Mathematics

January 2020

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Math update

- Curriculum documents
- Problem solving across the school (including Exemplars)
- Collaboration and coaching between Math coordinators and teachers
- Professional development:
  - Work with outside math consultants
  - Off site professional development
- Math Workshop

Pacing Guide

	Grade 2	Grade 5
Unit 1	Addition and Subtraction	Multiplication, Division and Order of Operations
Unit 2	Geometry	Decimals and Measurement
Unit 3	Place Value	Addition and subtraction of fractions
Unit 4	Using Place Value Understanding to Add	Multiplication and division of fractions
Unit 5	Time and Money	Measurement Part 2
Unit 6	Using Place Value Understanding to Subtract	Volume
Unit 7	Measurement and Data	Geometry
Unit 8		Logical and Algebraic Reasoning

Curriculum document

WOODBRIDGE SCHOOL DISTRICT MATH UNIT ORGANIZER

Grade: 3
Unit Title: Unit 2 Understanding Multiplication and Division
Overview of Unit: Multiplication, Division, Area concepts
Pacing: 10 Weeks

Background Information For The Teacher
Rationale: Third grade students explore the meaning of multiplication as finding the total number of objects when they know the number of groups and the number of items in each group. The relationship between multiplication and division helps students understand that when dividing, they are finding the number of groups when they know the total count or the number of items in a group. Problem solving situations and activities that include a variety of representations showing equal-sized groups, arrays, and area models lay the foundation for multiplication and division of whole numbers. **Key Learning** **Changes from Past Practice:** Area has been added to this unit. Students will connect the concept of area of multiplication by decomposing rectangles into rectangular arrays of squares.

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Essential Questions (and Corresponding Big Ideas)

Essential Questions (Corresponding Big Ideas)

1. How do we show the meaning of multiplication and division of whole numbers? Why is it important?
    - We can show the meaning of multiplication and division through models, arrays, drawings, pictures, and equations with a symbol for the unknown number.
2. How can we use multiplication and division in our daily lives?
    - By understanding multiplication and division, we can apply multiplication and division to … (students complete through lesson activities).
3. How do we use various strategies to multiply and divide fluently?
    - Flexible methods of computation involve taking apart and combining numbers in a wide variety of ways. Most of the partitions of numbers are based on place value. Flexible methods of computation requires a good understanding of the operations and properties of the operations, especially the commutative property, associative and the distributive property for multiplication. How the operations are related - addition to subtraction, addition to multiplication, subtraction to division, multiplication to division - is also an important ingredient (Van de Walle, 2001).
4. How are multiplication and division related?
    - We can use multiplication facts to help solve division equations.
    - We can use fact families to solve equations involving multiplication and division.
5. What is area? How do we apply it to real life situations?
    - Area is a measurement of the amount of flat space an object covers. The area of shapes can be found by covering without gaps or overlaps, by counting squares, tiling, and decomposing into non-overlapping rectangles. We measure the area of … (students will answer).

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Correlating Standard in Previous Year	Number Sequence & Standard	Correlating Standard in Following Year
2.OA.4: Work with equal groups of objects to gain foundations for multiplication. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.	3.OA.1 Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objects can be expressed as 5 × 7.	4.OA.1: Use the four operations with whole numbers to solve problems Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 x 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations

Correlating Standard in Previous Year	Number Sequence & Standard	Correlating Standard in Following Year
no correlated standard	3.OA.2 Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.	4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

Correlating Standard in Previous Year	Number Sequence & Standard	Correlating Standard in Following Year
2.OA.1 Use addition and subtraction within 100 to solve one-and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing with unknowns in all positions, e.g. by using drawings and equations with a symbol for the unknown number to represent the problem. 2.OA.4 Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.	3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.	4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. 4.OA.3 Solve multi-step word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a

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Core Content Standards	Explanations and Examples (Developed by Arizona DOE)

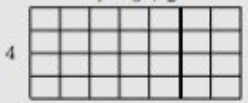
3.OA.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.

