $-2 > -5$, confusing a number's distance from zero with its actual value.
- **Subtraction Signs:** Students may misread expressions like $-2.6 - (-12/4)$, missing the significance of multiple negative signs.

See teacher's guide for specific misconceptions aligned to each lesson.

Connections to Prior Units:	Connections to Future Units:
<p>Essential prior concepts to engage with this unit:</p> <ul style="list-style-type: none"> rational numbers absolute value notation understanding of the coordinate plane <p>Relevant Unit(s)/Lesson(s) to Review:</p> <ul style="list-style-type: none"> Grade 6 Unit 7: Rational Numbers 	<p>Acts as a natural lead-in to solving equations and simplifying expressions in Unit 6. Prepares students for solving equations in the form $px+q=r$</p>

Differentiation through [Universal Design for Learning](#)

Engagement:

- Provide tools like calculators to facilitate information processing and focus on key ideas (Lesson 8, Activity 2 Launch)
- LT3: Apply multiplication and division of signed numbers to represent situations and solve problems

Representation:

- Use physical objects or demonstrations of movement like walking east and west (Lesson 3, Activity 1 Launch)
- LT1: Apply addition and subtraction of signed numbers to represent situations and solve problems

Action & Expression:

- Support communication by identifying connections between different equations that represent the same scenario (Lesson 16, Activity 2 Synthesis)
- LT7: Write an equation of the form $x+p=q$ or $px=q$ to represent a situation

Supporting Multilingual Learners

Math Language Routines

The Illustrative Mathematics curriculum incorporates eight Mathematical Language Routines (MLRs) that support English Language Learners:

- MLR1: *Stronger and Clearer Each Time* - Students revise and refine their mathematical language through multiple drafts
- MLR2: *Collect and Display* - Students capture and organize language in visual displays
- MLR3: *Clarify, Critique, Correct* - Students analyze mathematical writing/talk
- MLR4: *Information Gap* - Students share information to solve problems
- MLR5: *Co-Craft Questions* - Students create and improve questions
- MLR6: *Three Reads* - Students analyze complex mathematical text
- MLR7: *Compare and Connect* - Students connect different mathematical representations
- MLR8: *Discussion Supports* - Students participate in mathematical discussions

In this unit:

- MLR1: Stronger and Clearer Each Time (Lessons 3, 6, 9, 17)
- MLR2: Collect and Display (Lessons 1, 3, 4, 9, 13, 15)
- MLR3: Critique, Correct, Clarify (Lesson 16)
- MLR4: Information Gap (Lesson 10)
- MLR5: Co-Craft Questions (Lessons 1, 3, 8, 14, 15)
- MLR6: Three Reads (Lessons 8, 12, 14, 17)
- MLR7: Compare and Connect (Lessons 2, 5, 7, 11, 15, 16)
- MLR8: Discussion Supports (Lessons 1, 2, 3, 4, 6, 9, 10, 13, 14, 15, 16)

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes such as interpreting, representing, and generalizing. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Interpret

- situations involving signed numbers (throughout Unit)
- tables with signed numbers (Lesson 3)
- bank statements with signed numbers (Lesson 4)

Represent

- addition of signed numbers on a number line (Lesson 2)
- situations involving signed numbers (Lessons 3 and 11)
- changes in elevation (Lesson 6)
- position, speed, and direction (Lesson 8)

Generalize

- about subtracting and adding signed numbers (Lesson 5)
- about differences and magnitude (Lesson 6)
- about multiplying negative numbers (Lesson 9)
- about additive and multiplicative inverses (Lesson 15)

Sentence Frames and Stems

Section A

- To add/subtract _____ and/from _____ on the number line, first I _____ then I...
- The equation _____ represents this situation because ...
- The sum of _____ and _____ is _____ because ...
- The difference between _____ and _____ is _____ because ...
- I used the number _____ to represent this situation because ...

Section B

- The product/quotient of _____ and _____ is _____ because ...
- The equation _____ fits this situation because ...
- _____ and _____ are opposites because ...
- The value of the expression _____ will be positive/negative because ...

Section C

- The expressions _____ and _____ are equivalent because ...
- To find the difference between _____ and _____, I can ...
- _____ is the multiplicative inverse of _____ because ...
- The equation _____ fits this situation, where the variable _____ represents ...
- The solution to the equation _____ is _____ because ...

Section D

- The equation _____ can be used to find the missing value _____ for company _____.
- The company saw their shares increase/decrease by _____ percent.

Unit Outline

In this unit, students perform operations on rational numbers, which are all numbers that can be written as a positive or negative fraction. This builds on grade 6 work with interpreting, comparing, and plotting rational numbers. It prepares students for a later unit when they will solve equations of the form $px+q=r$ or $p(x+q)=r$, where p , q , and r are rational numbers.

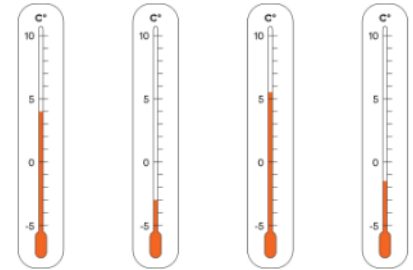
Students begin by revisiting how signed numbers are used to represent quantities above and below a reference point, such as measurements of temperature and elevation. They use tables and number line diagrams to represent changes in temperature or elevation. They extend addition and subtraction from fractions to all rational numbers. And they see that $a-b$ is equivalent to $a+(-b)$.

Next, students examine multiplication and division. They work with constant velocity, which is a signed number that indicates direction and speed. This allows products of signed numbers to be interpreted in terms of position, direction of movement, and time before or after a specific point. Students use the relationship between multiplication and division to understand how division extends to rational numbers.

Then students work with expressions that use the four operations on rational numbers. They also solve problems that involve interpreting negative numbers in context. They solve linear equations of the form $p+x=q$ or $px=q$, where p and q rational numbers. The focus of these lessons is representing situations with equations and what it means for a number to be a solution for an equation, rather than methods for solving equations. Such methods are the focus of a later unit

A note on using the terms "expression," "equation," and "signed number":

In these materials, an expression is built from numbers, variables, operation symbols ($+$, $-$, \cdot , \div), parentheses, and exponents. (Exponents—in particular, negative exponents—are not a focus of this unit. Students work with integer exponents in grade 8 and noninteger exponents in high school.) An equation is a statement that two expressions are equal, thus it always has an equal sign. Signed numbers include all rational numbers, written as decimals or in the form a/b .



Lesson Sequence	Learning Target(s)	Success Criteria/Assessment
Section A Adding and Subtracting Rational Numbers (Lessons 1-7)	Learning Target #1: Apply addition and subtraction of signed numbers to represent situations and solve problems. Learning Target #2	Lesson 1 Interpreting <ul style="list-style-type: none"> • I can compare rational numbers. • I can use rational numbers to describe temperature and elevation. Lesson 2 Changing Temperatures <ul style="list-style-type: none"> • I can use a number line to add positive and negative numbers. Lesson 3 Changing Elevation

	Calculate the sum or difference of two rational numbers.	<ul style="list-style-type: none"> I can add positive and negative numbers. Lesson 4 Money and Debts <ul style="list-style-type: none"> I understand what positive and negative numbers mean in a situation involving money. Lesson 5 Representing Subtraction <ul style="list-style-type: none"> I can explain the relationship between addition and subtraction of rational numbers. I can use a number line to subtract positive and negative numbers. Lesson 6 Finding Differences <ul style="list-style-type: none"> I can subtract positive and negative numbers. Lesson 7 Adding and Subtracting to Solve Problems <ul style="list-style-type: none"> I can solve problems that involve adding and subtracting rational numbers. <p>NOTE: Lesson 7 activity 1 directly related to EOU Question</p>
Checkpoint A	Responding to Student Thinking <ul style="list-style-type: none"> Problem 1: Points to Emphasize If most students struggle with adding or subtracting rational numbers, revisit this concept as opportunities arise over the next several lessons. Problem 2: More Chances Students will have more opportunities to develop this understanding in later units. There is no need to slow down or add additional work to review this concept at this time. 	
Section B Multiplying and Dividing Rational Numbers (Lessons 8-12)	Learning Target #3 Apply multiplication and division of signed numbers to represent situations and solve problems Learning Target #4 Calculate the product of quotient of two rational numbers	Lesson 8 Multiplying Rational Numbers (Part 1) <ul style="list-style-type: none"> I can multiply a positive number with a negative number Lesson 9 Multiplying Rational Numbers (Part 2) <ul style="list-style-type: none"> I can explain what it means when time is represented with a negative number in a situation about speed and direction I can multiply multiply two negative numbers Lesson 10 Multiply <ul style="list-style-type: none"> I can solve problems that involve multiplying rational numbers Lesson 11 Dividing Rational Numbers <ul style="list-style-type: none"> I can divide rational numbers Lesson 12 Negative Rates <ul style="list-style-type: none"> I can solve problems that involve multiplying and dividing rational numbers I can solve problems that involve negative rates <p>NOTE: Activity 1 in Lesson 12 is directly related to EOU question</p>
Checkpoint B	Responding to Student Thinking <ul style="list-style-type: none"> Problem 1: Points to Emphasize If most students struggle with multiplying or dividing rational numbers, revisit this concept as opportunities arise over the next several lessons. Problem 2: More Chances Students will have more opportunities to develop this understanding in later units. There is no need to slow down or add additional work to review this concept at this time. 	
Section C Four operations with Rational Numbers (Lessons 13-16)	Learning Target #5 Apply the four operations with rational numbers to solve problems Learning Target #6 Solve an equation of the form $x+p=q$ or $px=q$ where p, q and x are rational numbers. Learning Target #7 Write an equation of the form $x+p=q$ or $px=q$ (where p, q and x are rational numbers to represent a situation.	Lesson 13 Expressions with Rational Numbers <ul style="list-style-type: none"> I can add, subtract, multiply and divide rational numbers I can evaluate expressions that involve rational numbers Lesson 14 Solving Problems with Rational Numbers <ul style="list-style-type: none"> I can represent situations with expressions that include rational numbers I can solve problems using the four operations with rational numbers Lesson 15 Solving Equations with Rational Numbers <ul style="list-style-type: none"> I can solve equations that include rational numbers and have rational solutions Lesson 16 Representing Contexts with Equations <ul style="list-style-type: none"> I can explain what the solution to an equation means for the situation I can write and solve equations to represent situations that involve rational numbers
Checkpoint C	Responding to Student Thinking <ul style="list-style-type: none"> Problem 1: Press Pause By this point in the unit, there should be some student mastery of solving equations of the form $p + x = q$ or $px=q$. If most students struggle, make time to revisit related work in the activities referred to here. 	

