

Course Title:	Content Area:	Grade Level:	Credit (if applicable)
AP Precalculus	Mathematics	10-12	1.0
Course Description:			
<p>AP Precalculus centers on functions modeling dynamic phenomena. This research-based exploration of functions is designed to better prepare students for college-level calculus and provide grounding for other mathematics and science courses. In this course, students study a broad spectrum of function types that are foundational for careers in mathematics, physics, biology, health science, business, social science, and data science. Furthermore, as AP Precalculus may be the last mathematics course of a student's secondary education, the course is structured to provide a coherent capstone experience rather than exclusively focusing on preparation for future courses.</p> <p>Throughout this course, students develop and hone symbolic manipulation skills, including solving equations and manipulating expressions, for the many function types throughout the course. Students also learn that functions and their compositions, inverses, and transformations are understood through graphical, numerical, analytical, and verbal representations, which reveal different attributes of the functions and are useful for solving problems in mathematical and applied contexts. In turn, the skills learned in this course are widely applicable to situations that involve quantitative reasoning.</p> <p>AP Precalculus fosters the development of a deep conceptual understanding of functions. Students learn that a function is a mathematical relation that maps a set of input values— the domain—to a set of output values—the range—such that each input value is uniquely mapped to an output value. Students understand functions and their graphs as embodying dynamic covariation of quantities, a key idea in preparing for calculus. With each function type, students develop and validate function models based on the characteristics of a bivariate data set, characteristics of covarying quantities and their relative rates of change, or a set of characteristics such as zeros, asymptotes, and extrema. These models are used to interpolate, extrapolate, and interpret information with different degrees of accuracy for a given context or data set. Additionally, students also learn that every model is subject to assumptions and limitations related to the context. As a result of examining functions from many perspectives, students develop a conceptual understanding not only of specific function types but also of functions in general.</p>			
Aligned Core Resources:		Connection to the BPS Vision of the Graduate	
AP Classroom (digital-access)		<p>CONTENT MASTERY</p> <ul style="list-style-type: none"> Develop and draw from a baseline understanding of knowledge in academic disciplines from our Bristol curriculum. <p>CRITICAL THINKING AND PROBLEM SOLVING</p> <ul style="list-style-type: none"> Collect, assess and analyze relevant information Reason effectively. Use systems thinking. Make sound judgments and decisions. Identify, define and solve authentic problems and essential questions. Reflect critically on learning experience, processes and solutions. Transfer knowledge to other situations. 	
Additional Course Information: <i>Knowledge/Skill Dependent courses/prerequisites</i>		Link to Completed Equity Audit	
		AP Precalculus Equity Audit	
Standard Matrix			

AP/College Board Mathematical Practices			
	Unit 1	Unit 2	Unit 3
Practice 1: Procedural and Symbolic Fluency			
1.A Solve equations and inequalities represented analytically with and without technology.			X
1.B Express function equations, or expressions in analytically equivalent forms that are useful in given mathematical or applied content.			X
1.C Construct new functions, using transformations, compositions, inverses, or regressions, that may be useful in modeling contexts, criteria, or data with and without technology.			X
Practice 2: Multiple Representations			
2.A Identify information from graphical, numerical, analytical and verbal representations to answer a question or construct a model with or without technology.		X	
2.B Construct equivalent graphical, numerical, analytical, and verbal representations of functions that are useful in a given mathematical or applied context, with and without technology.		X	X
Practice 3: Communication and Reasoning			
3.A Describe the characteristics of a function with varying levels of precision, depending on the function representation and available mathematical tools.	X		
3.B Apply numerical results given mathematical or applied context.	X		
3.C Support conclusions or choices with a logical rationale or appropriate data.			X
Unit Links			
<p><i>If unit headings are formatted as a heading, then we can link a Table of Contents to better organize and provide faster access to each unit</i></p> <p>Polynomial and Rational Functions</p> <p>Exponential and Logarithmic Functions</p> <p>Trigonometric and Polar Functions</p>			

