Course Title	Content Area	Grade Level	Credit (if applicable)
Grade 6 Mathematics	**Mathematics**	**Grade 6**	**N/A**

Course Description

iM Grade 6 begins with an exploration of area and surface area—an invitation for students to engage with novel ideas that they can represent concretely and visually, and reason about in intuitive ways. Starting with geometry also creates opportunities to elicit close observation, sense- and connection-making, and the exchange of ideas—elements of a healthy learning community.

The next two units introduce ratios and rates, concepts that are also new. Students learn to represent, make sense of, and solve problems about equivalent ratios, rates, unit rates, and percentages. The mathematical reasoning here constitutes major work of the grade.

In the two units that follow, students expand and deepen their prior knowledge of numbers and operations. In one unit, students explore division involving fractions, and work toward dividing a fraction by fraction. In the other, they learn to multiply and divide multi-digit, base-ten numbers, including decimals, using the standard algorithm for each operation. Building fluency with algorithms takes time and continues beyond the two units.

Next, students further their understanding of equations and expressions, including those with variables. Students consider ways to represent, justify, and generate equivalent expressions. They also use expressions and equations to describe the relationship between quantities.

From there, students are introduced to rational numbers. Students learn about negative numbers, and represent negative numbers on the number line and on the coordinate plane. They analyze and write inequalities that compare rational numbers.

Toward the end of the course, students examine data sets and distributions. They learn about statistical questions, categorical data, and numerical data. They also explore ways to describe the center and the distribution of a data set. *(https://accessim.org/6-8/grade-6/course-guide/scope-and-sequence?a=teacher#narrative)*

Illustrative Math Information for Families: https://accessim.org/6-8/grade-6/course-guide/information-for-families?a=teacher

Aligned Core Resources

https://accessim.org

CT Core Standards

National Common Core Standards

Imagine Learning iM Resources (BPS teacher login through ClassLink)

https://accessim.org/6-8/grade-6/course-guide/further-reading?a=teacher

- Empowering All Storytellers: Tips for Engaging Special Populations Using IM® v.360 for Grade 6–12
- Tackling Wordy Problems: How the Three Reads Math Language Routine Supports Access for All Learners
- Think Pair Share
- Making Sense of Story Problems
- Math Language Routines: Discourse with a Purpose

Connection to the *BPS Vision of the Graduate*

Common Core State Standards: Math Practice (MP) Standards

MP 1: Make sense of problems and persevere in solving them.
MP 2: Reason abstractly and quantitatively.
MP 3: Construct viable arguments and critique the reasoning of others.
MP 4: Model with mathematics.
MP 5: Use appropriate tools strategically.
MP 6: Attend to precision.
MP 7: Look for and make use of structure.
MP 8: Look for and express regularity in repeated reasoning.

	Lessons that Showcase Math Practice Standards							
	Unit 1	**Unit 2**	**Unit 3**	**Unit 4**	**Unit 5**	**Unit 6**	**Unit 7**	**Unit 8**
MP 1	1, 11, 12, 15	1, 13–17	8, 9, 14, 17	8, 9, 15, 16	10, 11, 14	6	7, 12, 18	17
MP 2	4, 15	5, 6, 9, 12, 13, 15	1, 4, 5, 7–9, 11, 12	1–4, 6, 7, 9, 12, 16	4, 9, 11–14	5, 7, 9, 10, 16, 17, 19	1, 2, 5, 6, 8–10, 13, 16, 17	1, 2, 5, 9, 13, 15, 16, 18
MP 3	1–3, 9, 19	2, 10	1, 5	13, 16, 17	8	4, 6, 11, 13, 14, 19	3, 4, 10	2, 4, 12
MP 4	12, 19	17	4, 17	13, 17				18
MP 5	1, 19	16		6, 8				3
MP 6	1, 2, 6, 11, 13, 15–19	1, 2, 5–8, 10, 11, 13–15	6, 8–14, 16	1, 3, 6–8, 11–13	1, 6–8, 10–15	1, 4–7, 9, 11, 13, 16, 19	3–5, 7, 12, 19	2, 7, 8, 10, 17
MP 7	1, 3, 6–8, 10, 11, 13–15, 18	1, 3, 4, 6, 8, 11	3, 5–7, 11, 13, 16	1, 8, 10, 11, 13	2–8, 12–14	2–5, 8–13, 15, 18, 19	1–3, 8, 11, 15, 16	2, 5, 6, 8–10, 14, 16
MP 8	4, 5, 9, 17, 18	5	2, 6, 13, 15, 16	5, 10, 14	5	4, 6–8, 12	14	11