Decomposing 4×7

$$\begin{aligned} 4 \times 7 &= 4 \times (5+2) \\ &= (4 \times 5) + (4 \times 2) \\ &= 20 + 8 \\ &= 28 \end{aligned}$$

Supporting reasoning with area diagram

$7 = 5 + 2$



$20 + 8 = 28$

$4 \times 5 + 4 \times 2 = 4 \times 7$

The $5 + n$ pattern for multiplying the numbers 4, 6, and 8

n	$5+n$	$4 \times n$		$6 \times n$		$8 \times n$	
1	5 + 1	4	24	6	36	8	48
2	5 + 2	8	28	12	42	16	56
3	5 + 3	12	32	18	48	24	64
4	5 + 4	16	36	24	54	32	72
5	5 + 5	20	40	30	60	40	80

Patterns in multiples of 9

$1 \times 9 = 9$

$2 \times 9 = 2 \times (10-1) = (2 \times 10) - (2 \times 1) = 20 - 2 = 18$

$3 \times 9 = 3 \times (10-1) = (3 \times 10) - (3 \times 1) = 30 - 3 = 27,$ etc

Standards for Mathematical Practice	Explanations and Examples
1. Make sense of problems and persevere in solving them	In third grade, mathematically proficient students know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Third grade students may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" Students listen to other students' strategies and are able to make connections between various methods for a given problem.
2. Reason abstractly and quantitatively	Mathematically proficient third grade students should recognize that a number represents a specific quantity. They connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities.
3. Construct viable arguments and critique the reasoning of others	In third grade, mathematically proficient students may construct arguments using concrete referents, such as objects, pictures, and drawings. They refine their mathematical communication skills as they participate in mathematical discussions that the teacher facilities by asking questions such as "How did you get that?" and "Why is that true?" They explain their thinking to others and respond to others' thinking.
4. Model with mathematics	Mathematically proficient students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart, list, or graph, creating equations, etc. Students require extensive opportunities to generate various mathematical representations and to both equations and story problems, and explain connections between representations as well as between representations and equations. Students should be able to use all of these representations as needed. They should evaluate their results in the context of the situation and reflect on whether the results make sense.
5. Use appropriate tools strategically	Mathematically proficient third grader students consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use graph paper to find all the possible rectangles that have a given perimeter. They compile the possibilities into an organized list or a table, and determine whether they have all the possible rectangles.

K-U-D

KNOW
Facts, formulas, information, vocabulary

Students will **know:**

- **Multiplication** is when objects are arranged in **equal groups**
- Multiplication is repeated addition
- **Division i**s repeated subtraction
- Division and multiplication are **inverse operations**
- Division is partitioning into **equal shares**
- Each multiplication and division situation involves three quantities each of which can be unknown.
- We can use strategies such as patterns, properties of operations, and **fact families**
- Numbers can be broken apart in various ways.
- One number (**factor**) represents the number of groups and the other number (factor)represents the size of each group.
- Know from memory all products of one-digit numbers
- When multiplying by a **multiple** of 10 the product is 10 times larger. Example: 3 x 50 = 3 x (5 x10) ; 3 x (5 x 10) = (3 x 5) x 10; (3 x 5) x 10 = 15 x 10
- **Area** is the amount of **square units** covering a **plane figure**.
- You can use the distributive property to find the area of a rectangular shape.
- Area is additive. Example: 5 x 7= (5x5) + (5x2)

DO
Skills of the discipline

Students will **be able to:**

- Represent an **equation** visually
- Look at a picture and write the equation
- Write a story problem to match an equation (context)
- Use a symbol for the **unknown number**
- Determine the unknown using models and equations
- Apply properties of operations to multiplication and division
- Solve equations and story problems using various strategies
- Use the strategy that works best for you to fluently multiply and divide within 100.
- Solve two-step word problems using the four operations.
- Demonstrate with models/pictures/skip counting the place value shift when multiplying by multiples of 10
- Determine that area is length times width using various strategies
- Count square units to measure the area of a plane figure
- Decompose a shape into smaller rectangles to find the area
- Solve real-world and mathematical problems of rectangles with the same area and different perimeters or the same perimeter and different areas

UNDERSTAND
Big ideas, generalizations, principles, concepts, ideas that transfer across situations

Students will **understand**:

- Properties of operations and the relationship between multiplication and division
- What operations are required to solve word problems
- The context of real world problems as they relate to multiplication and division
- There is a relationship between multiplication and addition as well as division and subtraction
- There are a variety of strategies to fluently multiply and divide
- Perimeter is a boundary while area covers a plane figure
- The area is a measurement of covering a polygon

Common Student Misconceptions for this Unit
• Teaching shortcuts (adding a zero to the product of the two non-zero whole numbers) rather than understanding the relationship between the product and its place value does not establish the underlying importance of place value in multiplication. Understanding that multiplying 4 x 30 means I have 4 groups of 3 tens and that is 12 tens or 120 (rather than multiply 4 x 3 and "add a zero at the end" is fundamental to ongoing work with multiplication and working with partial products. Students who recognize and can explain a pattern rather than following a rule begin to understand the structure of multiplication rather than a meaningless shortcut.