- | | |
|--|---|
| | <ul style="list-style-type: none">• Problem 2: More Chances Students will have more opportunities to develop this understanding in later units. There is no need to slow down or add additional work to review this concept at this time.• Problem 3: Points to Emphasize If most students struggle with solving problems that involve the four operations with rational numbers, revisit this concept as opportunities arise over the next several lessons. |
|--|---|

End of Unit Assessment

NOTES

Remove question 5c and replace with

$$\frac{15}{8} + c = \frac{51}{8}$$

Unit Title:

Unit 6: Expressions, Equations, and Inequalities

Relevant Standards: Bold indicates priority

Lesson	Standards	Lesson	Standards
Lesson 1		Lesson 12	7.EE.A.2, 7.EE.B.3, 7.EE.B.4, 7.EE.B.4.a
Lesson 2	7.EE.B.3	Lesson 13	7.EE.B.4
Lesson 3	7.EE.B.3	Lesson 14	7.EE.B.4.b
Lesson 4	7.EE.B.3, 7.EE.B.4.a	Lesson 15	7.EE.B.4
Lesson 5	7.EE.B.3, 7.EE.B.4, 7.EE.B.4.a	Lesson 16	7.EE.B.4.b
Lesson 6	7.EE.B.3	Lesson 17	7.EE.B.4.b
Lesson 7	7.EE.B.4.a	Lesson 18	7.EE.A.1, 7.NS.A.1, 7.NS.A.1.c
Lesson 8	7.EE.B.4.a	Lesson 19	7.EE.A.1
Lesson 9	7.EE.B.4, 7.EE.B.4.a	Lesson 20	7.EE.A.1
Lesson 10	7.EE.B.4.a	Lesson 21	7.EE.A.1
Lesson 11	7.EE.B.3, 7.EE.B.4, 7.EE.B.4.a	Lesson 22	—

Essential Question(s):	Enduring Understanding(s):
<ul style="list-style-type: none"> How can equations represent complex real-world situations? What does it mean to find a solution to an equation or inequality? How can I determine if two different algebraic expressions are equivalent? 	<ul style="list-style-type: none"> Situations involving a constant rate of change and a starting amount can be represented by equations of the form $px+q=r$ or $p(x+q)=r$ Solving an equation or inequality means finding all values for a variable that make the mathematical statement true, often by performing the same operations on each side to maintain balance. Expressions are equivalent if they yield the same value for any substitution of the variable, which can be proven using properties such as the distributive, commutative, and associative properties

Demonstration of Learning:	Pacing for Unit
CFA 1: Checkpoint A (after Lesson 6) CFA 2: Checkpoint B (after Lesson 12) Unit 6 Mid-Unit Assessment CFA 3: Checkpoint C (after Lesson 17) CFA 4: Checkpoint D (after Lesson 21) Unit 6 End of Unit Assessment	27 Days Lessons to Add/Review: <ul style="list-style-type: none"> Practice Solving Equations and Representing Situations with Equations (6.6.4) The Distributive Property, Part 2 (6.6.10) Combine Two Related Quantities, Part 1 & 2 (6.6.16 and 6.6.17) Lesson Modifications: <ul style="list-style-type: none"> Combine 7.6.4 and 7.6.5: Parts 1 and 2 of Reasoning about Equations and Tape Diagrams Combine 7.6.7 and 7.6.8: Parts 1 and 2 of Reasoning about Solving Equations Move 7.6.22 to outside of class: In this culminating lesson students investigate several real-world situations that can be represented by an expression with a variable. A discussion could take place in class.

Family Overview	Integration of Technology:
https://accessim.org/6-8/grade-7/unit-6?a=family	<ul style="list-style-type: none"> Desmos Online Graphing Calculator Pear Assessment (Edulastic) iM v.360 Digital Applets (see below)

Unit-specific Vocabulary:	Aligned Unit Materials, Resources, and Technology											
<table border="1"> <thead> <tr> <th rowspan="2">Lesson</th> <th colspan="2">New Terminology</th> </tr> <tr> <th>receptive</th> <th>productive</th> </tr> </thead> <tbody> <tr> <td>7.6.2</td> <td>unknown amount</td> <td></td> </tr> <tr> <td>7.6.3</td> <td>equivalent expressions</td> <td></td> </tr> </tbody> </table>	Lesson	New Terminology		receptive	productive	7.6.2	unknown amount		7.6.3	equivalent expressions		Digital Applets <ul style="list-style-type: none"> 6.2 Every Story Needs a Picture 6.3 Drawing Tape Diagrams to Represent Equations 6.4 Situations and Diagrams 6.5 More Situations and Diagrams
Lesson		New Terminology										
	receptive	productive										
7.6.2	unknown amount											
7.6.3	equivalent expressions											

	commutative (property)		Provide access as needed throughout the unit: <ul style="list-style-type: none"> • Index cards • Math Community Chart • Sticky notes • Tools for creating a visual display
7.6.4		unknown amount relationship	
7.6.6		variable	
7.6.7	balanced hanger each side (of an equation)		
7.6.8		equivalent expression each side (of an equation)	
7.6.9		operation solve	
7.6.10	distribute substitute		
7.6.13	inequality less than or equal to greater than or equal to open / closed circle	less than greater than	
7.6.14	solution to an inequality boundary direction (of an inequality)	less than or equal to greater than or equal to substitute	
7.6.15		open / closed circle	
7.6.16		solution to an inequality	
7.6.17		inequality	
7.6.18	term		
7.6.19	factor (an expression) expand (an expression)		
7.6.20	combine like terms	term commutative (property)	
7.6.21		distribute	
7.6.22	associative property	factor (an expression) expand (an expression)	
Opportunities for Interdisciplinary Connections:			Anticipated misconceptions:
<ul style="list-style-type: none"> • Aerospace Engineering: The unit features "CubeSats" (small satellites) to provide a context for complex algebraic modeling • Social Science: Algebraic expressions model participation in various school club activities and community festivals 			<ul style="list-style-type: none"> • Term Combination: Students might incorrectly combine terms like $7x+2$ into $9x$, failing to recognize they are not like terms • Distribution Errors: In equations like $2(x-9)=10$, students may add 9 to both sides before dealing with the multiplier of 2 <p>See teacher's guide for specific misconceptions aligned to each lesson.</p>
Connections to Prior Units:			Connections to Future Units:
Essential prior concepts to engage with this unit: <ul style="list-style-type: none"> • linear equations with variables • tape diagrams • exponential equations Relevant Unit(s)/Lesson(s) to Review: <ul style="list-style-type: none"> • Grade 6 Unit 6: Expressions and Equations 			Prepares students for Grade 8 work with variables on both sides of an equation and systems of equations.
Differentiation through <i>Universal Design for Learning</i>			
Engagement: <ul style="list-style-type: none"> • Provide rubrics or checklists for multi-step projects like creating visual displays (Lesson 16, Activity 2 Launch) LT7: Write an inequality of the form $px+q>r$ or $px+q<r$ to represent a situation with a constraint Representation: <ul style="list-style-type: none"> • Provide blank templates of tape diagrams to help organize information (Lesson 3, Activity 2 Launch) 			

LT1: Create diagrams and equations in the form $px+q=r$ and $p(x+q)=r$ to represent situations

Action & Expression:

- Provide alternatives to writing by allowing students to share learning orally or through physical manipulatives (Lesson 20, Activity 1 Launch)
- LT8: Apply properties of operations to write an expression with fewer terms that is equivalent to a given expression

Supporting Multilingual Learners

Math Language Routines

The Illustrative Mathematics curriculum incorporates eight Mathematical Language Routines (MLRs) that support English Language Learners:

- MLR1: *Stronger and Clearer Each Time* - Students revise and refine their mathematical language through multiple drafts
- MLR2: *Collect and Display* - Students capture and organize language in visual displays
- MLR3: *Clarify, Critique, Correct* - Students analyze mathematical writing/talk
- MLR4: *Information Gap* - Students share information to solve problems
- MLR5: *Co-Craft Questions* - Students create and improve questions
- MLR6: *Three Reads* - Students analyze complex mathematical text
- MLR7: *Compare and Connect* - Students connect different mathematical representations
- MLR8: *Discussion Supports* - Students participate in mathematical discussions

In this unit:

- MLR1: Stronger and Clearer Each Time (Lessons 3, 8, 15, 18, 22)
- MLR2: Collect and Display (Lessons 3, 7, 10, 13, 16, 20, 21)
- MLR3: Critique, Correct, Clarify (Lessons 2, 4, 10, 13, 14, 15, 16, 17, 19, 21, 22)
- MLR4: Information Gap
- MLR5: Co-Craft Questions (Lesson 14)
- MLR6: Three Reads (Lessons 1, 6, 11, 12, 14, 17)
- MLR7: Compare and Connect (Lessons 1, 3, 5, 8, 9, 11, 15, 17, 22)
- MLR8: Discussion Supports (Occurs throughout)

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes such as comparing, explaining, and justifying. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Compare

- stories with corresponding tape diagrams (Lesson 2)
- tape diagrams with corresponding equations (Lesson 3)
- hanger diagrams and equations (Lesson 7)
- solution pathways (especially Lesson 10)
- descriptions of situations with corresponding inequalities (Lesson 16)

Explain

- strategies for using hanger diagrams to solve equations (Lesson 8)
- different strategies for solving equations (Lesson 9) and inequalities (Lesson 14)
- reasoning about situations, tape diagrams, and equations (Lesson 12)
- strategies for identifying and writing equivalent expressions (Lesson 22)

Justify

- reasoning about inequalities (Lesson 13)
- reasoning about solutions to inequalities (Lesson 15)
- the need for specific information in order to write and solve inequalities (Lesson 17)
- reasoning about the distributive property (Lesson 19)
- whether different sequences of calculations give the same result (Lesson 23)

Sentence Frames and Stems

Section A

- In this situation, I notice _____.
- The equation _____ represents this situation because ...
- To represent this equation, I used _____ (strategy/tool) to show _____ because ...
- The tape diagram represents the equation _____ because ...
- I noticed _____, which means the equation _____ is equivalent to the equation _____.
- The solution to the equation _____ is _____ because ...

Section B

- The equation _____ represents this situation because ...
- To find the unknown weight on the hanger diagram, first I _____, then I ...

- The solution to the equation _____ is _____ because ...
- I began solving the equation by _____ on both sides. Then, I found the solution by _____.
- The solution _____ to the equation _____ makes sense because ...

Section C

- The solution to the inequality _____ can be modeled on a number line diagram by ...
- The solution is _____. This means any number _____ makes the inequality true.
- The value _____ is a solution to the inequality _____ because ...
- The inequality _____ represents this situation because ...

Section D

- I used the _____ property to simplify the expression from _____ to _____.
- I combined like terms ... to simplify the expression. My new expression is _____.
- The distributive property allowed me to expand/factor the expression _____ to _____.
- The expression _____ is equivalent to the expression _____ because ...

Section E

- I used the expression _____ to represent the situation _____ because ...
- The expression _____ and the expression _____ are equivalent.

Unit Outline

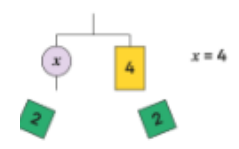
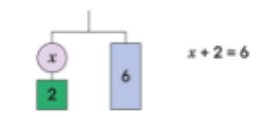
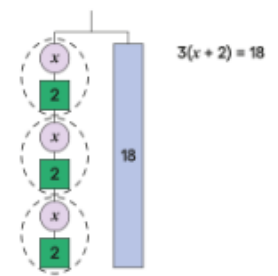
In this unit, students deepen their algebraic reasoning as they write and solve equations of the forms $px+q=r$ and $p(x+q)=r$ and inequalities of the forms $px+q>r$ and $p(x+q)<r$. Students also work with equivalent expressions that are more complex than what they have seen previously. This builds on grade 6 work with equations of the form $p+x=q$ or $px=q$ and with simpler equivalent expressions. Students will build on this work in grade 8 when they solve equations that have a variable on both sides of the equal sign and when they work with systems of equations.

Students begin the unit by making sense of situations that involve both multiplication and addition. They represent such situations with tape diagrams and with equations. They see that different diagrams and equations can represent the same situation, and they use diagrams to find solutions to equations.

Next, students consider hanger diagrams as another way to represent equations. The diagrams help students understand solving equations in terms of “doing the same thing to each side of the equation.” Students examine different pathways for solving the same equation and consider whether one method is more efficient than another.

Then students apply what they have learned about equations to inequalities. They write inequalities to represent situations and solve inequalities by reasoning about the related equation. The inequality symbols \geq and \leq are introduced.

Lastly, students work with equivalent linear expressions that are more complex due to having more terms, more parentheses, and negative rational numbers. Students use properties of operations to justify why the expressions are equivalent.



Lesson Sequence	Learning Target(s)	Success Criteria/Assessment
<p>Section A Representing Situations of the Form $px+q=r$ and $p(x+q)=r$ (Lessons 1-6)</p>	<p>Learning Target #1: Create diagrams and equations in the form $px+q=r$ and $p(x+q)=r$ to represent situations.</p> <p>Learning Target #2 Interpret equations in the form $px+q=r$ and $p(x+q)=r$ that represent relationships in diagrams and situations.</p>	<p>Lesson 1 Relationships between Quantities</p> <ul style="list-style-type: none"> • I can think of ways to solve some more complicated word problems. <p>Lesson 2 Reasoning about Contexts with Tape Diagrams</p> <ul style="list-style-type: none"> • I can explain how a tape diagram represents parts of a situation and relationships between them. • I can use a tape diagram to find an unknown amount in a situation. <p>Lesson 3 Reasoning about Equations with Tape Diagrams</p> <ul style="list-style-type: none"> • I can match equations and tape diagrams that represent the same situation. • If I have an equation, I can draw a tape diagram that shows the same relationship. <p>Lesson 4 Reasoning about Equations and Tape Diagrams (Part 1)</p> <ul style="list-style-type: none"> • I can draw a tape diagram to represent a situation where there is a known amount and several copies of an unknown amount and explain what the parts of the diagram represent. • I can find a solution to an equation by reasoning about a tape diagram or about what value would make the equation true.

		<p>Lesson 5 Reasoning about Equations and Tape Diagrams (Part 2)</p> <ul style="list-style-type: none"> I can draw a tape diagram to represent a situation where there is more than one copy of the same sum and explain what the parts of the diagram represent. I can find a solution to an equation by reasoning about a tape diagram or about what value would make the equation true. <p>Lesson 6 Reasoning to Find Area</p> <ul style="list-style-type: none"> I understand the similarities and differences between the two main types of equations we are studying in this unit. When I have a situation or a tape diagram, I can represent it with an equation.
Checkpoint A	<p>Responding to Student Thinking</p> <ul style="list-style-type: none"> Problem 1: Points to Emphasize If most students struggle with coordinating the parts of the equation with the diagram, focus on the ways students see each part of the equation when students interpret hanger diagrams in the referenced lessons. Problem 2: More Chances Students will have more opportunities to develop this understanding in later lessons. There is no need to slow down or add additional work to review this concept at this time. 	
Section B Parallelograms (Lessons 7-12)	<p>Learning Target #3 Solve equations of the form $px+q=r$ and $p(x+q)=r$, including those that involve fractions, decimals, and negative numbers and explain the solution method</p> <p>Learning Target #4 Solve word problems leading to equations of the form $px+q=r$ or $p(x+q)=r$</p>	<p>Lesson 7 Parallelograms</p> <ul style="list-style-type: none"> I can use reasoning strategies and what I know about the area of a rectangle to find the area of a parallelogram. I know how to describe the characteristics of a parallelogram using mathematical vocabulary. <p>Lesson 5 Bases and Heights of Parallelograms</p> <ul style="list-style-type: none"> I can identify pairs of base and height of a parallelogram. I can write and explain the formula for the area of a parallelogram. I know what the terms "base" and "height" refer to in a parallelogram. <p>Lesson 6 Area of Parallelograms</p> <ul style="list-style-type: none"> I can use the area formula to find the area of any parallelogram.
Checkpoint B	<p>Responding to Student Thinking</p> <ul style="list-style-type: none"> Problem 1: Press Pause By this point in the unit, there should be some student mastery of solving equations. If most students struggle with solving equations of the form $px + q = r$, make time to revisit related work in the Sections referred to here. Problem 2: Points to Emphasize If most students struggle with representing and solving the problem, focus on connecting diagrams and equations to the relationships in problems throughout the next section. For example, invite multiple students to share how the tape diagram matches the stories in the referenced practice problem. Encourage students to use and explain diagrams and equations as needed when representing and solving problems in lesson activities and practice problems throughout the section. 	
Section C Inequalities (Lessons 13-17)	<p>Learning Target #5 Draw and label a graph on a number line that represents all the solutions to an inequality.</p> <p>Learning Target #6 Generalize the process for finding the area of a triangle.</p> <p>Learning Target #7 Compare and contrast different strategies for finding areas of polygons.</p>	<p>Lesson 13 Reintroducing Inequalities</p> <ul style="list-style-type: none"> I can explain what the symbols \leq and \geq mean I can represent an inequality on a number line. I understand what it means for a number to make an inequality true. <p>Lesson 14 Finding Solutions to Inequalities in Context</p> <ul style="list-style-type: none"> I can describe the solutions to an inequality by solving a related equation and then reasoning about values that make the inequality true. I can write an inequality to represent a situation. <p>Lesson 15 Efficiently Solving Inequalities</p> <ul style="list-style-type: none"> I can graph the solutions to an inequality on a number line. I can solve inequalities by solving a related equation and then checking which values are solutions to the original inequality. <p>Lesson 16 Interpreting Inequalities</p> <ul style="list-style-type: none"> I can match an inequality to a situation it represents, solve it, and then explain what the solution means in the situation. If I have a situation and an inequality that represents it, I can explain what the parts of the inequality mean in the situation. <p>Lesson 17 Modeling with Inequalities</p> <ul style="list-style-type: none"> I can use what I know about inequalities to solve real-world problems.
Checkpoint C	<p>Responding to Student Thinking</p>	