Unit Title:	
Polynomial and Rational Functions	
Relevant Standards: Bold indicates priority	
<p>3.A 3.B</p> <p>Throughout the course, students should practice communicating mathematics and developing notational fluency—and that practice should begin in Unit 1. Students should use precise language such as, “On the closed interval 0 to 1, as the value of x increases, the value of y increases then decreases.” To the fullest extent possible, students should work on functions presented in contextual scenarios such as graphs showing distance vs. time, tables showing velocity vs. time, or scenarios involving volume vs. time. In these contexts, students should use clear language when referring to variables and functions, including units of measure as appropriate. For example, when considering a problem of filling a pool with water, a student may write, “The input values of the function V are times in minutes, and the output values are volumes in cubic meters. The average rate of change of the function V over the time interval t equals 2 minutes to t equals 5 minutes is 0.4 cubic meters per minute.” Practicing communicating with precise language can help students clarify their thinking and make important connections while revealing misconceptions.</p>	
Essential Question(s):	Enduring Understanding(s):
<ul style="list-style-type: none"> • How do we model the intensity of light from its source? • How can I use data and graphs to figure out the best time to purchase event tickets? • How can we adjust known projectile motion models to account for changes in conditions? 	<p>After studying Unit 1, students should be able to describe, represent, and model polynomial and rational functions and their additive and multiplicative transformations. Because part of the exam relies on technology, students should be able to identify zeros, points of intersection, and extrema using graphing calculator technology. Students should be able to calculate linear, quadratic, cubic, and quartic regressions to model a data set. In the free-response section of the exam, students will not only be required to arrive at a solution but also explain and provide rationales for their conclusions. Students should practice providing reasons for conclusions throughout the unit in both spoken and written form and continually refine their explanations to improve precision.</p>
Demonstration of Learning:	Pacing for Unit
Certified AP assessments and released items.	8 Weeks
Family Overview (link below)	Integration of Technology:
AP Classroom Resources (requires student login)	<i>Intentionally aligned use of digital tools and resources to support acquisition of content, researching, organizing and communicating learning</i>
Unit-specific Vocabulary:	
Graphical, Numerical, Analytical, Verbal, Rate of change, Average rate of change, Function, Domain, Range, Even/Odd Function, Increasing, Decreasing, Input, Output, Independent, Dependent, Linear, Quadratic, Polynomial, Rational, Asymptote, Horizontal, Vertical, Zeros, Intersections, Roots, Interval,	

Secant Line, Tangent Line, Equal-Length , Consecutive, Covariation, Variable, Distinct, Extrema, Absolute, Global, Maximum, Minimum, Local, Relative, Multiplicity, Complex, Conjugate, Multiplicative transformation, Additive transformation, Contextual, Limit, Dilation, Continuous, Parent Function, Quadratic, Cubic, Quartic, End Behavior , Degree, Regression, Boundedness, Successive differences, Inflection point, Multiplicity of zeros, Concavity

Differentiation through *Universal Design for Learning*

UDL Indicator	Teacher Actions:
<p>Comprehension:</p> <ul style="list-style-type: none"> • Activate or supply background knowledge 	<ul style="list-style-type: none"> • Anchor instruction by linking to and activating relevant prior knowledge (e.g., using visual imagery, concept anchoring, or concept mastery routines) • Use advanced organizers (e.g., KWL methods, concept maps) • Pre-teach critical prerequisite concepts through demonstration or models • Bridge concepts with relevant analogies and metaphors

Supporting Multilingual/English Learners

Related <i>CELP standards:</i>	Learning Targets:
<p>An EL can construct grade appropriate oral and written claims and support them with reasoning and evidence.</p>	<p>All learning targets in this unit support CELP standard 9-12.</p>