Bristol Public Schools Vision of the Graduate

Problem Solving

- iM's focus on real-world modeling and problem-solving strategies
- Multiple solution pathways are encouraged and explored
- Students develop perseverance through challenging tasks

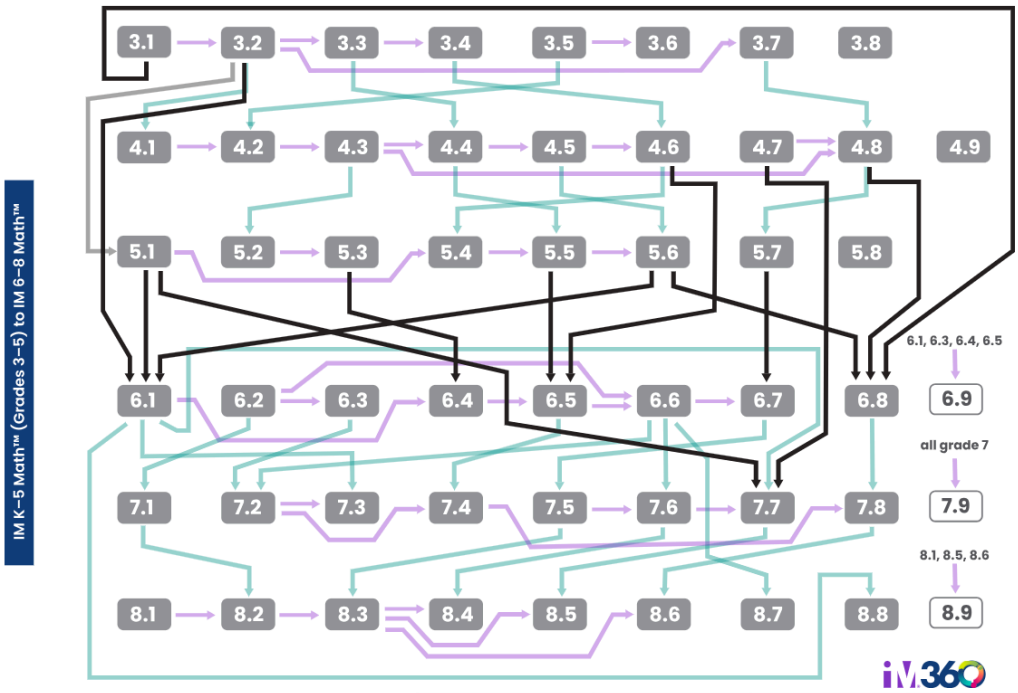
- [Unlocking Learners' Thinking Using the Mathematical Language Routines]
- [A Circumference By Any Other Name . . .]

Critical Thinking
- Students analyze mathematical relationships and justify their reasoning
- Regular opportunities to critique others' reasoning
- Emphasis on understanding "why" not just "how"

Communication and Collaboration
- Structured mathematical discourse is built into lessons
- Students explain their thinking both verbally and in writing
- Many activities involve partner and group work

Additional Course Information:
Knowledge/Skill Dependent courses/prerequisites

The following chart shows unit dependencies across the curriculum for IM Grades 3–8.



