Pre-Calculus
Content Standards Revision
2022

Course Title:	Pre-Calculus
Course/Unit Credit:	1
Course Number:	433000
Teacher Licensure:	Please refer to the Course Code Management System (https://adedata.arkansas.gov/ccms/) for the most current licensure codes.
Grades:	9-12
Prerequisites:	Algebra I, Geometry, Algebra II

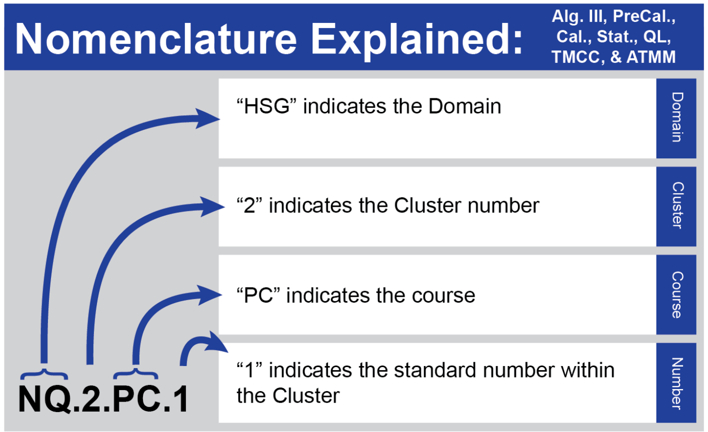
Course Description: Pre-Calculus will emphasize a study of trigonometric functions and identities as well as applications of right triangle trigonometry and periodic functions. Students will use symbolic reasoning and analytical methods to represent mathematical situations, express generalizations, and study mathematical concepts and the relationships among them. Students will use functions and equations as tools for expressing generalizations. Pre-Calculus does not require Arkansas Department of Education approval.

Introduction to Secondary Arkansas Mathematics Standards

When the Division of Elementary and Secondary Education (DESE) began the process of revising math standards, a diverse group of qualified educators from across the state came together to craft Arkansas standards specific to the schools and students in the state. The result of this work, the Arkansas Mathematics Standards, is contained in this document. These standards reflect what educators across the state know to be best for Arkansas students.

Standards Organization: The revision committee maintained the organizational structure and nomenclature of the previous standards. Secondary Arkansas Mathematics Standards are categorized into domains, clusters, and standards.

- **Domains** represent the big ideas to be studied in each course and sometimes across grade bands. These big ideas support educators in determining the proper amount of focus and instructional time to be given to each of these topics.
- **Clusters** represent collections of standards grouped to help educators understand the building blocks of rich and meaningful instructional units. These units help students make connections within clusters and avoid seeing mathematics as a discrete list of skills they must master.
- **Standards** represent the foundational building blocks of math instruction. The standards outlined in this document work together to ensure that students are college and career ready and on track for success.



Standards Support: The revision of the Arkansas Mathematics Standards represent the work of the committee to provide greater clarity, strength, and support of the standards. Additionally, the revised mathematics standards are designed to help educators better understand the areas of emphasis and the focus within the standards. Educators should address the bulleted content as more than a checklist of items that they must teach individually. Content is bulleted to provide specificity of learning expectations included within some extensive standards. In some instances, the standard document includes Arkansas examples, teacher notes, specifications, and italicized words to assist educators with planning, teaching, and student learning.

- **Examples** included in the original standards were either changed for clarity or separated from the body of the actual standard. The examples included in the body of the standards document in no way reflect all of the possible examples. Likewise, these examples do not mandate curriculum or problem types. Local districts are free to select the high-quality curricula and instructional methods that best meet the needs of their students.
- **Teacher notes** offer clarification of the standards. These notes are intended to clarify, for teachers, what the expectations are for the learner. Likewise, these notes provide instructional guidance and limitations so that educators can better understand the scope of the standard. This will help with determining what is developmentally appropriate for students when working with specific standards.
- **Standard specifications** are to strengthen standards. The specifications are precise statements highlighting the need for mastery or function-type parameters for specific standards. This will assist educators in pinpointing the best opportunities for students to gain and master the knowledge and skills needed to succeed in a progression.
- **Asterisks (*)** are denoted to represent the modeling component of the standards. These standards should be presented in a modeling context which allows students to engage in the modeling process that is outlined in the Standards for Mathematical Process. (See Appendix A)
- **Italicized words** are defined in the glossary.

Finally, the Arkansas Mathematics Standards will be a living document. As these standards are implemented across schools in the state, DESE welcomes further suggestions related to notes of clarification, examples, professional development needs, and future revisions of the standards.