Assessment	
Performance Assessment (Overview only; full task in separate document)	**Other** (Quizzes, tests, etc.)
	Letter home about multiplication and division facts Grade 3 multiplication fact fluency assessment Grade 3 division fact fluency assessment Grade 3 Unit 2 pre assessment EL Unit 2 Assessment Unit 2 Assessment

Vocabulary

area, array, associative property (familiarize), commutative property (familiarize), distributive property (familiarize), division, equal groups, equal shares, equation, fact families, factors, inverse operations, length, models, multiples, multiplication, operations, patterns, place value, plane figure, product, quotient, repeated addition, repeated subtraction, square unit, unknown number, width

Math Vocabulary Cards. Retrieved from: http://www.graniteschools.org/mathvocabulary/vocabulary-cards/ Vocabulary cards aligned to the Common Core Standards for Mathematics K-12, in English, French, Spanish and Chinese. Each card features a vocabulary word three ways: word, word with image, word with image and definition.

Literature/Literacy Connections

What Comes in 2's, 3's, and 4's by Aker, S.
Cloudy with a Chance of Meatballs by Barrett, J.

Suggested Learning Plan	
In order to learn: (learnings identified in KUD)	**Students might:** (one or more learning experiences, including references to relevant lessons in texts/programs)
• how to determine the unknown using models and equations	• Lesson 1.1: Many Things Come in Groups • Circle and Stars: Example: Two groups of 4 then show addition 4 + 4, then show 2 X 4 , picture Circle and Stars Lesson Can use dice, digit cards • Introducing the concepts of Multiplication and Division (Mastering the Basic Math Facts book: pages 16 to 19) Begin with the investigation at the bottom of page 16. The focus is on the mathematical process and symbolic representations. Then do the investigation: How Many Fish? This can be found on pages 18, 19 Fish Bowl Template Fishbowl Activity Worksheet These investigations are important to build conceptual understanding. This gives them more practice representing and solving multiplication problems. They need to become comfortable with the numeric representations. • Lesson 1.3: Solving Multiplication Problems (missing factor problems) K-5 Identify arithmetic patterns and explain them using properties of operations OA.9 • Lesson 1.4: Solving Problems About Pictures (Exit Slip) K-5 Interpret products of whole numbers OA.1 K-5 Use multiplication and division to solve word problems OA.3 **Mastering the Basic Math Facts in Multiplication and Division Book**. Mastering the Basic Math Facts book resources Multiplication chart made of arrays (click on links on the right) Quadruple Bump (note: this game helps students link x2 facts with x4 facts) Sextuple Bump (helps link x3 facts with x6 facts)

Possible Enrichment Activities:
Greg Tang Tangy Tuesday
Marcy Cook tile activities
Grade 3 Area Project
Grade 3 Array Project
Multiplication, Division and Perimeter Projects

Extra practice:
Greg Tang Resource: Great Times Volume 2
New Mark Common Core Practice book: Multiplication and Division practice: pages 26 to 66
Area practice: pages 126 to 130
Common Core Sheets - Multiplication (website)
Common Core Sheets - Multiplication (folder)
Multiplication and Division fact practice partner games

Journal Activities: K-5 Journal Prompts

Songs: Area Song

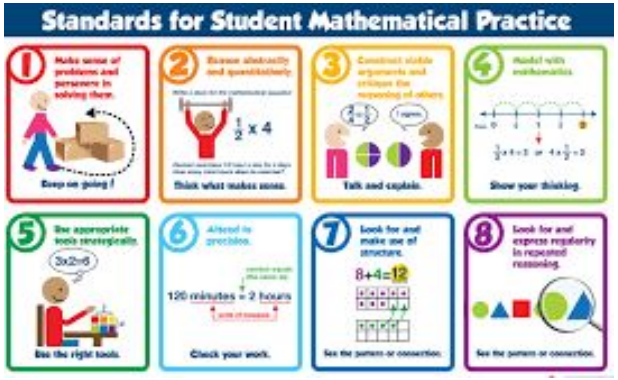
Games: Down to Earth Multiplication Game
Lights Out Division Fact Game
Skyscraper Multiplication Game
Weekly Multiplication Games: can be sent home for homework
Multiplication Tic-Tac-Toe 1-9
Multiplication Tic-Tac-Toe 1-10

Technology: Starfall on ipads: has area and multiplication practice

Suggested Formative Assessment Practices/Processes
If you choose to identify formative assessment practices/processes here (in addition to in lesson plans), make them specific to this unit of study. *3.NBT.3 Multiplying by 10 and multiples of 10* *3.OA.1 I can interpret products in multiplication* *3.OA.2 I can explain division and interpret quotients* *3.OA.3 I can interpret when to multiply and divide* *3.OA.4 I can intepret the unknown*

Resources

COMMON CORE
STATE STANDARDS INITIATIVE
PREPARING AMERICA'S STUDENTS FOR COLLEGE & CAREER



Curriculum vs. Resource

Resources:

- Investigations
- Big Ideas
- Various others

BIG IDEAS MATH

NCTM | NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Exemplars® LIBRARY
We Set the Standards!

