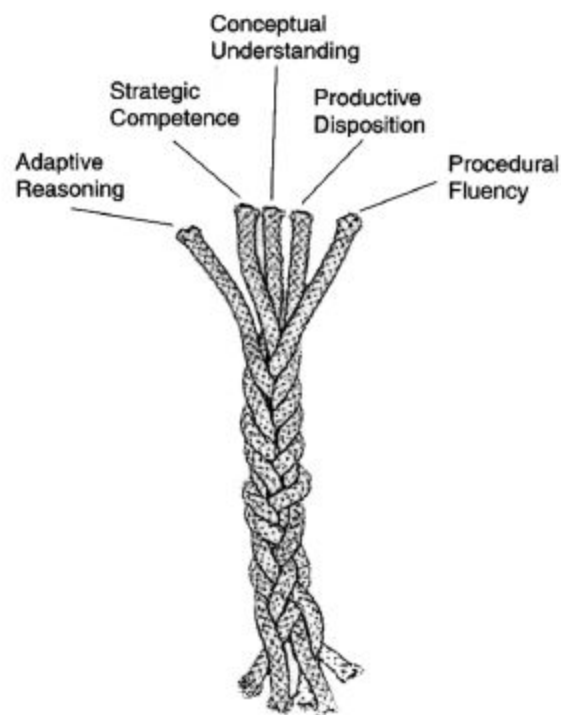


THE STRANDS OF MATHEMATICAL PROFICIENCY

Mathematical proficiency, as we see it, has five components, or *strands*:



Intertwined Strands of Proficiency

- *conceptual understanding*—comprehension of mathematical concepts, operations, and relations
- *procedural fluency*—skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- *strategic competence*—ability to formulate, represent, and solve mathematical problems
- *adaptive reasoning*—capacity for logical thought, reflection, explanation, and justification
- *productive disposition*—habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

These strands are not independent; they represent different aspects of a complex whole. The most important observation we make here, is that ***the five strands are interwoven and interdependent in the development of proficiency in mathematics***. Mathematical proficiency is not a one-dimensional trait, and it cannot be achieved by focusing on just one or two of these strands.

Kilpatrick, J., Swafford, J., & Findell, B. (2009). *Adding it up: helping children learn mathematics*. Washington: National Academy Press.

K-12 Mathematical Process Standards

The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course.

(1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

- (A) apply mathematics to problems arising in everyday life, society, and the workplace;
- (B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;
- (C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;
- (D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;
- (E) create and use representations to organize, record, and communicate mathematical ideas;
- (F) analyze mathematical relationships to connect and communicate mathematical ideas; and
- (G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

The student uses mathematical processes to **acquire** and **demonstrate** mathematical understanding.

<p><i>Context</i></p> <p>A. Apply mathematics to problems arising in everyday life, society, and the workplace</p>	<p>B. Use a problem solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution</p> <p>C. Select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems</p>	<p><i>Modeling and Using Tools</i></p>
	<p>E. Create and use representations to organize, record, and communicate mathematical ideas</p> <p>F. Analyze mathematical relationships to connect and communicate mathematical ideas</p>	<p><i>Representing and Connecting</i></p>
	<p>D. Communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate</p> <p>G. Display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication</p>	<p><i>Communicating and Justifying</i></p>