Lesson Sequence	Learning Target	Success Criteria/Assessment
<p>1 Change in Tandem</p>	<p>I can describe how the input and output values of a function vary together by comparing function values.</p> <p>I can construct a graph representing two quantities that vary with respect to each other in a contextual scenario.</p>	<ul style="list-style-type: none"> • I can identify the domain and range of a function. • I can compare two quantities that are changing. • I can identify the intervals of increase and decrease of a function given a table or graph. • I can identify over which intervals the graph of a function is concave up or concave down and justify my reasoning.
<p>2 Rates of Change</p>	<p>I can compare the rates of change at two points using average rates of change near the points.</p> <p>I can describe how two quantities vary together at different points and over different intervals of a function</p>	<ul style="list-style-type: none"> • I can determine the average rate of change of a function given a table and an equation. • I can determine the average rate of change of a function at a point given an equation. • I can use the average rate of change to describe how the function behaves over an interval or at a point.
<p>3 Polynomial Functions and Rates of Change</p>	<p>I can determine the average rates of change or sequences and functions, including linear, quadratic, and other function types.</p> <p>I can determine the change in the average rates of change for linear,</p>	<ul style="list-style-type: none"> • I can identify a linear function by finding the average rate of change from a table. • I can identify a quadratic function by finding the average rate of change from a table. • I can determine if a function is concave up or down by calculating the average rate of change over equal length intervals from a table.

	quadratic, and other function types.	
4 Polynomial Function and Rates of Change	I can identify key characteristics of polynomial functions related to rates of change.	<ul style="list-style-type: none"> I can identify the degree, relative extrema, absolute extrema, and inflection points of a function given a graph. I can determine if a polynomial function is increasing or decreasing by calculating the average rate of change over various intervals.
5 Polynomial Functions and Complex Zeros	<p>I can identify key characteristics of a polynomial function related to its zeros when suitable factorizations are available or with technology.</p> <p>I can determine if a polynomial function is even or odd</p>	<ul style="list-style-type: none"> I can determine the number of complex zeros of a polynomial function. I can find the zeros of a polynomial function. I can determine the multiplicity of a zero of a polynomial function given its graph. I can determine if a function is even or odd given a table, graph, or equation.
6 Polynomial Functions and End Behavior	I can describe end behaviors of polynomial functions.	<ul style="list-style-type: none"> I can state the end behavior of a function using limit notation given a graph, table, or equation.
7 Rational Functions and End Behavior	I can describe the end behaviors of rational functions.	<ul style="list-style-type: none"> I can analyze the degree of the numerator and denominator of a rational function to determine if a function has a horizontal/slant asymptote. I can determine the end behavior of a rational function given a table, graph or an equation using limit notation.
8 Rational Functions and Zeros	I can determine the zeros of rational functions	<ul style="list-style-type: none"> I can simplify the equation of a rational function by factoring its numerator and denominator. I can determine the zeros of a rational function by analyzing the numerator of its simplified form. I can use a sign chart to solve a rational inequality.
9 Rational Functions and Vertical Asymptotes	I can determine vertical asymptotes of graphs of rational functions	<ul style="list-style-type: none"> I can determine the zeros of a rational function by simplifying the function and analyzing the denominator. I can determine where the values of a rational function increase or decrease without bound using limit notation from a graph or table.
10 Rational Functions and Holes	I can determine holes in graphs of rational functions.	<ul style="list-style-type: none"> I can determine the holes of a rational function by factoring the function and analyzing both the numerator and denominator. I can determine the location of a hole at $x = c$ by examining the behavior of a rational function as x approaches c using limit notation.
11 Equivalent Representations of	<p>I can rewrite polynomial and rational expressions in equivalent forms.</p> <p>I can determine the quotient of two</p>	<ul style="list-style-type: none"> I can convert a polynomial function between standard form and factored form. I can convert a rational function between standard form and factored form.