Workshop

- Approach to teaching that requires student-centered, responsive, assessment-based instruction

- Emphasis on creating life-long mathematicians who display independence and perseverance in future endeavors

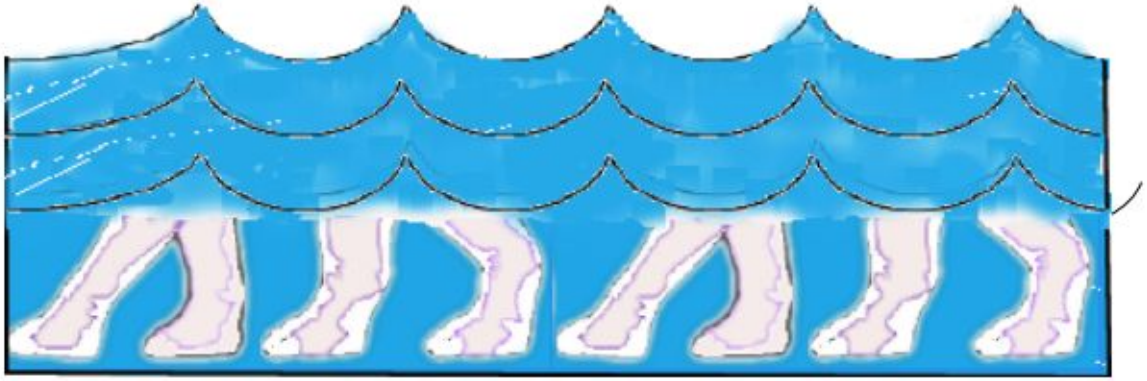
How many letters are in your name? Build a cube tower with one cube for each letter.

Problem Solving

Digging Dinosaurs

Level A:

You are swimming under water in a lake and you see dinosaur feet in the water. You don't want to go to the surface in case they are not friendly dinosaurs. Below is a picture of what you see.



How many dinosaurs are standing in the lake?
Explain how you know. Use words and mathematical language to explain your solution.

Continued:

Level B:

You want to go with your sister to the museum to see the dinosaur exhibit. The museum has three different plans to pay for going to see the dinosaurs.

Museum Rate Plans

Plan A: Pay $ 3.00 per person to visit the Museum.

Plan B: Monthly membership is $8.00 for each person, but you can go as many times as you like during the month.

Plan C: A family membership for a month is $17.00. Everyone in your family can go as often as they like for a month.

You and your sister want to go see the dinosaur exhibit three times this month. Which plan should you buy to save money?

Explain your reasoning.

Level C:

It is summer vacation and you can go to the museum more often. The rates change for a summer special.

Museum Summer Rate Plans

Plan A: Pay $ 2.75 per person to visit the museum.

Plan B: Monthly membership is $7.50 for each person, but you can go as many times as you like during the month.

Plan C: A family membership for a month is $15.25. Everyone in your family can go as often as they like for a month.

If you and your brother want to go to the museum eight times during the three months of summer, which one plan should you use and when should you go to save the most money?