	<ul style="list-style-type: none"> ● Problem 1: Press Pause By this point in the unit, there should be some student mastery of writing and solving inequalities. If most students struggle, make time to revisit related work in the referenced sections.
<p>Section D Writing Equivalent Expressions (Lessons 18-21)</p>	<p>Learning Target #8 Apply properties of operations to write an expression with fewer terms that is equivalent to a given expression.</p> <p>Learning Target #9 Apply the distributive property to factor or expand an expression.</p> <p>Lesson 18 Subtracting in Equivalent Expressions</p> <ul style="list-style-type: none"> ● I can organize my work when I use the distributive property. ● I can rewrite subtraction as adding the opposite and then rearrange terms in an expression. <p>Lesson 19 Expanding and Factoring</p> <ul style="list-style-type: none"> ● I can organize my work when I use the distributive property. ● I can use the distributive property to rewrite expressions with positive and negative numbers. ● I understand that “factoring” and “expanding” are words used to describe using the distributive property to write equivalent expressions. <p>Lesson 20 Combining Like Terms (Part 1)</p> <ul style="list-style-type: none"> ● I can figure out whether two expressions are equivalent to each other. ● When possible, I can write an equivalent expression that has fewer terms. <p>Lesson 21 Combining Like Terms (Part 2)</p> <ul style="list-style-type: none"> ● I am aware of some common errors when writing equivalent expressions, and I can avoid them. ● When possible, I can write an equivalent expression that has fewer terms.
<p>Checkpoint D</p>	<p>Responding to Student Thinking</p> <ul style="list-style-type: none"> ● Problem 1: Points to Emphasize If students struggle with writing an equivalent expression with fewer terms, revisit this concept when opportunities arise over the next several lessons. ● Problem 2: Points to Emphasize If students struggle with using the distributive property to factor or expand an expression, revisit this concept when opportunities arise over the next several lessons.
<p>End of Unit Assessment</p>	

Unit Title:			
Unit 7: Angles, Triangles, and Prisms			
Relevant Standards: Bold indicates priority			
Lesson	Standards	Lesson	Standards
Lesson 1	7.G.A, 7.G.B	Lesson 10	7.G.A.2
Lesson 2	7.G.B, 7.G.B.5	Lesson 11	7.G.A.3
Lesson 3	7.EE.A, 7.G.B.5	Lesson 12	7.G.B.6
Lesson 4	7.G.A, 7.G.B, 7.G.B.5	Lesson 13	7.G.A.3, 7.G.B.6
Lesson 5	7.EE.B.4, 7.G.B.5	Lesson 14	7.G.B.6
Lesson 6	7.G.A.2	Lesson 15	7.G.B, 7.G.B.6
Lesson 7	7.G.A.2	Lesson 16	7.G.B, 7.G.B.6, 7.RP.A
Lesson 8	7.G.A.2	Lesson 17	7.G.A.2, 7.G.B.6
Lesson 9	7.G.A, 7.G.A.2		
Essential Question(s):		Enduring Understanding(s):	
<ul style="list-style-type: none"> How do geometric relationships help us find unknown angle measures? What conditions determine if a triangle can be built or if it is unique? How do volume and surface area describe three-dimensional objects differently? 		<ul style="list-style-type: none"> Knowledge of complementary (90°), supplementary (180°), and vertical (equal) angle relationships allows us to write and solve equations for unknown measures The sum of the two shorter side lengths must be greater than the longest side to form a triangle, and specific combinations of side and angle measures can determine one unique triangle, more than one, or none at all Volume measures the total space inside a three-dimensional object (Area of Base \times Height), while surface area measures the total area of all its outer faces 	
Demonstration of Learning:		Pacing for Unit	
CFA 1: Checkpoint A (after Lesson 5) CFA 2: Checkpoint B (after Lesson 10) CFA 3 Checkpoint C (after Lesson 16) Unit 7 End of Unit Assessment		21 Days Lessons to Add/Review: <ul style="list-style-type: none"> None Lesson Modifications to Condense Unit if Needed: <ul style="list-style-type: none"> Remove optional Lesson 1, Activity 3. Remove optional Lesson 3, Activity 3. Remove optional Lesson 6, Activity 3. Remove optional Lesson 11, Activity 3. Remove optional Lesson 12, Activity 3. Remove optional Lesson 15, Activity 3. Remove optional Lesson 17. 	
Family Overview		Integration of Technology:	
https://accessim.org/6-8/grade-7/unit-7?a=family		<ul style="list-style-type: none"> Desmos Online Graphing Calculator Pear Assessment (Edulastic) iM v.360 Digital Applets (see below) 	
Unit-specific Vocabulary:		Aligned Unit Materials, Resources, and Technology	
New Terminology			
Lesson	receptive	productive	Digital Applets <ul style="list-style-type: none"> 7.1 Visualizing Angles, Pattern Block Angles, More Pattern Block Angles 7.6 What Can You Build?, Building Shapes 7.7 Where is Lin?, How Long is the Third Side?, Swinging the Sides Around 7.9 Does Your Triangle Match Theirs? 7.10 Revisiting How Many Can You Draw?, Three Angles
7.7.1	straight angle adjacent angles degree	right angle	
7.7.2	supplementary complementary angle measure	measurement error degrees	