<p>Polynomial and Rational Expressions</p>	<p>polynomial functions using long division.</p> <p>I can rewrite the repeated product of binomial using the binomial theorem.</p>	<ul style="list-style-type: none"> ● I can use long division to determine the equations of a slant asymptote of a rational function. ● I can use Pascal's triangle to perform a binomial expansion.
<p>12 Transformations of Functions</p>	<p>I can construct a function that is an additive and/or multiplicative transformation of another function.</p>	<ul style="list-style-type: none"> ● I can vertically/horizontally translate a function given an equation or a graph. ● I can vertically/ horizontally dilate a function given a graph or an equation. ● I can reflect a function given a graph or an equation. ● I can determine the domain or range of a transformed function.
<p>13 Function Model Selection and Assumption Articulation</p>	<p>I can identify an appropriate function type to construct a function model given a scenario.</p> <p>I can describe assumptions and restrictions related to building a function model.</p>	<ul style="list-style-type: none"> ● I can determine a linear function to model data that demonstrates a roughly constant rate of change. ● I can determine a quadratic function to model data that demonstrates roughly linear rate of change. ● I can determine a cubic function to model data regarding volume or three dimensions. ● I can determine a polynomial to model scenarios with multiple real zeros or multiple maxima or minima. ● I can determine a piecewise function to model contextual scenarios that demonstrate different characteristics over different intervals.
<p>14 Function Model Construction and Application</p>	<p>I can construct a linear, quadratic, cubic, quartic, polynomial of degree n, or related piecewise-defined function model.</p> <p>I can construct a rational function model based on a context.</p> <p>I can apply a function model to answer questions about a data set or contextual scenario.</p>	<ul style="list-style-type: none"> ● I can use technology to determine a polynomial (linear, quadratic, cubic, or quartic) model to represent a set of data. ● I can use a model to draw conclusions about a data set or contextual scenario.

Unit Title:	
Exponential and Logarithmic Functions	
Relevant Standards: Bold indicates priority	
2.A; 2.B Students should learn to communicate differences and similarities among arithmetic sequences, linear functions, geometric sequences, and exponential functions. Students can develop a deeper understanding of these four function types by considering how each would be represented in a graph, in a table, in an analytical representation, and through verbal descriptions of related scenarios. Examining multiple representations is also powerful in understanding composition of functions and relationships between functions and their inverse functions. In this unit, multiple representations should be used to explore the inverse relationship between exponential and logarithmic functions.	
Essential Question(s):	Enduring Understanding(s):
<ul style="list-style-type: none"> • How can I make a single model that merges the interest I earn from my bank with the taxes that are due so I can know how much I will have in the end? • How can we adjust the scale of distance for a model of planets in the solar system so the relationships among the planets are easier to see? • If different functions can be used to model data, how do we pick which one is best? 	In Unit 2, students build an understanding of exponential and logarithmic functions. Exponential and logarithmic function models are widespread in the natural and social sciences. When an aspect of a phenomenon changes proportionally to the existing amount, exponential and logarithmic models are employed to harness the information. Exponential functions are key to modeling population growth, radioactive decay, interest rates, and the amount of medication in a patient. Logarithmic functions are useful in modeling sound intensity and frequency, the magnitude of earthquakes, the pH scale in chemistry, and the working memory in humans. The study of these two function types touches careers in business, medicine, chemistry, physics, education, and human geography, among others.
Demonstration of Learning:	Pacing for Unit
Certified AP assessments and released items.	10 Weeks
Family Overview (link below)	Integration of Technology:
	<i>Intentionally aligned use of digital tools and resources to support acquisition of content, researching, organizing and communicating learning</i>
Unit-specific Vocabulary:	
Sequence, Discrete, Arithmetic, Geometric, Common Difference, Constant Rate, Common ratio, Constant proportional change, Validate, Exponential, Logarithmic, Proportional Output Value, Logarithmically scaled, Semi-Log Plot, Residual, Linearization, Composition, Decomposition, Inverse	
Differentiation through Universal Design for Learning	
UDL Indicator	Teacher Actions:

<p>Comprehension: Highlight patterns, critical features, big ideas, and relationships</p>	<ul style="list-style-type: none"> ● Highlight or emphasize key elements in text, graphics, diagrams, formulas ● Use outlines, graphic organizers, unit organizer routines, concept organizer routines, and concept mastery routines to emphasize key ideas and relationships ● Use multiple examples and non-examples to emphasize critical features ● Use cues and prompts to draw attention to critical features ● Highlight previously learned skills that can be used to solve unfamiliar problems
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Supporting Multilingual/English Learners

<p>Related <i>CELP standards:</i></p>	<p>Learning Targets:</p>
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<p>An EL can construct grade appropriate oral and written claims and support them with reasoning and evidence.</p>	<p>All learning targets in this unit support CELP standard 9-12.</p>
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Lesson Sequence	Learning Target	Success Criteria/ Assessment
<p>1 Change in Arithmetic and Geometric Sequences</p>	<p>I can express arithmetic sequences found in mathematical and contextual scenarios as functions of the whole numbers. I can express geometric sequences found in mathematical and contextual scenarios as functions of the whole numbers.</p>	<ul style="list-style-type: none"> ● I can find a common difference of an arithmetic sequence and use it to write the equation for the sequence. ● I can find a common ratio of a geometric sequence and use it to write the equation for the sequence.
<p>2 Change in Linear and Exponential Functions</p>	<p>I can construct functions of the real numbers that are comparable to arithmetic and geometric sequences. I can describe similarities and differences between linear and exponential functions.</p>	<ul style="list-style-type: none"> ● I can write a linear function that is comparable to an arithmetic sequence. ● I can write a linear function that is comparable to a geometric sequence. ● I can determine if a function is linear or exponential from a table, graph or an equation by examining a rate of change.
<p>3 Exponential Functions</p>	<p>I can identify key characteristics of exponential functions.</p>	<ul style="list-style-type: none"> ● I can find the initial value of an exponential function. ● I can find the growth factor of an exponential function. ● I can determine if an exponential function represents growth or decay given a table, graph or an equation. ● I can use limit notation to describe the end behavior of an exponential function. ● I can identify exponential functions that has been vertically translated.
<p>4 Exponential Function Manipulation</p>	<p>I can rewrite exponential expressions in equivalent forms.</p>	<ul style="list-style-type: none"> ● I can use the product property for exponents to rewrite an exponential expression. ● I can use the power property for exponents to rewrite an exponential expression.

		<ul style="list-style-type: none"> • I can use the negative exponent property for exponents to rewrite an exponential expression. • I can rewrite an exponential expression containing a rational exponent as a radical expression.
5 Exponential Function Context and Data Modeling	<p>I can construct a model for situations involving proportional output values over equal length input value intervals.</p> <p>I can apply exponential models to answer questions about a data set or contextual scenario.</p>	<ul style="list-style-type: none"> • I can use exponential functions to model growth patterns. • I can construct an exponential model from a ratio and initial value or given two input output pairs. • I can transform $f(x) = ab^x$ to create an exponential model based on a scenario or data set. • I can use technology to determine exponential regression models. • I can rewrite an exponential function to determine growth rates over different intervals of time.
6 Competing Function Model Validation	<p>I can construct linear, quadratic, and exponential models based on a data set.</p> <p>I can validate a model constructed from a data set.</p>	<ul style="list-style-type: none"> • I can determine the most appropriate model (linear, quadratic or exponential) for a data set. • I can determine the residual of a data value. • I can validate a model by creating a residual plot.
7 Composition of Functions	<p>I can evaluate the composition of two or more functions for given values.</p> <p>I can construct a representation of the composition of two or more functions.</p> <p>I can rewrite a given function as a composition of two or more functions.</p>	<ul style="list-style-type: none"> • I can determine the composition of two functions. • I can calculate values of a composite function from tables, graphs or equations. • I can decompose a function into two or more less complicated functions.
8 Inverse Functions	<p>I can determine the input-output pairs of the inverse of a function.</p> <p>I can determine the inverse of a function on an invertible domain.</p>	<ul style="list-style-type: none"> • I can determine if a function is invertible from its graph. • I can determine an inverse function value given a table, graph or equation. • I can determine the equation of a function's inverse. • I can show that two functions are inverses of each other by composing them.
9 Logarithmic Expressions	I can evaluate logarithmic expressions.	<ul style="list-style-type: none"> • I can evaluate logarithmic expressions with or without the use of technology.
10 Inverses of Exponential Functions	I can construct representations of the inverse of an exponential function with an initial value of 1.	<ul style="list-style-type: none"> • I can show that logarithms and exponentials are inverses of each other.
11. Logarithmic Functions	I can identify key characteristics of logarithmic functions.	<ul style="list-style-type: none"> • I can determine the domain and range of logarithmic functions. • I can determine if a logarithmic function is increasing or decreasing. • I can determine the concavity of a logarithmic function. • I can determine the end behavior of a logarithmic function given a table, graph or an equation using