K - 12 Standards for Mathematical Practices	
1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics.*	5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Pre-Calculus Standards: Overview	
Abbreviations: The following abbreviations are for the domains for the Arkansas Mathematics Standards.	
Number and Quantity - NQ	
	1. Students will perform operations with complex numbers
	2. Students will perform operations with vectors and use those skills to solve problems.
Trigonometry - T	
	3. Students will develop and apply the definitions of the six trigonometric functions and use the definitions to solve problems and verify identities.
	4. Students will solve trigonometric equations and sketch the graph of periodic trigonometric functions.
Conic Sections - CS	
	5. Students will identify, analyze, and sketch the graphs of the conic sections and relate their equations and graphs.
Functions - F	
	6. Students will be able to perform operations on functions to build new functions from existing functions.
	7. Students will be able to interpret different types of functions and their key characteristics including polynomial, exponential, logarithmic, power, trigonometric, rational, and other types of functions.

Number and Quantity	
Cluster 1: Students will perform operations with complex numbers	
NQ.1.PC.1	Perform the following operations. ● Find the conjugate of a complex number. ● Use conjugates to find quotients of complex numbers
Cluster 2: Students will perform operations with vectors and use those skills to solve problems.	
NQ.2.PC.1	Understand and represent with vector quantities. ● Recognize vector quantities as having both magnitude and direction. ● Represent vector quantities by directed line segments, and use appropriate symbols for *vectors* and their magnitudes (e.g. vector, ***v***; magnitude ||***v***|| or |***v***| and *scalar* multiple, *c*).
NQ.2.PC.2	Find the components of a *vector* by subtracting the coordinates of an initial point from the coordinates of a terminal point.
NQ.2.PC.3	Solve problems involving velocity and other quantities that can be represented by *vectors.*
NQ.2.PC.4	Add and subtract *vectors*. ● Add *vectors* end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two *vectors* is typically not the sum of the magnitudes. ● Given two *vectors* in magnitude and direction form, determine the magnitude and direction of their sum. ● Understand *vector* subtraction ***v*** - ***w*** as ***v*** + (-***w***), where -***w*** is the additive inverse of ***w***, with the same magnitude as ***w*** and pointing in the opposite direction. ● Represent *vector* subtraction graphically by connecting the tips in the appropriate order. ● Perform *vector* subtraction component-wise.
NQ.2.PC.5	Multiply a *vector* by a *scalar*. ● Represent *scalar* multiplication graphically by scaling *vectors* and possibly reversing their direction. ● Perform *scalar* multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$. ● Compute the magnitude of a *scalar* multiple ***cv*** using ||*c**v***|| = |*c*|***v***. ● Compute the direction of cv knowing that when |*c*|***v*** ≠ 0, the direction of *c**v*** is either along ***v*** (for c > 0) or against ***v*** (for c < 0).

Trigonometry	
Cluster 3: Students will develop and apply the definitions of the six trigonometric functions and use the definitions to solve problems and verify identities.	
T.3.PC.1	Understand *radian* measure of an angle as the length of the arc on the unit circle subtended by the angle. Teacher Note: Students are introduced to the idea of a *radian* in geometry. Teachers may need to reinforce converting between *radian* to degree measure.
T.3.PC.2	Explain how the unit circle in the coordinate plane enables the extension of *trigonometric functions* to all real numbers, interpreted as *radian* measures of angles traversed around the unit circle (e.g., reference angles, coterminal angles).
T.3.PC.3	Extend the domain of trigonometric functions using the unit circle. ● Use special right triangles to determine geometrically the exact values of sine, cosine, tangent for $\frac{\pi}{3}, \frac{\pi}{4}, \frac{\pi}{6}, and\ \frac{\pi}{2}$. ● Use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their exact values for x, where x is one of these values: $\frac{\pi}{3}, \frac{\pi}{4}, \frac{\pi}{6}, and\ \frac{\pi}{2}$.
T.3.PC.4	Develop and apply Pythagorean identity. ● Develop the Pythagorean identity, $sin^2(\theta) + cos^2(\theta) = 1$. ● Given $sin(\theta)$, $cos(\theta)$, or $tan(\theta)$ and the quadrant of the angle, use the Pythagorean identity to find the remaining five *trigonometric functions*. Teacher Note: Algebraic methods may be used to obtain alternate versions. ● $1 + cot^2(\theta) = csc^2(\theta)$ ● $tan^2(\theta) + 1 = sec^2(\theta)$
T.3.PC.5	Develop addition, subtraction, double, and half-angle formulas for sine, cosine, and tangent and use them to solve problems including verifying other identities.
T.3.PC.6	Derive the formula $A = \left(\frac{1}{2}\right)ab\ sin\ C$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
T.3.PC.7	Prove the *Law of Sines* and the *Law of Cosines* and use them to solve problems.