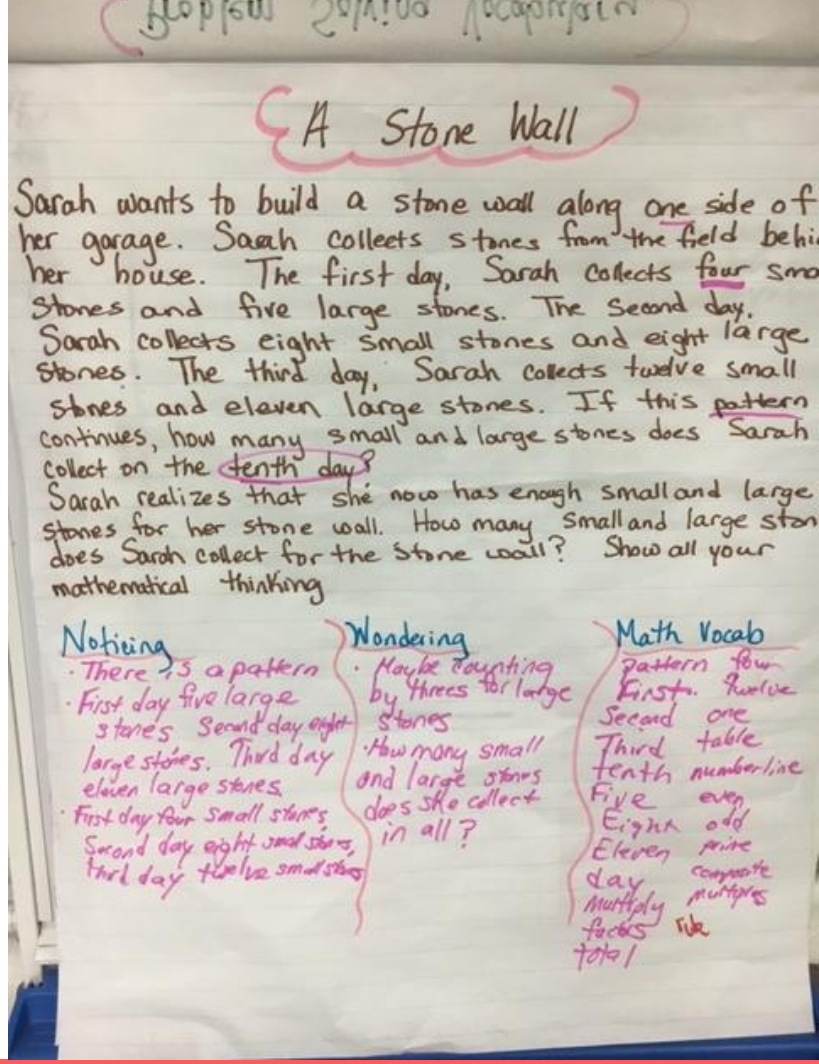
What if you can't go as you originally planned? What other plans might you use? State when you would attend and the best plan(s) to use. Explain your thinking.

Still more....