	protractor		<ul style="list-style-type: none"> 7.11 What's the Cross-Section?, Drawing Cross-Sections 7.12 Finding Volume with Cubes, Can You Find the Volume? <p>Provide access as needed throughout the unit:</p> <ul style="list-style-type: none"> Blank paper Compasses Copies of blackline masters Fruits or vegetables Geometry toolkits Knife Materials assembled from the blackline master Math Community Chart Metal paper fasteners Brass brads Paint Pattern blocks Pre-assembled polyhedra Pre-printed cards, cut from copies of the blackline master Pre-printed slips, cut from copies of the blackline master Protractors Clear protractors with no holes and with radial lines printed on them are recommended. Rulers marked with centimeters Scissors Snap cubes Straightedges <p>A rigid edge that can be used for drawing line segments. Sometimes a ruler is okay to use as a straightedge, but sometimes it is preferable to use an unruled straightedge, like a blank index card.</p>																			
7.7.3	vertical angles intersect vertex (of an angle)																					
7.7.4		supplementary vertical angles																				
7.7.5	perpendicular	complementary																				
7.7.6	identical copy condition	angle measure side length quadrilateral																				
7.7.7	compass different triangle	intersect identical copy segment																				
7.7.8		condition different triangle																				
7.7.9	unique triangle parallel																					
7.7.10		protractor compass																				
7.7.11	cross section base (of a prism or pyramid) vertex (of a pyramid) face	prism pyramid perpendicular parallel																				
7.7.12		volume cross section base (of a prism or pyramid)																				
7.7.14		face perimeter																				
7.7.15		surface area																				
				<table border="1"> <thead> <tr> <th>Lesson</th> <th>Materials to Gather</th> <th>Materials to Copy</th> </tr> </thead> <tbody> <tr> <td>1</td> <td> <ul style="list-style-type: none"> Blank paper: Warm-up, Activity 1 Scissors: Warm-up, Activity 1 Pattern blocks: Activity 1, Activity 2 Straightedges: Activity 1 Protractors: Activity 3 </td> <td></td> </tr> <tr> <td>2</td> <td> <ul style="list-style-type: none"> Geometry toolkits: Activity 1 Protractors: Activity 1 Scissors: Activity 1 Straightedges: Activity 1 </td> <td>Cutting Rectangles (1 copy for every 2 students): Activity 1</td> </tr> <tr> <td>3</td> <td> <ul style="list-style-type: none"> Geometry toolkits: Activity 1, Activity 2 </td> <td></td> </tr> <tr> <td>4</td> <td> <ul style="list-style-type: none"> Math Community Chart: Activity 1 Pre-printed slips, cut from copies of the blackline master: Activity 1 </td> <td>Angle Finding Cards (1 copy for every 2 students): Activity 1</td> </tr> <tr> <td>6</td> <td> <ul style="list-style-type: none"> Geometry toolkits: Activity 1, Activity 2, Activity 3 Metal paper fasteners: Activity 1, Activity 2, Activity 3 Pre-printed slips, cut from copies of the blackline master: Activity 1, Activity 2, Activity 3 </td> <td>What Can You Build? Cutouts (1 copy for every 2 students): Activity 1</td> </tr> </tbody> </table>	Lesson	Materials to Gather	Materials to Copy	1	<ul style="list-style-type: none"> Blank paper: Warm-up, Activity 1 Scissors: Warm-up, Activity 1 Pattern blocks: Activity 1, Activity 2 Straightedges: Activity 1 Protractors: Activity 3 		2	<ul style="list-style-type: none"> Geometry toolkits: Activity 1 Protractors: Activity 1 Scissors: Activity 1 Straightedges: Activity 1 	Cutting Rectangles (1 copy for every 2 students): Activity 1	3	<ul style="list-style-type: none"> Geometry toolkits: Activity 1, Activity 2 		4	<ul style="list-style-type: none"> Math Community Chart: Activity 1 Pre-printed slips, cut from copies of the blackline master: Activity 1 	Angle Finding Cards (1 copy for every 2 students): Activity 1	6	<ul style="list-style-type: none"> Geometry toolkits: Activity 1, Activity 2, Activity 3 Metal paper fasteners: Activity 1, Activity 2, Activity 3 Pre-printed slips, cut from copies of the blackline master: Activity 1, Activity 2, Activity 3 	What Can You Build? Cutouts (1 copy for every 2 students): Activity 1
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	7	<ul style="list-style-type: none"> • Geometry toolkits: Warm-up, Activity 1, Activity 2 • Compasses: Activity 1, Activity 2 • Metal paper fasteners: Activity 1, Activity 2 • Copies of blackline masters: Activity 2 	Swinging the Sides Around Handout (1 copy for every student) Activity 2
	8	Geometry toolkits: Activity 1, Activity 2	
	9	Geometry toolkits: Activity 1, Activity 2	
	10	<ul style="list-style-type: none"> • Compasses: Warm-up, Activity 1, Activity 2 • Copies of blackline masters: Activity 1 • Geometry toolkits: Activity 1, Activity 2 	
	11	<ul style="list-style-type: none"> • Fruits or vegetables: Activity 1 • Knife: Activity 1 • Paint: Activity 1 • Pre-printed cards, cut from copies of the blackline master: Activity 2 	Cross Sections Cards (1 copy for every 3 students): Activity 2
	12	<ul style="list-style-type: none"> • Copies of blackline masters: Activity 1, Activity 2 • Snap cubes: Activity 1 • Pre-assembled polyhedra: Activity 2 • Rulers marked with centimeters: Activity 2 	<ul style="list-style-type: none"> • Finding Volume with Cubes Handout (1 copy for every 6 students): Activity 1 • Can You Find the Volume Cutouts (1 copy for every 3 students): Activity 2
	14	<ul style="list-style-type: none"> • Materials assembled from the blackline master: Lesson • Materials assembled from the blackline master: Warm-up 	Multifaceted Cutouts (1 copy for every 1 student): Warm-up
	15	Pre-printed slips, cut from copies of the blackline master: Activity 2	Surface Area or Volume Cards (1 copy for every 2 students): Activity 2
	17	<ul style="list-style-type: none"> • Compasses: Activity 1 • Geometry toolkits: Activity 1, Activity 2 • Rulers marked with centimeters: Activity 1, Activity 2 • Copies of blackline masters: Activity 2 	

Opportunities for Interdisciplinary Connections:	Anticipated misconceptions:
<ul style="list-style-type: none"> • Construction Engineering: Students study cross-sections and volume through the lens of 3-D Printed Houses • Manufacturing: Volume and surface area calculations are applied to designing packaging, such as octagonal or heart-shaped boxes 	<ul style="list-style-type: none"> • Protractor Reading: Students may read the wrong scale on a protractor (e.g., 140° instead of 40°). • Triangle Inequality: Students may believe any three lengths can form a triangle, not realizing the two shorter sides must sum to more than the longest side <p>See teacher's guide for specific misconceptions aligned to each</p>

	lesson.
Connections to Prior Units:	Connections to Future Units:
Essential prior concepts to engage with this unit: <ul style="list-style-type: none"> drawing angles, measuring angles, identifying angles Relevant Unit(s)/Lesson(s) to Review: <ul style="list-style-type: none"> None 	Builds understanding of unique triangle conditions needed for Grade 8 and high school congruence proofs.
Differentiation through Universal Design for Learning	
Engagement: <ul style="list-style-type: none"> Foster interest by providing choices in how to label and solve for unknown angle measures (Lesson 5, Activity 1 Launch) LT1: Solve multi-step problems involving complementary, supplementary, and vertical angles Representation: <ul style="list-style-type: none"> Use virtual or concrete manipulatives like cardboard strips and fasteners to associate symbols with side lengths (Lesson 10, Activity 1 Launch) LT3: Draw triangles with two given angle measures and one side length, one given angle measure and two side lengths, or three side lengths Action & Expression: <ul style="list-style-type: none"> Reduce barriers by providing access to digital applets for drawing cross-sections (Lesson 11, Activity 3 Launch) LT5: Calculate the surface area and volume of a prism 	
Supporting Multilingual Learners	
Math Language Routines	
The Illustrative Mathematics curriculum incorporates eight Mathematical Language Routines (MLRs) that support English Language Learners: <ul style="list-style-type: none"> MLR1: <i>Stronger and Clearer Each Time</i> - Students revise and refine their mathematical language through multiple drafts MLR2: <i>Collect and Display</i> - Students capture and organize language in visual displays MLR3: <i>Clarify, Critique, Correct</i> - Students analyze mathematical writing/talk MLR4: <i>Information Gap</i> - Students share information to solve problems MLR5: <i>Co-Craft Questions</i> - Students create and improve questions MLR6: <i>Three Reads</i> - Students analyze complex mathematical text MLR7: <i>Compare and Connect</i> - Students connect different mathematical representations MLR8: <i>Discussion Supports</i> - Students participate in mathematical discussions In this unit: <ul style="list-style-type: none"> MLR1: Stronger and Clearer Each Time (Lessons 10, 12, 15) MLR2: Collect and Display (Lessons 1, 2, 3, 11) MLR3: Critique, Correct, Clarify (Lessons 2, 8, 11, 12, 16) MLR4: Information Gap (Lesson 4) MLR5: Co-Craft Questions (Lesson 13) MLR6: Three Reads (Lessons 12, 14) MLR7: Compare and Connect (Lessons 6, 7, 13, 15, 16) MLR8: Discussion Supports (Lessons 2, 3, 4, 5, 7, 9, 11, 14, 15, 17) 	
Progression of Disciplinary Language	
In this unit, teachers can anticipate students using language for mathematical purposes such as critiquing, explaining, interpreting, and justifying. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:	
Critique <ul style="list-style-type: none"> reasoning about measuring angles (Lesson 1) reasoning about decomposition of prisms (Lesson 13) reasoning about surface area of prisms (Lesson 14) 	
Explain <ul style="list-style-type: none"> how to measure angles (Lesson 2) how to find unknown angle measurements (Lessons 4 and 5) how to find the volume of prisms (Lessons 12 and 13) how to find the surface area of prisms (Lesson 14) 	
Interpret <ul style="list-style-type: none"> situations involving intersecting lines in order to form a conjecture (Lesson 3) which information is relevant to answer questions (Lesson 4) equations representing angle measurements (Lesson 5) situations involving volume and surface area (Lesson 15 and 16) 	

Justify

- whether or not shapes are identical copies (Lesson 6)
- whether or not measurements determine identical copies (Lesson 9)
- whether or not measurements determine unique triangles (Lesson 10)

Sentence Frames and Stems

Section A

- Angle _____ (name e.g PQR) is an acute/obtuse angle because ...
- Angles _____ and _____ (names) are complementary/supplementary angles because ...
- Since angles _____ and _____ (names) are complementary/supplementary, the missing angle measure is _____ because ...
- Lines _____ and _____ (names) cross each other, so angles _____ and _____ (names) are _____ ...
- I used the equation _____ for the relationship between angles _____ and _____ (names) because ...

Section B

- Side lengths _____, _____ and _____ will/will not form a triangle because ...
- Two side lengths of the triangle are _____ and _____. The length of the third side of the triangle will be longer than _____ but shorter than _____ because ...
- Triangles _____ and _____ (names) are/ are not identical because ...
- I think that angles _____ and _____ (names) with side length _____ will/will not form a unique triangle because ...

Section C

- The shape of the cross section is _____ because...
- The base area of the prism is _____ and the height is _____, so the volume of the prism is _____. I calculated it by ...
- Figure _____ is/is not a prism because ...
- The surface area of the prism is _____. I calculated it by ...
- In this situation, I need to find _____ (e.g. volume, base, surface area) because ...