T.3.PC.8	Understand and apply the *Law of Sines* and the *Law of Cosines* to find unknown measurements in right and non-right triangles. Teacher Note: Examples should include but are not limited to surveying problems and problems related to resultant forces.
T.3.PC.9	Define and use reciprocal functions, cosecant, secant, and cotangent to solve problems.
Cluster 4: Students will solve trigonometric equations and sketch the graph of periodic trigonometric functions.	
T.4.PC.1	Use the unit circle to explain symmetry (odd and even) and *periodicity* of the graph of *trigonometric functions.*
T.4.PC.2*	Choose *trigonometric functions* to model *periodic* phenomena with specified *amplitude*, frequency, and *midline.*
T.4.PC.3	Understand that restricting a *trigonometric function* to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. Teacher Note: Recognizing that the domain requires restriction because the function is not one-to-one, is acceptable for Algebra 2. Whereas, knowledge of how to restrict the domain and find the inverse is usually reserved for a fourth year mathematics course.
T.4.PC.4*	Use *inverse functions* to: ● Solve trigonometric equations that arise in modeling context(s). ● Evaluate the solutions of trigonometric equations, with or without technology. ● Interpret the solutions of trigonometric equations in terms of the context(s).
T.4.PC.5	Recognize that some trigonometric equations have infinitely many solutions and be able to state a general formula to represent the infinite solutions.

Conic Sections	
Cluster 5: Students will identify, analyze, and sketch the graphs of the conic sections and relate their equations and graphs.	
CS.5.PC.1	Examine the equation of a circle. ● Derive the equation of a circle given the center and radius using the Pythagorean Theorem. ● Complete the square to find the center and radius of a circle given by an equation. Teacher Note: Students should also be able to identify the center and radius when given the equation of a circle and write the equation given a center and radius.

CS.5.PC.2	Derive the equation of a parabola given a focus and directrix.
CS.5.PC.3	Derive the equations of ellipses and hyperbolas given the foci; use the fact that the sum or difference of distances from the foci is constant.
CS.5.PC.4	Find the equations for the *asymptotes* of a hyperbola.
CS.5.PC.5	Complete the square in order to generate an equivalent form of an equation for any conic section; use that equivalent form to identify key characteristics of the conic section.
CS.5.PC.6	Identify, graph, write, and analyze equations of each type of conic section; use properties such as symmetry, intercepts, foci, *asymptotes*, and *eccentricity*, and using technology when appropriate.
CS.5.PC.7	Solve systems of equations and inequalities involving conics and other types of equations, with and without appropriate technology (including but not limited to conic-conic and conic-linear).

Functions	
Cluster 6: Students will be able to perform operations on functions to build new functions from existing functions.	
F.6.PC.1*	Write a function that describes a relationship between two quantities. ● From a context, determine an explicit expression, a recursive process, or steps for calculation. ● Combine standard function types using arithmetic operations (e.g., given that $f(x)$ and $g(x)$ are functions developed from a context, find $(f + g)(x)$, $(f - g)(x)$, $(fg)(x)$, $\left(\frac{f}{g}\right)(x)$, and any combination thereof, given $g(x) \neq 0$). ● Compose functions.
F.6.PC.2	Find *inverse functions*. ● Solve an equation of the form $y = f(x)$ for a simple function f that has an inverse and write an expression for the inverse. ○ For example, $f(x) = 2x^2$ or $f(x) = \frac{x+1}{x-1}$ for $x \neq 1$ ● Verify by composition that one function is the inverse of another. ● Read values of an inverse function from a graph or a table, given that the function has an inverse. ● Produce an invertible function from a non-invertible function by restricting the domain.
F.6.PC.3	Work with exponential functions and their inverses. ● Understand the inverse relationship between exponents and logarithms. ● Use the inverse relationship between exponents and logarithms to solve problems.

Cluster 7: Students will be able to interpret different types of functions and their key characteristics including polynomial, exponential, logarithmic, power, trigonometric, rational, and other types of functions.	
F.7.PC.1	Understand that sequences are functions, sometimes defined recursively, whose domains are a subset of the integers. Teacher Note: Example: The Fibonacci sequence is defined recursively by $(0) = (1) = 1$, $(n + 1) = (n) + (n - 1)$ for $n \geq 1$.
F.7.PC.2	Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems Teacher Note: Example: Calculate mortgage payments.
F.7.PC.3	Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined (e.g., Pascal's Triangle). Teacher Note: The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.
F.7.PC.4	For a function that models a relationship between two quantities. ● Interpret key features of graphs and tables in terms of the quantities. ● Sketch graphs showing key features given a verbal description of the relationship. Teacher Note: ● Key features may include but not limited to: domain and range; intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and *periodicity*. ● Connection: This standard is closely related to standard F.7.PC.6.
F.7.PC.5*	Use functions in problem-solving to: ● Calculate and interpret the average rate of change of a function (presented algebraically or as a table) over a specified interval. ● Estimate the rate of change from a graph.
F.7.PC.6	Graph functions expressed algebraically and show key features of the graph, with and without technology. ● Graph linear and quadratic functions and show intercepts, maxima, and minima, when applicable. ● Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