	Common Core Abbreviation	Grade 6	Grade 7	Grade 8
Geometry	G	Units 1 & 4	Units 1, 3, 7	Units 1, 2, 5, 8
Ratios & Proportional Relationships	RP	Units 2 & 3	Units 2–4	
Expressions & Equations	EE	Units 6 & 7	Units 5–7	Units 2–4, 7, 8
Statistics & Probability	SP	Unit 8	Unit 8	Unit 6
The Number System	NS	Units 4, 5, & 7	Unit 5	Unit 8
Functions	F			Units 5 & 6

Link to *Equity Audit*

[Equity Curriculum Review - Math 6 (2024-25)]

Standard Matrix

Standard	Lessons																				
6.EE.A	U1L17	U5L5																			
6.EE.A.1	U1L17	U1L18	U6L12	U6L13	U6L14	U6L15															
6.EE.A.2	U6L7	U6L10	U6L11	U6L19																	
6.EE.A.2.a	U1L5	U1L9	U1L18	U6L7																	
6.EE.A.2.b	U4L1	U7L10																			
6.EE.A.2.c	U1L5	U1L6	U1L9	U1L10	U6L14	U6L15															
6.EE.A.3	U6L10	U6L11																			
6.EE.A.4	U6L8	U6L10	U6L11																		
6.EE.B	U6L4																				
6.EE.B.5	U6L2	U6L3	U6L4	U6L5	U6L15	U7L9	U7L10														
6.EE.B.6	U6L2	U6L3	U6L5	U6L6	U6L7	U7L8	U7L10														
6.EE.B.7	U6L3	U6L4	U6L5	U6L6	U6L19																
6.EE.B.8	U7L8	U7L9	U7L10																		

6.EE.C.9	U6L16	U6L17	U6L18	U6L19																		
6.G.A	U9L1																					
6.G.A.1	U1L2	U1L3	U1L4	U1L5	U1L6	U1L7	U1L8	U1L9	U1L10	U1L11	U1L19	U4L14										
6.G.A.2	U1L15	U4L14	U4L15	U4L17																		
6.G.A.3	U7L15	U7L19																				
6.G.A.4	U1L12	U1L13	U1L14	U1L15	U1L16	U1L18	U1L19															
6.NS.A	U9L11																					
6.NS.A.1	U4L4	U4L5	U4L6	U4L7	U4L8	U4L9	U4L10	U4L11	U4L12	U4L13	U4L14	U4L16	U4L17									
6.NS.B	U5L5	U5L6	U9L1	U9L4																		
6.NS.B.2	U5L9	U5L10	U5L11	U5L12	U5L13	U5L14																
6.NS.B.3	U5L2	U5L3	U5L4	U5L7	U5L8	U5L9	U5L13	U5L14	U5L15	U6L4	U8L12	U9L8										
6.NS.B.4	U7L16	U7L17	U7L18																			
6.NS.C	U7L4																					
6.NS.C.5	U7L1	U7L5																				
6.NS.C.6	U7L1	U7L2	U7L4	U7L7	U7L14																	
6.NS.C.6.a	U7L2	U7L4																				
6.NS.C.6.b	U7L11	U7L12	U7L14																			
6.NS.C.6.c	U7L2	U7L11	U7L12	U7L13	U7L15																	
6.NS.C.7	U7L4	U7L6	U7L7																			
6.NS.C.7.a	U7L3	U7L7	U7L9																			
6.NS.C.7.b	U7L3	U7L8																				
6.NS.C.7.c	U7L6	U7L13																				
6.NS.C.7.d	U7L6	U7L7																				
6.NS.C.8	U7L11	U7L13	U7L14	U7L15	U7L19																	
6.RP.A	U2L17	U3L17	U9L1	U9L4																		
6.RP.A.1	U2L1	U2L2	U2L3	U2L4	U2L5	U9L5																
6.RP.A.2	U2L10	U3L4	U3L5	U3L6	U3L8	U9L8																
6.RP.A.3	U2L6	U2L7	U2L10	U2L11	U2L12	U2L13	U2L14	U2L15	U2L16	U3L5	U3L6	U3L7	U3L9	U3L15	U3L17	U9L2	U9L3	U9L5	U9L6	U9L8	U9L9	
6.RP.A.3.a	U2L11	U2L12	U2L13	U6L16	U6L17																	
6.RP.A.3.b	U2L8	U2L9	U2L10	U3L4	U3L5	U3L6	U3L7	U3L8	U3L9	U6L17												
6.RP.A.3.c	U3L10	U3L11	U3L12	U3L13	U3L14	U3L15	U3L16	U6L6	U9L5	U9L9												
6.RP.A.3.d	U3L2	U3L3	U9L2																			
6.SP.A	U8L2																					
6.SP.A.1	U8L2	U8L3	U8L6	U8L7	U8L17																	
6.SP.A.2	U8L4	U8L5	U8L7	U8L8	U8L11	U8L18																
6.SP.A.3	U8L6	U8L9	U8L10	U8L11																		
6.SP.B	U8L1	U8L2	U8L4	U8L5	U8L7	U8L9	U8L13	U8L18														
6.SP.B.4	U8L3	U8L4	U8L5	U8L6	U8L7	U8L8	U8L16	U8L17														
6.SP.B.5	U8L17																					
6.SP.B.5.a	U8L3	U8L4																				
6.SP.B.5.b	U8L2	U8L3	U8L5	U8L6	U8L7	U8L14																
6.SP.B.5.c	U8L9	U8L10	U8L11	U8L12	U8L13	U8L14	U8L15	U8L16	U8L18													
6.SP.B.5.d	U8L12	U8L14	U8L15	U8L16	U8L18																	