Section D

- The area of the triangle is _____.
- I chose this triangle because ...
- The volume/surface area of my prism is _____.

Unit Outline

In this unit, students investigate whether sets of angle and side length measurements determine unique triangles or multiple triangles, or fail to determine triangles. Students also study and apply angle relationships, learning to understand and use the terms “complementary,” “supplementary,” “vertical angles,” and “unique.” The work gives them practice working with rational numbers and equations for angle relationships. Students analyze and describe cross-sections of prisms, pyramids, and polyhedra. They understand and use the formula for the volume of a right rectangular prism and solve problems involving area, surface area, and volume. Students should have access to their geometry toolkits so that they have an opportunity to select and use appropriate tools strategically.

Note: It is not expected that students memorize which conditions result in a unique triangle, an impossible-to-create triangle, or multiple possible triangles. Understanding that, for example, side-side-side (SSS) information results in zero or exactly one triangle will be explored in high school geometry. At this level, students should attempt to draw triangles with the given information and notice that there is only one way to do it (or that it is impossible to do).

Lesson Sequence	Learning Target(s)	Success Criteria/Assessment
Section A Angle Relationships (Lessons 1-5)	Learning Target #1 Solve multi-step problems involving complementary, supplementary and vertical angles Learning Target #2 Write an equation to represent the relationship between angles and a given diagram	Lesson 1 Relationships of Angles <ul style="list-style-type: none"> • I can find unknown angle measurements by reasoning about adjacent angles with known measures • I can recognize when an angle measures 90°, 180°, or 360° Lesson 2 Adjacent Angles <ul style="list-style-type: none"> • I can find unknown angle measures by reasoning about complementary or supplementary angles • I can recognize when adjacent angles are complementary or supplementary Lesson 3 Nonadjacent Angles <ul style="list-style-type: none"> • I can determine if angles that are not adjacent are complimentary or supplementary • I can explain what vertical angles are in my own words Lesson 4 Solving for Unknown Angles <ul style="list-style-type: none"> • I can reason through multiple steps to find unknown angle measures • I can recognize when an equation represents a relationship between angle measures Lesson 5 Using Equations to Solve for Unknown Angles <ul style="list-style-type: none"> • I can write an equation to represent a relationship between angle measures and solve the equation to find unknown angle measures

Checkpoint A	<p><i>Responding to Student Thinking</i></p> <ul style="list-style-type: none"> ● Problem 1: Press Pause By this point in the unit, there should be some student mastery of representing angle relationships with equations. If most students struggle, make time to revisit related work in the lesson referred to here. 	
<p>Section B Drawing Polygons with Given Conditions (Lessons 6-10)</p>	<p>Learning Target #3 Draw triangles with two given angle measures and one side length, one given angle measure and two side lengths, or three side lengths.</p> <p>Learning Target #4 Justify whether 3 measures of angles or sides determine a unique triangle or more than one triangle, or if no triangle is possible.</p>	<p>Lesson 6 Building Polygons (Part 1)</p> <ul style="list-style-type: none"> ● I can show that the 3 side lengths that form a triangle cannot be rearranged to form a different triangle. ● I can show that the 4 side lengths that form a quadrilateral can be rearranged to form different quadrilaterals. <p>Lesson 7 Building Polygons (Part 2)</p> <ul style="list-style-type: none"> ● I can reason about a figure with an unknown angle. ● I can show whether or not 3 side lengths will make a triangle. <p>Lesson 8 Triangles with 3 Common Measures</p> <ul style="list-style-type: none"> ● I understand that changing which sides and angles are next to each other can make different triangles. <p>Lesson 9 Drawing Triangles (Part 1)</p> <ul style="list-style-type: none"> ● Given two angle measures and one side length, I can draw different triangles with these measurements or show that these measurements determine one unique triangle or no triangle. <p>Lesson 10 Drawing Triangles (Part 2)</p> <ul style="list-style-type: none"> ● Given two side lengths and one angle measure, I can draw different triangles with these measurements or show that these measurements determine one unique triangle or no triangle.
Checkpoint B	<p><i>Responding to Student Thinking</i></p> <ul style="list-style-type: none"> ● Problem 1: Press Pause By this point in the unit, there should be some student mastery of creating triangles with 3 given measures. If most students struggle, make time to revisit related work in the lesson referred to here. ● Problem 2: More Chances Students will have more opportunities to develop this understanding in later lessons. There is no need to slow down or add additional work to review this concept at this time. 	
<p>Section C Solid Geometry (Lessons 11-16)</p>	<p>Learning Target #5 Calculate the surface area and volume of a prism</p> <p>Learning Target #6 Decide whether to calculate the surface area or volume of a prism to solve a problem in a real-world situation</p>	<p>Lesson 11 Slicing Solids</p> <ul style="list-style-type: none"> ● I can explain that when a three-dimensional figure is sliced it creates a face that is two dimensional. ● I can picture different cross-sections of prisms and pyramids. <p>Lesson 12 Volume of Right Prisms</p> <ul style="list-style-type: none"> ● I can explain why the volume of a prism can be found by multiplying the area of the base by the height of the prism. <p>Lesson 13 Decomposing Bases for Area</p> <ul style="list-style-type: none"> ● I can calculate the volume of a prism with a complicated base by decomposing the base into quadrilaterals or triangles. <p>Lesson 14 Surface Area of Right Prisms</p> <ul style="list-style-type: none"> ● I can find and use shortcuts when calculating the surface area of a prism. ● I can picture the net of a prism to help me calculate its surface area. <p>Lesson 15 Distinguishing Volume and Surface Area</p> <ul style="list-style-type: none"> ● I can decide whether I need to find the surface area or the volume, when solving a problem about a real-world situation. <p>Lesson 16 Applying Volume and Surface Area</p> <ul style="list-style-type: none"> ● I can solve problems involving the volume and surface area of children's play structures.
Checkpoint C	<p><i>Responding to Student Thinking</i></p> <ul style="list-style-type: none"> ● Problem 1: More Chances Students will have more opportunities to develop this understanding in later lessons. There is no need to slow down or add additional work to review this concept at this time. ● Problem 2: Press Pause By this point in the unit, there should be some student mastery of finding surface area and volume of prisms. If most students struggle, make time to revisit related work in the lesson referred to here. 	
<p>End of Unit Assessment NOTES</p>		

#7 on EOU the line segment that is 1.5 ft long needs to be 2 ft long in order to break the shape up into rectangles and triangles. As currently written the 1.5 ft line would require students to find area of a trapezoid which they do not go into enough detail in the unit (we see it briefly during lesson 13 activity 1)

Unit Title:**Unit 8: Probability and Sampling****Relevant Standards: Bold indicates priority**

Lesson	Standards	Lesson	Standards
Lesson 1	7.SP.C.6	Lesson 11	7.SP.B, 7.SP.B.3
Lesson 2	7.SP.C.5	Lesson 12	7.SP.A.1, 7.SP.B
Lesson 3	7.SP.C.5, 7.SP.C.6, 7.SP.C.7, 7.SP.C.7.a	Lesson 13	7.SP.A, 7.SP.A.1, 7.SP.A.2
Lesson 4	7.RP.A, 7.SP.C.5, 7.SP.C.6, 7.SP.C.7, 7.SP.C.7.b	Lesson 14	7.SP.A.1, 7.SP.A.2, 7.SP.C.7
Lesson 5	7.SP.C.5, 7.SP.C.6, 7.SP.C.7, 7.SP.C.7.b	Lesson 15	7.SP.A.1, 7.SP.A.2, 7.SP.B.4
Lesson 6	7.SP.C, 7.SP.C.5, 7.SP.C.6, 7.SP.C.7.b, 7.SP.C.8.c	Lesson 16	7.NS.A.2.d, 7.RP.A, 7.SP.A, 7.SP.A.2, 7.SP.B.4
Lesson 7	7.RP.A, 7.SP.C.8.c	Lesson 17	7.SP.A, 7.SP.A.2
Lesson 8	7.SP.C.8.b	Lesson 18	7.SP.B.3, 7.SP.B.4
Lesson 9	7.SP.C.8.a, 7.SP.C.8.b	Lesson 19	7.SP.B.4
Lesson 10	7.SP.C.8.c	Lesson 20	7.RP.A, 7.SP.A, 7.SP.A.1, 7.SP.A.2, 7.SP.B.4, 7.SP.C.7.a

Essential Question(s):	Enduring Understanding(s):
<ul style="list-style-type: none"> How can we predict the likelihood of a future event? How can complex, multi-step events be modeled mathematically? How can a small sample be used to understand a much larger population? 	<ul style="list-style-type: none"> Probability is a number from 0 to 1 that describes the long-run relative frequency of an event, which can be estimated through repeated experiments or theoretical sample spaces Compound events can be modeled using tree diagrams, lists, or tables to track all possible outcomes and calculate probabilities A representative, random sample can provide valid inferences about a larger population, though results are subject to sampling variability