		limit notation.
12 Logarithmic Function Manipulation	I can rewrite logarithmic expressions in equivalent forms.	<ul style="list-style-type: none"> • I can use the product property for logarithms to rewrite a logarithmic expression. • I can use the power property for logarithms to rewrite a logarithmic expression. • I can use the change of base property for logarithms to rewrite a logarithmic expression.
13 Exponential and Logarithmic Equations and Inequalities	<p>I can solve exponential and logarithmic equations and inequalities.</p> <p>I can construct the inverse function or exponential and logarithmic functions.</p>	<ul style="list-style-type: none"> • I can use the properties of exponents and logarithms to solve exponential and logarithmic equations. • I can identify the extraneous solutions. • I can use the properties of exponents and logarithms to solve exponential and logarithmic inequalities. • I can determine the equation of the inverse of an exponential function that has been transformed. • I can determine the equation of the inverse of a logarithmic function that has been transformed.
14 Logarithmic Function Context and Data Modeling	I can construct a logarithmic function model.	<ul style="list-style-type: none"> • I can construct a logarithmic model from an appropriate proportion and real zero or from two input-output pairs. • I can transform the function $f(x) = a \cdot \log_b x$ to create logarithmic function models based on a problem scenario or data set. • I can use technology to construct logarithmic function models • I can use logarithmic function models to predict values for the dependent variable.
15 Semi-log Plots	<p>I can determine if an exponential model is appropriate by examining a semi-log plot of a data set.</p> <p>I can construct the linearization of exponential data.</p>	<ul style="list-style-type: none"> • I can construct a semi-log plot. • I can determine if a function is exponential by analyzing a semi log plot. • I can model exponential data using a linear model.

Unit Title:	
Trigonometric and Polar Functions	
Relevant Standards: Bold indicates priority	
1.A; 1.B; 1.C; 2.B; 3.A	
<p>Students should have multiple experiences transitioning among, and communicating about, the various representations of trigonometric functions, especially sinusoidal functions. It is important that, in addition to solving trigonometric equations and finding equivalent trigonometric expressions, students build sinusoidal models with and without technology and practice constructing different representations. As students transition to thinking in the polar plane, they will refine their communications related to characteristics of functions. The more casual language that students may have adopted such as “goes up” and “goes down” will need to be replaced with more careful language that addresses a function’s behavior related to angles and radii.</p>	
Essential Question(s):	Enduring Understanding(s):
<ul style="list-style-type: none"> • Since energy usage goes up and down through the year, how can I use trends in data to predict my monthly electricity bills when I get my first apartment? • How do we model aspects of circular and spinning objects without using complex equations from the x-y rectangular-based coordinate system? • How does right triangle trigonometry from geometry relate to trigonometric functions? 	<p>In Unit 3, students explore trigonometric functions and their relation to the angles and arcs of a circle. Since their output values repeat with every full revolution around the circle, trigonometric functions are ideal for modeling periodic, or repeated pattern phenomena, such as: the highs and lows of a wave, the blood pressure produced by a heart, and the angle from the North Pole to the Sun year to year. Furthermore, periodicity is found in human inventions and social phenomena. For example, moving parts of an analog clock are modeled by a trigonometric function with respect to each other or with respect to time; traffic flow at an intersection over the course of a week demonstrates daily periodicity; and demand for a particular product over the course of a year falls into an annually repeating pattern. Polar functions, which are also explored in this unit, have deep ties to trigonometric functions as they are both based on the circle. Polar functions are defined on the polar coordinate system that uses the circular concepts of radii and angles to describe location instead of rectangular concepts of left-right and up-down, which students have worked with previously. Trigonometry serves as the bridge between the two systems.</p>
Demonstration of Learning:	Pacing for Unit
Certified AP assessments and released items.	10 Weeks
Family Overview (link below)	Integration of Technology:
	<i>Intentionally aligned use of digital tools and resources to support acquisition of content, researching, organizing and communicating learning</i>