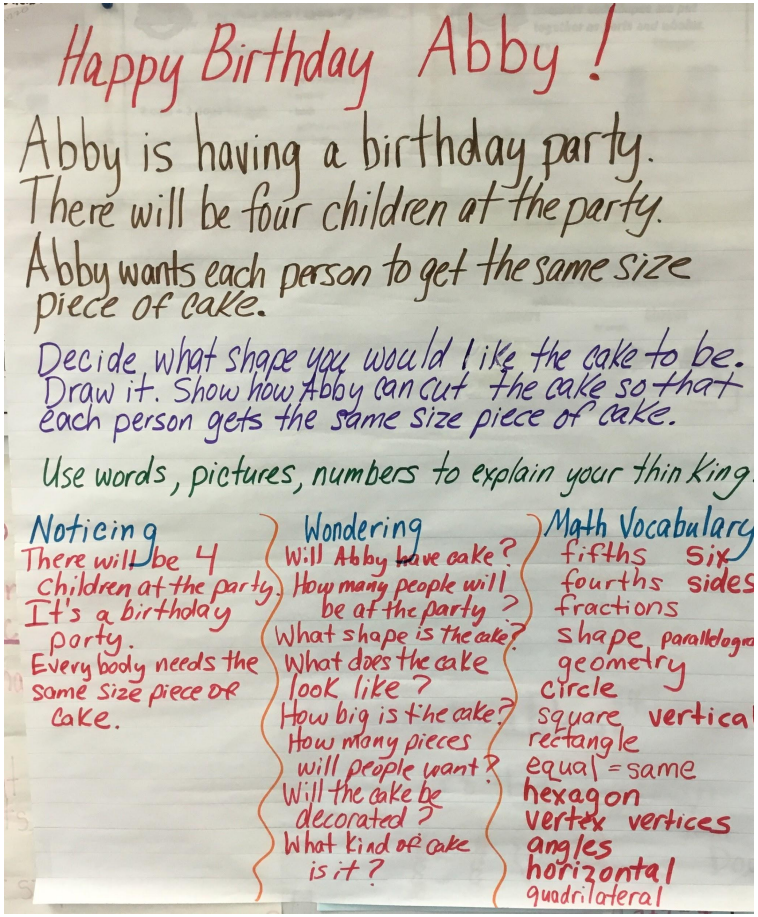
Exemplars

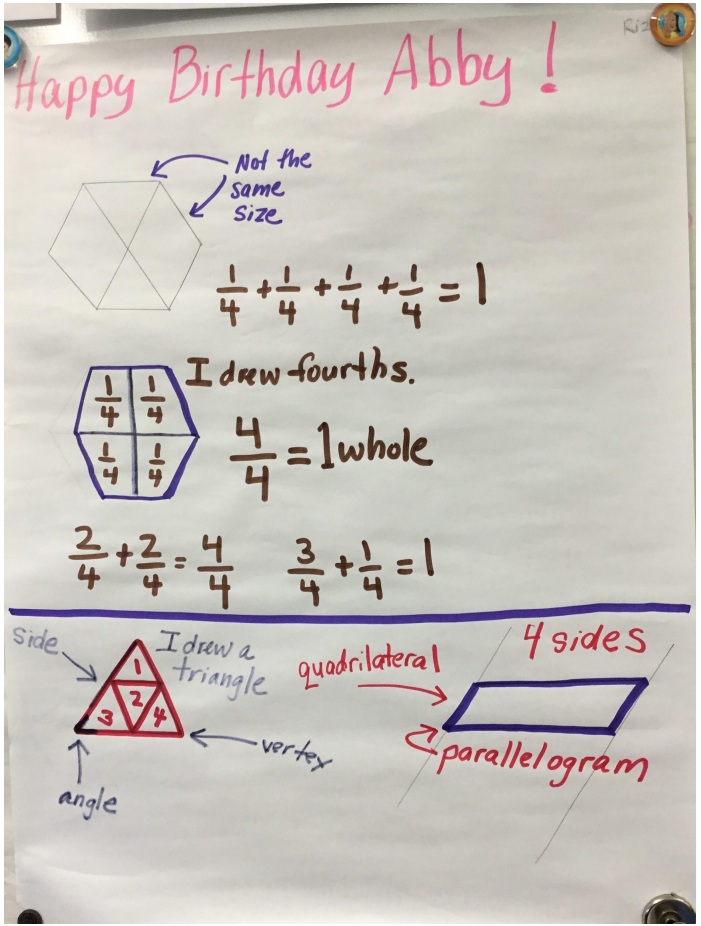
Students begin by identifying what they are noticing and wondering within the problem. This helps them to understand the problem. These are metacognition strategies they also use when they read a story. They then identify important math vocabulary they may need to access or explain the problem.

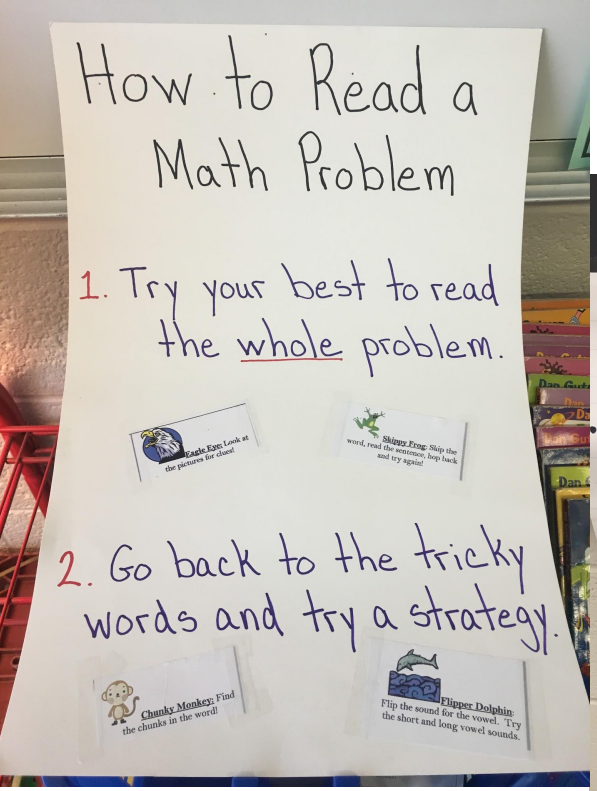
The students are then given an opportunity to work on a solution and an explanation.