Unit Links

Use of Instructional Time (181 School Days)

- ➔ 162 iM Content and Assessment Days
- ➔ 6 Climate and Culture Days: 2 days at start of year, 2 shortened days before breaks, and 2 days at end of year
- ➔ 9 IAB Days: 1 day Strategic Review and 2 day IAB in fall, winter, and spring
- ➔ 4 SBA Days: 1 day Strategic Review and 3 day SBA

Unit Title:

Unit 1: Area and Surface Area

Relevant Standards: Bold indicates priority

Lesson	Standards	Lesson	Standards
Lesson 1		Lesson 11	6.G.A.1
Lesson 2	6.G.A.1	Lesson 12	6.G.A.4
Lesson 3	6.G.A.1	Lesson 13	6.G.A.4
Lesson 4	6.G.A.1	Lesson 14	6.G.A.4
Lesson 5	6.EE.A.2.a 6.EE.A.2.c 6.G.A.1	Lesson 15	6.G.A.2 6.G.A.4
Lesson 6	6.EE.A.2.c 6.G.A.1	Lesson 16	6.G.A.4
Lesson 7	6.G.A.1	Lesson 17	6.EE.A 6.EE.A.1
Lesson 8	6.G.A.1	Lesson 18	6.EE.A.1 6.EE.A.2.a 6.G.A.4
Lesson 9	6.EE.A.2.a 6.EE.A.2.c 6.G.A.1	Lesson 19	6.G.A.1 6.G.A.4
Lesson 10	6.EE.A.2.c 6.G.A.1		

Essential Question(s):

- What is the meaning of "area," and how can it help us understand shapes and their measurements?
- What is the relationship of the base and height for parallelograms and triangles, and why is knowing the base and height essential to finding the area of a shape?
- What characteristics define polygons, polyhedra, and their nets, and how can we use these features to calculate surface area?
- How can we find the area of complex shapes by decomposing them into simpler parts and rearranging them?
- How do surface area and volume differ, and why might two objects with the same surface area have different volumes?

Enduring Understanding(s):

- Area represents the amount of two-dimensional space a shape covers and is a fundamental measurement in geometry and real-world contexts.
- Finding the base and height is necessary for calculating the area of parallelograms and triangles.
- Mathematical formulas, such as those for the areas of parallelograms and triangles and volume of cubes, are powerful tools derived from reasoning and patterns, enabling efficient and accurate calculations.
- Complex shapes can be broken down into simpler components, and their areas can be rearranged to better understand and calculate the total area.
- Surface area refers to the total area covering the outside of a shape, while volume measures the space inside it.

Demonstration of Learning:

Checkpoint A is an opportunity for feedback
CFA 1: Checkpoint B (after lesson 6)
CFA 2: Checkpoint C (after lesson 11)
MoU: Assessment A (after lesson 11)
Checkpoint D is an opportunity for feedback
CFA 3 Checkpoint E (after lesson 18)
EoU: Assessment A (after lesson 19)

Pacing for Unit

24 Days
Lesson Modifications:

- Combine 6.1.1 and 6.1.2
- Remove 6.1.16 - An optional lesson. Late unit exploration
- Move to outside of class 6.1.19 - Culminating lesson. Optional given time constraints.