Unit-specific Vocabulary:		
Periodic, Secant, Cosecant, Reciprocal, Pole, Reflect, Tangent, Cotangent, Origin, Cosine (horizontal displacement), Sine (vertical displacement), Unit Circle, Identity, Rectangular, Cartesian, Polar, Angle θ , Radius, Directed distance, Terminal Ray		
Differentiation through <i>Universal Design for Learning</i>		
UDL Indicator	Teacher Actions:	
Comprehension: Highlight patterns, critical features, big ideas, and relationships	<ul style="list-style-type: none"> • Highlight or emphasize key elements in text, graphics, diagrams, formulas • Use outlines, graphic organizers, unit organizer routines, concept organizer routines, and concept mastery routines to emphasize key ideas and relationships • Use multiple examples and non-examples to emphasize critical features • Use cues and prompts to draw attention to critical features • Highlight previously learned skills that can be used to solve unfamiliar problems 	
Supporting Multilingual/English Learners		
Related <i>CELP standards:</i>	Learning Targets:	
An EL can construct grade appropriate oral and written claims and support them with reasoning and evidence.	All learning targets in this unit support CELP standard 9-12.	
Lesson Sequence	Learning Target	Success Criteria
1 Periodic Phenomena	<p>I can construct graphs of periodic relationships based on verbal representations.</p> <p>I can describe key characteristics of a periodic function based on a verbal representation.</p>	<ul style="list-style-type: none"> • I can identify a period behavior between two variables. • I can graph period function from the single cycle of the relationship based on a verbal description. • I can find a period of a period function.
2 Sine, Cosine, and Tangent	I can determine the sine, cosine, and tangent of an angle using the unit circle.	<ul style="list-style-type: none"> • I can name and draw angles in standard position. • I can find coterminal angles of an angle. • I can determine sine, cosine and tangent ratio on a unit circle. • I can represent angles using radian measure.
3 Sine and Cosine Function Values	I can determine the coordinates of points on a circle centered at the origin.	<ul style="list-style-type: none"> • I can identify points on a circle centered at the origin. • I can use a unit circle to find the exact values of sine and cosine of angles that are multiples of $\frac{\pi}{4}$ and $\frac{\pi}{6}$.