Demonstration of Learning:	Pacing for Unit
CFA 1: Checkpoint A (after Lesson 6) CFA 2: Checkpoint B (after Lesson 10) Unit 8 Mid-Unit Assessment CFA 3 Checkpoint C (after Lesson 14) CFA 4: Checkpoint D (after Lesson 19) Unit 8 End of Unit Assessment	24 Days Lessons to Add/Review: <ul style="list-style-type: none"> Combine 6.8.2 and 6.8.3: introduce variability and distributions 6.8.5: describe distributions using center and spread Combine 6.8.6 and 6.8.7: focus on interpreting histograms Combine 6.8.9 and 6.8.10: focus on understanding mean Combine 6.8.11 and 6.8.12: focus on understanding MAD Combine 6.8.13 and 6.8.14: focus on understanding median Combine 6.8.15 and 6.8.16: focus on box plots and interquartile range Lesson Modifications: <ul style="list-style-type: none"> Remove 7.8.1: simulation introduction that has analogs later in the unit Remove 7.8.5: additional practice that could be done outside of class Remove 7.8.11: reminder of mean and median that will be more fresh after doing 6.8 lessons Remove 7.8.16: inferring another measure from the population based on a sample Remove 7.8.17: an optional lesson examining how sample size affects the variability of sample means Remove 7.8.19: a lesson comprised primarily of an info gap that practices comparing populations

- Move to outside of class 7.8.20: culminating lesson incorporating work from the unit
- BPS Modifications:
- Lesson 17 and 20 have historically been removed

Family Overview

<https://accessim.org/6-8/grade-8/unit-6=8?a=family>

Integration of Technology:

- Desmos Online Graphing Calculator
- Pear Assessment (Edulastic)
- iM v.360 Digital Applets (see below)

Unit-specific Vocabulary:

Aligned Unit Materials, Resources, and Technology

New Terminology

Lesson	receptive	productive
7.8.1		more likely less likely
7.8.2	event chance experiment outcome equally likely as not	likely unlikely impossible certain
7.8.3	probability random sample space	outcome
7.8.5	simulation	probability random
7.8.7		event simulation
7.8.8	tree (diagram)	sample space
7.8.9		tree (diagram)
7.8.11	mean absolute deviation (MAD) distribution very different overlap	mean median dot plot
7.8.12	population sample survey	mean absolute deviation (MAD)
7.8.13	representative sample measure of center	distribution center (of a distribution) spread
7.8.14	random sample	
7.8.15	interquartile range (IQR) measure of variability box plot	population sample random sample symmetric
7.8.16	proportion	representative sample
7.8.17		interquartile range (IQR) measure of variability
7.8.18	meaningful difference	overlap measure of center
7.8.20		meaningful difference

Digital Applets

- 8.2 Take a Chance
- 8.4 In the Long Run
- 8.5 Making My Head Spin
- 8.7 Alpine Zoom
- 8.10 Breeding Mice
- 8.13 Sampling the Fish Market

Provide access as needed throughout the unit:

- Coins (any fair, two-sided coins)
- Compasses
- Four-function calculators
- Graph paper
- Math Community Chart
- Number cubes (cubes with sides numbered from 1- 6)
- Paper bags
- Paper clips
- Paper cups
- Protractors
- Clear protractors with no holes and with radial lines printed on them are recommended.
- Rulers marked with inches
- Scissors
- Snap cubes
- Sticky notes
- Straightedges

Lesson	Materials to Gather	Materials to Copy
1	<ul style="list-style-type: none"> • Paper bags: Activity 1 • Snap cubes: Activity 1 	
2	Number cubes: Activity 1, Activity 2	Likelihood Cards (1 copy for every 2 students): Activity 3
3	Paper bags: Activity 2	What's in the Bag Cutouts (1 copy for every 8 students): Activity 2
4	<ul style="list-style-type: none"> • Graph paper: Activity 1 • Number cubes: Activity 1 	
5	<ul style="list-style-type: none"> • Paper bags: Activity 1, Activity 2 • Paper cups: Activity 1 • Snap cubes: Activity 2 	Making My Head Spin Handout (1 copy for every 4 students): Activity 1
6	<ul style="list-style-type: none"> • Number cubes: Activity 1 • Paper bags: Activity 1 • Paper clips: Activity 1 	Diego's Walk Cards (1 copy for every 18 students): Activity 1
7	<ul style="list-style-type: none"> • Activity 1 • Paper bags: Activity 2 • Snap cubes: Activity 2 	Alpine Zoom Handout (1 copy for every 6 students): Activity 1
10	<ul style="list-style-type: none"> • Activity 1 • Compasses: Activity 2 	

	<ul style="list-style-type: none"> • Math Community Chart: Activity 2 • Number cubes: Activity 2 • Paper bags: Activity 2 • Paper clips: Activity 2 • Protractors: Activity 2 • Scissors: Activity 2 • Snap cubes: Activity 2 • Straightedges: Activity 2 	
13	Four-function calculators: Activity 1	
14	<ul style="list-style-type: none"> • Paper bags: Activity 2 • Rulers marked with inches: Activity 2 • Straws: Activity 2 	
16	Paper bags: Activity 1	Reaction Times Cutouts (1 copy for every 4 students): Activity 1
17	Sticky notes: Activity 1	
19	Math Community Chart: Activity 1	Comparing Populations Cards (1 copy for every 2 students): Activity 1
20	<ul style="list-style-type: none"> • Paper bags: Activity 1, Activity 2 • Number cubes: Activity 2 • Paper clips: Activity 2 	Collecting a Sample Handout (1 copy for every 2 students): Activity 1

Opportunities for Interdisciplinary Connections:	Anticipated misconceptions:
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<ul style="list-style-type: none"> • Marine Biology: Students estimate probabilities using data from crabbing experiments • Anthropology: Statistical variability is explored by comparing the chemical properties of steel tools from different geographical regions • Psychology: Students collect data using memory tests to understand population sampling 	<p>Sample Bias: Students may not recognize that a sample of only their friends is biased and does not represent the whole school population</p> <p>See teacher's guide for specific misconceptions aligned to each lesson.</p>
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Connections to Prior Units:	Connections to Future Units:
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<p>Essential prior concepts to engage with this unit:</p> <ul style="list-style-type: none"> • interpreting dot plots, histograms, and box plots • describing distributions using measures of center (mean and median) and measures of variability (mean absolute deviation and interquartile range) <p>Relevant Unit(s)/Lesson(s) to Review:</p> <ul style="list-style-type: none"> • Grade 6, Unit 8: Data Sets and Distributions 	<p>Students will build on this by designing and using simulations in high school statistics and probability units.</p>
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Differentiation through Universal Design for Learning
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<p>Engagement:</p> <ul style="list-style-type: none"> • Develop effort by chunking activities into manageable parts and providing frequent check-ins (Lesson 11, Activity 2 Launch) <p>LT7: Determine whether two populations are likely to be meaningfully different by reasoning about center and spread</p> <p>Representation:</p> <ul style="list-style-type: none"> • Use gestures to explain tree diagrams by tracing paths of branches to highlight outcomes (Lesson 8, Activity 1 Synthesis) <p>LT4: Interpret or create a list, table, or tree diagram that represents the sample space of a compound event</p> <p>Action & Expression:</p> <ul style="list-style-type: none"> • Provide access to tools like graphing calculators or spreadsheet software to calculate statistics efficiently (Lesson 13, Activity 1 Launch) • LT5: Describe methods to obtain a random sample from a population
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Supporting Multilingual Learners

Math Language Routines
The Illustrative Mathematics curriculum incorporates eight Mathematical Language Routines (MLRs) that support English

Language Learners:

- MLR1: *Stronger and Clearer Each Time* - Students revise and refine their mathematical language through multiple drafts
- MLR2: *Collect and Display* - Students capture and organize language in visual displays
- MLR3: *Clarify, Critique, Correct* - Students analyze mathematical writing/talk
- MLR4: *Information Gap* - Students share information to solve problems
- MLR5: *Co-Craft Questions* - Students create and improve questions
- MLR6: *Three Reads* - Students analyze complex mathematical text
- MLR7: *Compare and Connect* - Students connect different mathematical representations
- MLR8: *Discussion Supports* - Students participate in mathematical discussions

In this unit:

- MLR1: Stronger and Clearer Each Time (Lessons 3, 16, 18, 22)
- MLR2: Collect and Display (Lessons 2, 3, 8, 12, 13)
- MLR3: Critique, Correct, Clarify (Lessons 8, 16, 18)
- MLR4: Information Gap (Lesson 19)
- MLR5: Co-Craft Questions (Lessons 5, 7, 15, 18)
- MLR6: Three Reads (Lessons 3, 9, 15, 17)
- MLR7: Compare and Connect (Lessons 4, 8, 16, 17, 18, 20)
- MLR8: Discussion Supports (Lessons 1, 2, 3, 5, 10, 11, 14, 17)

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes such as describing, explaining, justifying, and comparing. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Describe

- observations and predictions during a game (Lesson 1)
- patterns observed in repeated experiments (Lesson 4)
- chance experiments to model situations (Lessons 6 and 7)
- a simulation used to model a situation (Lesson 10)
- observations about data sets (Lessons 11 and 17)

Explain

- predictions (Lesson 2)
- how to determine which events are more likely (Lesson 3)
- possible differences in experimental and theoretical probability (Lesson 5)
- how to use simulations to estimate probability (Lesson 7)
- how to use a simulation to answer questions about the situation (Lesson 10)

Justify

- whether situations are surprising and possible (Lesson 4)
- which samples are or are not representative of a larger population (Lesson 13)
- which samples correspond with each show, which show is most appropriate for a commercial, and whether a movie is eligible for an award (Lesson 15)
- reasoning about samples and populations (Lesson 16)
- whether or not differences between samples are meaningful (Lesson 18, 19, and 20)

Compare

- sample spaces and probably of outcomes for different spinners (Lesson 5)
- methods for writing sample spaces (Lesson 8)
- heights of two groups (Lesson 11)
- measures of center with samples (Lesson 13)
- sampling methods (Lesson 14)
- populations based on samples (Lessons 18 and 20)

Sentence Frames and Stems

Section A

- In this situation, the likelihood that _____ will occur is _____ because ...
- I used _____ to estimate the probability that _____ will occur is _____ because ...
- There are _____ possible outcomes in this experiment and the sample space is ...