	● Graph power and polynomial functions, identify zeros when suitable factorizations are available, and show end behavior. ● Graph rational functions, identify zeros and *asymptotes* when suitable factorizations are available, and show end behavior. ● Graph *exponential* and *logarithmic function*s, and show intercepts and end behavior. ● Graph *trigonometric functions*, and show *period*, *midline*, and *amplitude.*
F.7.PC.7	Compare properties (key features) of two functions each represented in a different way (e.g., algebraically, graphically, numerically in tables, verbal descriptions).
F.7.PC.8*	Build functions to model real-world applications using algebraic operations on functions and composition, with and without appropriate technology (e.g., profit functions as well as volume and surface area, optimization subject to constraints).

Functions	**Students will be able to interpret different types of functions and their key characteristics including polynomial, exponential, logarithmic, power, trigonometric, rational, and other types of functions.**
F.7.PC.1	Understand that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers For example: The Fibonacci sequence is defined recursively by $(0) = (1) = 1$, $(n + 1) = (n) + (n - 1)$ for $n \geq 1$.
F.7.PC.2	Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems For example: Calculate mortgage payments.
F.7.PC.3	Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle Note: The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.

<table>
<tr><td>F.7.PC.4</td><td>For a function that models a relationship between two quantities:
Interpret key features of graphs and tables in terms of the quantities, and
Sketch graphs showing key features given a verbal description of the relationship
Note: Key features may include but not limited to: domain and range; intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

Connection: This standard is closely related to standard F.7.PC.6.</td></tr>
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Graph linear and quadratic functions and, when applicable, show intercepts, maxima, and minima
Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions
Graph power and polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior
Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior
Graph exponential and logarithmic functions, showing intercepts and end behavior
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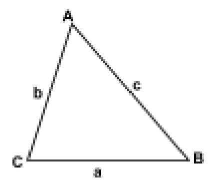
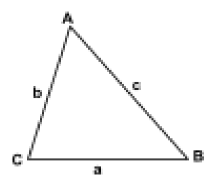
</table>

Glossary

Amplitude	Half the difference between the minimum and maximum values of the range; only periodic functions with a bounded range have an amplitude.
Asymptote(s)	Line(s) to which a graph becomes arbitrarily close as the value of x or y increases or decreases without bound (e.g., vertical, horizontal, slant)
Eccentricity	A number that indicates how drawn out or attenuated a conic section is; eccentricity is represented by the letter e (no relation to $e = 2.718...$)
Exponential Function(s)	Function(s) in which the variable(s) occurs in the exponent [e.g., $f(x) = ab^{x}, b > 0$]
Inverse Function(s)	Two functions f and g are inverse functions, if and only if both their compositions yield the identity function {e.g., $[f \circ g](x) = x$ and $[g \circ f](x) = x$}
Law of Cosines	An equation relating the cosine of an interior angle and the lengths of the sides of a triangle; the Pythagorean theorem is a corollary of the Law of Cosines $c^2 = a^2 + b^2 - 2ab \cos C$ $b^2 = a^2 + c^2 - 2ac \cos B$ $a^2 = b^2 + c^2 - 2bc \cos A$ 
Law of Sines	Equations relating the sines of the interior angles of a triangle and the corresponding opposite sides $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ 
Logarithmic Functions	Function of the form $y = \log_b x$, where $b > 0$, $x > 0$ and $b \neq 1$
Midline	The horizontal line that a sine or cosine graph oscillates above and below. The midline is halfway between the maximum and minimum of a sine or cosine graph.
Period	The length of one cycle of a periodic graph.

Periodic/Periodicity	The behavior of a function to periodically repeat values over a fixed interval.
Radian	A unit of measure for angles; the angle made at the center of a circle by an arc whose length is equal to the radius of the circle has a measure of 1 radian
Scalar	Any real number, or any quantity that can be measured using a single real number; temperature, length, and mass are all scalars; a scalar is said to have magnitude but no direction
Trigonometric Function(s)	The six functions are sine, cosine, tangent, cosecant, secant, and cotangent
Vector(s)	Quantity or quantities with magnitude and direction in the plane or in space, defined by an ordered pair or triple of real numbers

Appendix A.

Mathematical Modeling Cycle

The basic modeling cycle is summarized in this diagram. It involves: (1) identifying variables in the situation and selecting those that represent essential features; (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables; (3) analyzing and performing operations on these relationships to draw conclusions; (4) interpreting the results of the mathematics in terms of the original situation; (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable; (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.