4 Sine and Cosine Function Graphs	I can construct representations of sine and cosine functions using the unit circle.	<ul style="list-style-type: none"> ● I can graph sine and cosine functions as functions of θ using a unit circle. ● I can determine the domain and range of a sine and cosine function.
5 Sinusoidal Functions	I can identify key characteristics of the sine and cosine functions.	<ul style="list-style-type: none"> ● I can find a period of a sinusoidal function given an equation or a graph. ● I can find an amplitude of a sinusoidal function given an equation or a graph. ● I can find a midline of a sine and cosine function given an equation or a graph. ● I can find the intervals of change in concavity for sinusoidal functions. ● I can understand that the frequency of a sinusoidal function is a reciprocal of a period.
6 Sinusoidal Transformations	I can identify the amplitude, vertical shift, period, and phase shift of a sinusoidal function.	<ul style="list-style-type: none"> ● I can identify a period and an amplitude of a sinusoidal function given an equation or a graph. ● I can identify the vertical shift of a sinusoidal function given an equation or a graph. ● I can identify a phase shift of a sinusoidal function given an equation or a graph. ● I can write an equation of a sinusoidal function given a graph.
7 Sinusoidal Function Context and Data Modeling	I can construct sinusoidal function models of periodic phenomena.	<ul style="list-style-type: none"> ● I can write a sinusoidal function modeling a contextual scenario using key points. ● I can use sinusoidal regression to find a sinusoidal model or a data set. ● I can use the sinusoidal model to answer questions about a contextual scenario.
8 The Tangent Function	<p>I can construct representations of the tangent function using the unit circle.</p> <p>I can describe key characteristics of the tangent function.</p> <p>I can describe additive and multiplicative transformations involving the tangent function.</p>	<ul style="list-style-type: none"> ● I can define a tangent function on a unit circle. ● I can find a period of a tangent function. ● I can find the domain and range of a tangent function. ● I can find the intervals of concavity of a tangent function given a graph. ● I can graph a tangent function and its transformations.
9 Inverse Trigonometric Functions	I can construct analytical and graphical representations of the inverse of the sine, cosine, and tangent functions over a restricted domain.	<ul style="list-style-type: none"> ● I can understand that restricted domain is necessary to find inverses of trigonometric functions. ● I can evaluate the exact value of inverse trigonometric functions. ● I can evaluate the value of inverse trigonometric functions using technology.
10 Trigonometric Equations and	I can solve equations and inequalities involving trigonometric functions.	<ul style="list-style-type: none"> ● I can solve trigonometric equations on restricted and unrestricted domains. ● I can solve trigonometric inequalities. ● I can solve trigonometric equations and inequalities

Inequalities		arising from contextual scenarios and apply appropriate domain restrictions for those scenarios.
11 The Secant, Cosecant, and Cotangent Functions	I can identify key characteristics of functions that involve quotients of the sine and cosine functions.	<ul style="list-style-type: none"> • I can define secant, cosecant and cotangent functions. • I can graph secant, cosecant and cotangent functions. • I can find the domain and range of secant, cosecant and cotangent functions.
12 Equivalent Representations of Trigonometric Functions	<p>I can rewrite trigonometric expressions in equivalent forms with the Pythagorean identity.</p> <p>I can rewrite trigonometric expressions in equivalent forms with sine and cosine sum identities.</p> <p>I can solve equations using equivalent analytic representations of trigonometric functions.</p>	<ul style="list-style-type: none"> • I can use Pythagorean trigonometric identities to rewrite trigonometric expressions in equivalent forms. • I can use sum identities to rewrite trigonometric expressions in equivalent forms. • I can solve trigonometric equations and identities using trigonometric identities.
13 Trigonometry and Polar Coordinates	I can determine the location of a point in the plane using both rectangular and polar coordinates.	<ul style="list-style-type: none"> • I can plot points using polar coordinates • I can convert coordinates from polar to rectangular forms and vice versa. • I can represent complex numbers in a complex plane. • I can write complex numbers in trigonometric form.
14 Polar function Graphs	I can construct graphs of polar functions.	<ul style="list-style-type: none"> • I can create a table of input-output pairs of polar functions. • I can graph polar functions in the polar plane.
15 Rates of Change in Polar Functions	I can describe characteristics of the graph of a polar function.	<ul style="list-style-type: none"> • I can determine the intervals on which a polar function is positive and increasing or negative and decreasing and vice versa. • I can determine the intervals on which the distance between a polar function and the origin is increasing or decreasing. • I can find the relative extrema of a polar function. • I can find an average rate of change of r values over an interval of θ. • I can use the average rate of change of r with respect to θ to estimate values of the function on an interval.