Section B

- There are _____ possible outcomes in this experiment and the sample space is ...
- I used a _____ (method/tool) to represent the sample space because ...
- The probability of _____ is _____ because ...
- A simulation is useful to _____ when ...

Section C

- To obtain a random sample of _____, I _____. I chose this method because ...
- To compare _____ (population) and _____ (population), I found the _____ to determine that ...
- This sample is/is not representative of the population because ...
- To gather data about _____ (population), at least _____ should be sampled by _____.
- The shape of the data from the sample is _____ which might mean ...

Section D

- For this data set, the _____ is the most appropriate measure of center to describe _____ because ...
- I analyzed the data on a _____ (model) to determine the variability is _____.
- Using the data provided in a sample, I found the probability of _____ to be _____ because ...
- The difference between _____ (population) and _____ (population) can be described as ... because ...

Section E

- We chose to use the _____ to describe the center of the data because ...
- A random sample is important because ...

Unit Outline

In this unit, students work with probability and sampling. They use their understanding of basic chance experiments to quantify how likely events are to happen and develop a working understanding of probability. Then they design and use simulations to further understand probability as the frequency of the event occurring when repeating an experiment many times. Students represent sample spaces using tables, tree diagrams, and lists, and use the number of outcomes in a sample space to calculate an expected probability.



Next, students examine different ways to collect data from samples within a population to understand why random selection is useful. Then students generate samples and estimate information about the population from sample data. Finally, students compare two groups by examining the measures of center and measures of variability calculated from sample data representing each group.

Lesson Sequence	Learning Target(s)	Success Criteria/Assessment
<p>Section A Probabilities of Single Step Events (Lessons 1-6)</p>	<p>Learning Target #1 Use the results from a repeated experiment to estimate the probability of an event, and justify the estimate</p> <p>Learning Target #2 Use the sample space to determine the probability of an event, and express it as a fraction, decimal, or percentage</p>	<p>Lesson 1 Mystery Bags</p> <ul style="list-style-type: none"> • I can get an idea for the likelihood of an event by using results from previous experiments <p>Lesson 2 Chance Experiments</p> <ul style="list-style-type: none"> • I can describe the likelihood of events using the words “impossible,” “unlikely,” “equally likely as not,” “likely” and “certain” • I can tell which event is more likely when the chances of different events are expressed as fractions, decimals, or percentages <p>Lesson 3 What are Probabilities?</p> <ul style="list-style-type: none"> • I can use the sample space to calculate the probability of an event when all outcomes are equally as likely • I can write out the sample space for a simple chance experiment <p>Lesson 4 Estimating Probabilities Through Repeated Experiments</p> <ul style="list-style-type: none"> • I can estimate the probability of an event based on the results from repeating an experiment • I can explain whether certain results from repeated experiments would be surprising or not <p>Lesson 5 More Estimating Probabilities</p> <ul style="list-style-type: none"> • I can calculate the probability of an event when the outcomes in the sample space are not equally likely • I can explain why results from a repeating experiment may not exactly match the expected probability for an event <p>Lesson 6 Estimating Probabilities Using Simulation</p> <ul style="list-style-type: none"> • I can simulate a real world situation using a simple experiment that reflects the probability of an actual event
<p>Checkpoint A</p>	<p><i>Responding to Student Thinking</i></p>	

	<ul style="list-style-type: none"> ● Problem #1: More Chances Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons. ● Problem #2: More Chances Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons. 	
Section B Probabilities of Multi-step Events (Lessons 7-10)	Learning Target #3 Describe a multi-step experiment that could be used to simulate a compound event in a real-world situation, and justify that it represents the situation.	Lesson 7 Simulating Multi-step Experiments <ul style="list-style-type: none"> ● I can use a simulation to estimate the probability of a multi-step event.
	Learning Target #4 Interpret or create a list, table, or tree diagram that represents the sample space of a compound event, and use the sample space to write the probability for an event.	Lesson 8 Keeping Track of All Possible Outcomes <ul style="list-style-type: none"> ● I can write out the sample space for a multi-step experiment, using a list, table, or tree diagram. Lesson 9 Multi-step Experiments <ul style="list-style-type: none"> ● I can use the sample space to calculate the probability of an event in a multi-step experiment. Lesson 10 Designing Simulations <ul style="list-style-type: none"> ● I can design a simulation to estimate the probability of a multi-step real-world situation.
Mid Unit Assessment		
Checkpoint B	Responding to Student Thinking <ul style="list-style-type: none"> ● Problem #1: Press Pause By this point in the unit, there should be some student mastery of finding probability. If most students struggle, make time to revisit related work in the lesson referred to here. ● Problem #2: Press Pause By this point in the unit, there should be some student mastery of designing simulations to estimate probability. If most students struggle, make time to revisit related work in the lesson referred to here. 	
Section C Sampling (Lessons 11-14)	Learning Target #5 Describe methods to obtain a random sample from a population, and explain why it is representative of the population.	Lesson 11 Comparing Groups <ul style="list-style-type: none"> ● I can calculate the difference between two means as a multiple of the mean absolute deviation. ● When looking at a pair of dot plots, I can determine whether the distributions are very different or have a lot of overlap.
	Learning Target #6 Explain why samples are necessary and describe a sample and population for a given statistical question.	Lesson 12 Larger Populations <ul style="list-style-type: none"> ● I can explain why it may be useful to gather data on a sample of a population. ● When I read or hear a statistical question, I can name the population of interest and give an example of a sample for that population. Lesson 13 What Makes A Good Sample? <ul style="list-style-type: none"> ● I can determine whether a sample is representative of a population by considering the shape, center, and spread of each of them. ● I know that some samples may represent the population better than others. ● I remember that when a distribution is not symmetric, the median is a better estimate of a typical value than the mean. Lesson 14 Sampling in a Fair Way <ul style="list-style-type: none"> ● I can describe ways to get a random sample from a population. ● I know that selecting a sample at random is usually a good way to get a representative sample.
Checkpoint C	Responding to Student Thinking <ul style="list-style-type: none"> ● Problem #1: Points to Emphasize If students struggle to identify populations and samples, emphasize the concepts as they come up later in the unit. For example, as students work with samples in the practice problem referred to here, ask them to identify the population and how a sample might have been collected. ● Problem #2: Points to Emphasize If students struggle to explain that random sampling is useful for obtaining representative samples, emphasize the idea as students work through the next section. For example, in the activity referred to here, as students analyze data from a sample to estimate information about the population, ask them why it is important that the sample was collected using a random process. 	
Section D	Learning Target #7	Lesson 15 Estimating Populations Measures of Center

<p>Using Samples (Lessons 15-19)</p>	<p>Determine whether two populations are likely to be meaningfully different by reasoning about center and spread.</p> <p>Learning Target #8 Use the proportion of a random sample that is within a certain category to make inferences about the population, and explain the reasoning.</p>	<ul style="list-style-type: none"> I can consider the variability of a sample to get an idea for how accurate my estimate is. I can estimate the mean or median of a population based on a sample of the population. <p>Lesson 16 Estimating Population Proportions</p> <ul style="list-style-type: none"> I can estimate the proportion of population data that are in a certain category based on a sample. <p>Lesson 17 More About Sampling Variability</p> <ul style="list-style-type: none"> I can use the means from many samples to judge how accurate an estimate for the population mean is. I know that as the sample size gets bigger, the sample mean is more likely to be close to the population mean. <p>Lesson 18 Comparing Populations Using Samples</p> <ul style="list-style-type: none"> I can calculate the difference between two medians as a multiple of the interquartile range. I can determine whether there is a meaningful difference between two populations based on a sample from each population. <p>Lesson 19 Comparing Populations with Friends</p> <ul style="list-style-type: none"> I can decide what information I need to know to be able to compare two populations based on a sample from each.
<p>Checkpoint D</p>	<p>Responding to Student Thinking</p> <ul style="list-style-type: none"> Problem #1: Press Pause By this point in the unit, there should be some student mastery of finding and interpreting proportions. If most students struggle, make time to revisit related work in the lesson referred to here. Problem #2: Press Pause By this point in the unit, there should be some student mastery of comparing populations from samples. If most students struggle, make time to revisit related work in the lesson referred to here 	
<p>End of Unit Assessment</p>		