Family Overview

Family Resources: Area and Surface Area
Recursos Familiares: Área y área de superficie

Integration of Technology:

- Desmos Online Graphing Calculator
- Pear Assessment (Edulastic)
- iM v.360 Digital Applets (see below)

Unit-specific Vocabulary:

	New Terminology	
Lesson	receptive	productive
6.1.1	area, region, plane, gap, overlap	
6.1.2	area, compose, decompose, rearrange, two-dimensional	
6.1.3	shaded, strategy	

Aligned Unit Materials, Resources, and Technology

Lesson 1: Tiling the Plane
Interactive pattern exploration applet that allows students to:

- View patterns on a triangular grid
- Frame repeating larger hexagons
- Isolate and manipulate individual shapes

Lesson 2: Finding Area by Decomposing and Rearranging
Visualization applet for:

6.1.4	parallelogram, opposite (sides or angles)	quadrilateral
6.1.5	base (of a parallelogram or triangle), height, corresponding, expression, represent	
6.1.6	horizontal, vertical	
6.1.7	identical	parallelogram
6.1.8	diagram	base (of a parallelogram or triangle), height, compose, decompose, rearrange
6.1.9	opposite vertex	
6.1.10	vertex, edge	
6.1.11	polygon	horizontal, vertical
6.1.12	face, surface area	area, region
6.1.13	polyhedron, net, prism, pyramid, base (of a prism or pyramid), 3D	polygon, vertex, edge, face
6.1.15		prism, pyramid
6.1.16	volume, appropriate, quantity	two-dimensional, three-dimensional
6.1.17	squared, cubed, exponent, edge length	
6.1.18	value (of an expression)	squared, cubed, net
6.1.19	estimate, description	surface area, volume

- Manipulating rectangles and right triangles
- Exploring decomposition and rearrangement of shapes
- Understanding area conservation

Lesson 4: Parallelograms

Area exploration applet featuring:

- Pre-loaded rectangles and right triangles
- Tools for visualizing decomposition and rearrangement
- Support for understanding parallelogram area

Lesson 13-14: Polyhedra and Nets

3D visualization tools for:

- Manipulating and rotating polyhedra
- Exploring nets and their relationships to 3D shapes
- Assembling nets into polyhedra

These applets support key learning goals around **area, shape composition/decomposition**, and **spatial visualization**.

Students are likely to need physical tools to support their reasoning:

Tracing paper is an excellent tool for verifying that figures "match up exactly." At all times in the unit, each student should have access to a geometry toolkit, which contains tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles. Access to the toolkit also enables students to practice selecting appropriate tools and using them strategically (MP5). In a digitally enhanced classroom, apps and simulations should be considered additions to their toolkits, not replacements for physical tools.

Opportunities for Interdisciplinary Connections:

CTE

- Design efficient packaging solutions
- Calculate material needs for construction
- Optimize surface area for solar panels
- Plan conservation of materials
- Design efficient room layouts
- Plan webpage layouts

Art

- Create nets for 3D sculptures
- Plan murals for walls

Social Studies

- Calculate land area on maps
- Study population density
- Compare pricing for different areas

Anticipated misconceptions:

- Students incorrectly identify a height as the slanted side in relationship to the base.
- Students fail to divide by 2 or multiply by ½ when finding the area of a triangle.
- Students confuse finding the area of a shape with finding the perimeter.
- Students find area instead of surface area and vice versa when solving a problem.
- Students use square units to label volume .
- Students use one-dimensional units for area and surface area.

See teacher's guide for specific misconceptions aligned to each lesson.

Connections to Prior Units:

In grade 3, students found the area of rectangles with whole-number side lengths. They also found the area of rectilinear figures by decomposing them into non-overlapping rectangles and adding those areas. Students used a formula for the area of rectangles in grade 4 and found the area of rectangles with fractional side lengths in grade 5.