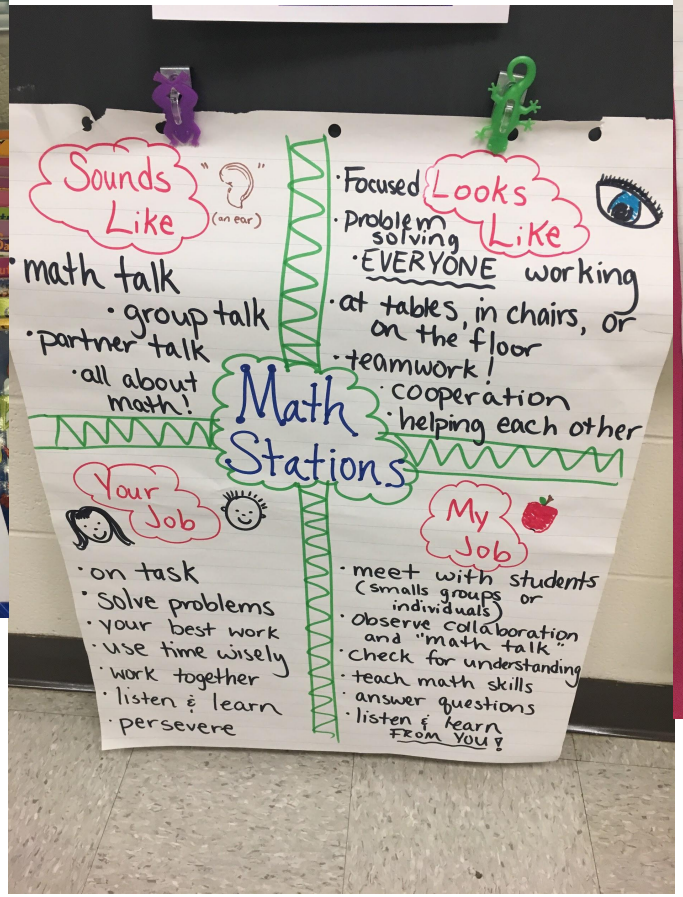


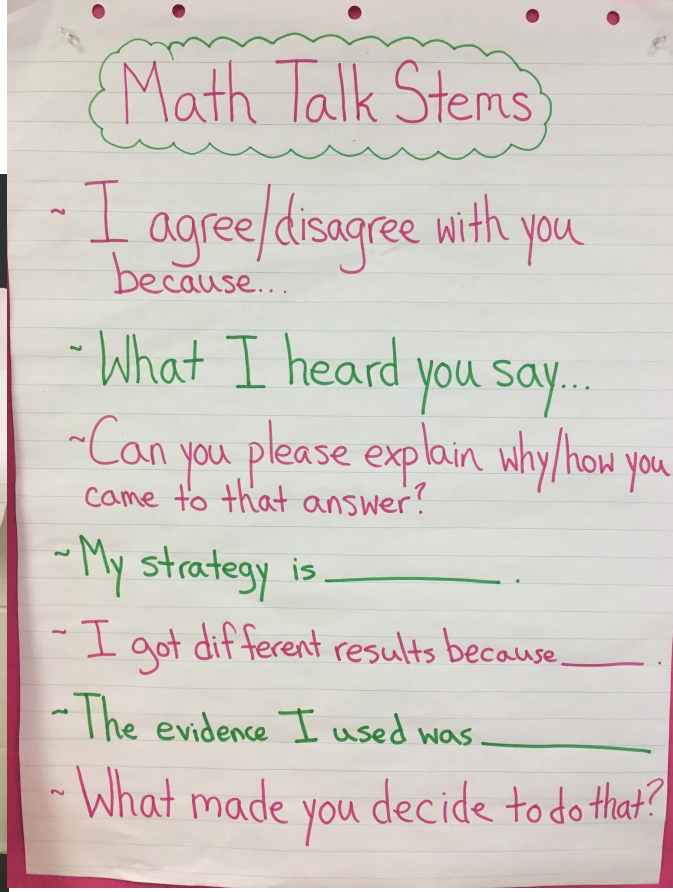
Exemplars


Happy Birthday Abby!
Abby is having a birthday party.
There will be four children at the party.
Abby wants each person to get the same size piece of cake.
Decide what shape you would like the cake to be. Draw it. Show how Abby can cut the cake so that each person gets the same size piece of cake.
Use words, pictures, numbers to explain your thinking.
Noticing
There will be 4 children at the party.
It's a birthday party.
Everybody needs the same size piece of cake.
Wondering
Will Abby have cake?
How many people will be at the party?
What shape is the cake?
What does the cake look like?
How big is the cake?
How many pieces will people want?
Will the cake be decorated?
What kind of cake is it?
Math Vocabulary
fifths
six
fourths
sides
fractions
shape
parallelogram
geometry
circle
square
vertical
rectangle
equal = same
hexagon
vertex vertices
angles
horizontal
quadrilateral


Happy Birthday Abby!
Not the same size
1/4 + 1/4 + 1/4 + 1/4 = 1
I drew fourths.
4/4 = 1 whole
2/4 + 2/4 = 4/4
3/4 + 1/4 = 1
Side
I drew a triangle
vertex
angle
quadrilateral
4 sides
parallelogram


How to Read a Math Problem
1. Try your best to read the whole problem.
Eagle Eyes: Look at the pictures for clues!
Skippy Frog: Skip the word, read the sentence, hop back and try again!
2. Go back to the tricky words and try a strategy.
Chunky Monkey: Find the chunks in the word!
Flipper Dolphin: Flip the sound for the vowel. Try the short and long vowel sounds.

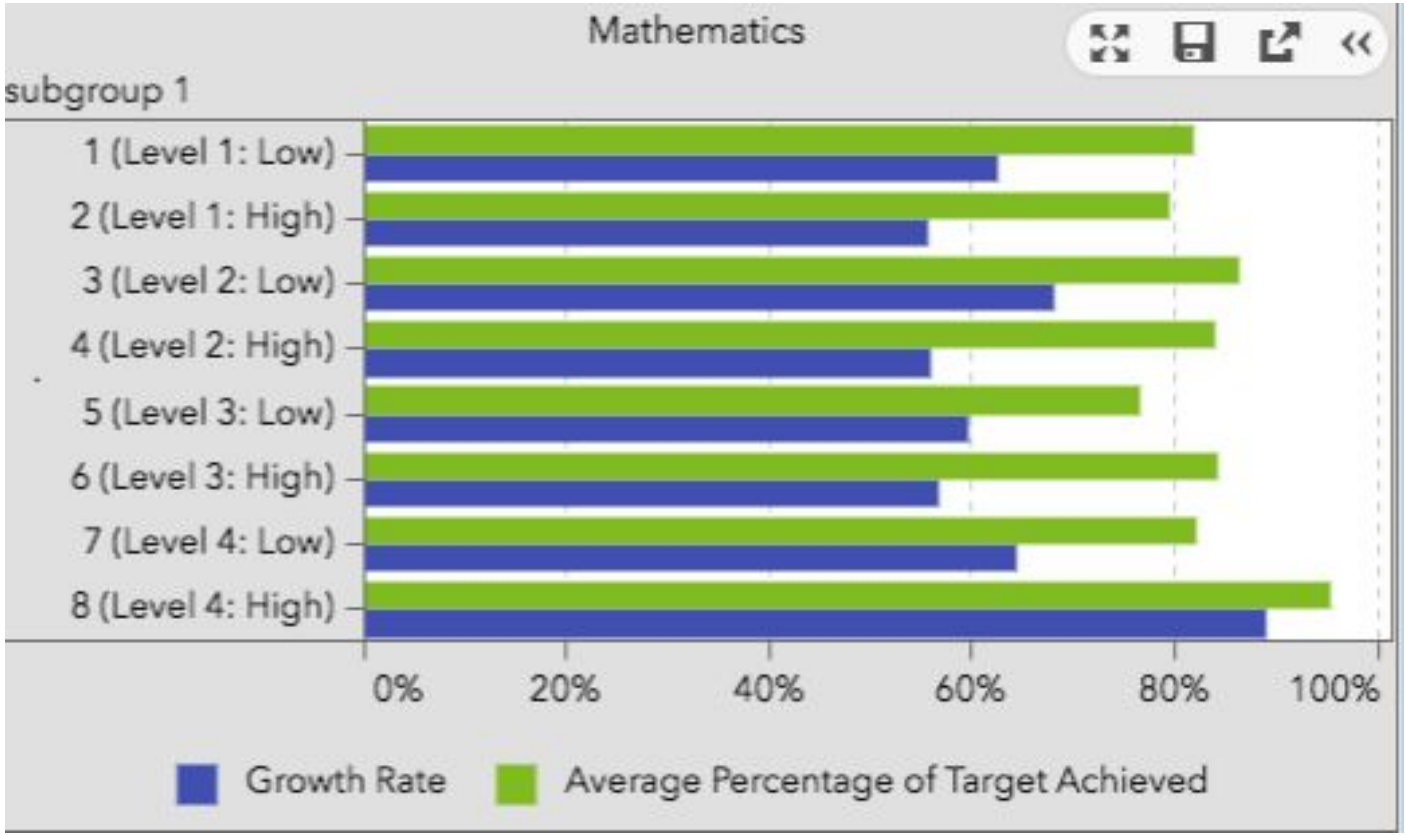

Sounds Like
(an ear)
math talk
group talk
partner talk
all about math!
Looks Like
Focused
problem solving
EVERYONE working
at tables, in chairs, or on the floor
teamwork!
cooperation
helping each other
Math Stations
Your Job
on task
Solve problems
your best work
use time wisely
work together
listen & learn
persevere
My Job
meet with students (smalls groups or individuals)
observe collaboration and "math talk"
Check for understanding
teach math skills
answer questions
listen & learn From YOU!


Math Talk Stems
~ I agree/disagree with you because...
~ What I heard you say...
~ Can you please explain why/how you came to that answer?
~ My strategy is _____.
~ I got different results because _____.
~ The evidence I used was _____
~ What made you decide to do that?

Coaching and professional development

- ➔ Coaching with Math Specialists
- ➔ Professional development:
  - ◆ Off site: Group Coaching, Greg Tang, Columbia
  - ◆ On site: Differentiation across a domain, curriculum development work

SBA Data: Growth rate



Next steps.....

Thank you!