Essential prior concepts to engage with this unit:

- Properties of 2-D and 3-D figures
- Concept of area using rectangular prisms

Relevant Unit(s)/Lesson(s) to Review:

N/A

Connections to Future Units:

Students will draw on the work here to further study exponents later in grade 6 and to find volumes of prisms and pyramids in grade 7. Their understanding of "two figures that match up exactly" will support their work on congruence and rigid motions in grade 8.

See Adaptation Pack in the IL Classroom (6.1 Plan) for more suggestions on connections, pacing modifications, modified plans based on Check Your Readiness Assessment, and a priority list of lessons.

Differentiation through *Universal Design for Learning*

Multiple Representations & Tools (Multiple Means of Representation)

- Physical tools (tracing paper, graph paper, scissors)
- Visual models (nets, diagrams)
- Digital tools (see iM provided applets)
- Geometric manipulatives

Compare, Explain, Describe (Action & Expression)

- Comparing geometric patterns and shapes
- Explaining area-finding strategies
- Describing features of polyhedra

Progressive Language Development (Engagement & Expression)

- Building from informal to formal geometric vocabulary
- Connecting visual and verbal descriptions
- Developing precise language about 2D and 3D shapes

The routines in this unit particularly support:

- Hands-on exploration with multiple tools
- Building conceptual understanding through visualization
- Developing geometric reasoning and communication

Related *CELP standards* aligned to Learning Targets:

In this unit, teachers can anticipate students using language for mathematical purposes, such as comparing, explaining, and describing. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Compare

- Geometric patterns and shapes (Lesson 1).
- Strategies for finding areas of shapes (Lesson 3) and polygons (Lesson 11).
- The characteristics of prisms and pyramids (Lesson 13).
- The measurements and units of 1-, 2-, and 3-dimensional attributes (Lesson 16).
- Representations of area and volume (Lesson 17).

Explain

- How to find areas by composing (Lesson 3).
- Strategies used to find areas of parallelograms (Lesson 4) and triangles (Lesson 8).
- How to determine the area of a triangle using its base and height (Lesson 9).
- Strategies to find surface areas of polyhedra (Lesson 14).

Describe

- Observations about decomposition of parallelograms (Lesson 7).
- Information needed to find the surface area of rectangular prisms (Lesson 12).
- The features of polyhedra and their nets (Lesson 13).
- The features of polyhedra (Lesson 15).
- Relationships among features of a tent and the amount of fabric needed for the tent (Lesson 19).

	Emerging	Expanding	Bridging
LT1	Count unit squares Label area units Complete "Area means __"	Explain area concept "Area is the space that __"	Define area precisely Justify area measurements Use mathematical language
LT2	Use area guides Follow solution steps Show "First I __, then __"	Choose strategies with support "I found area by..."	Select efficient strategies Justify approach choices Analyze shape properties
LT3	Label parallel sides Match similar shapes Point to key features	Describe properties Explain characteristics "A parallelogram has..."	Analyze shape properties Define precisely Connect to other shapes
LT4	Label base and height Match measurements to parts Show "I need __ and __"	Explain measurement choices Describe calculation needs "To find area, I need..."	Justify measurement selection Connect to area formula Analyze measurement relationships
LT5	Follow area steps Use formula guide Complete "Area = __"	Explain formula reasoning Show why formula works "The formula works because..."	Derive area formula Justify formula logic Connect to transformations
LT6	Match related shapes Show shape parts Label "Half of __"	Explain relationships Describe connections "The triangle is related by..."	Analyze shape relationships Prove connections Use precise reasoning
LT7	Follow area steps Use visual models Show "Area = __ × __ ÷ 2"	Explain area method Connect to parallelograms "Triangle area works by..."	Derive triangle formula Justify method fully Connect multiple approaches

LT8	Match methods to shapes List solution steps Show "This way works for..."	Compare approaches Explain method choices "This strategy is better when..."	Evaluate strategies Justify method selection Analyze efficiency
LT9	Count face squares Label outside parts Complete "Surface area is __"	Explain surface area concept Describe measurement process "Surface area means..."	Define surface area precisely Connect to 2D concepts Justify measurement approach
LT10	Label faces and edges Match nets to shapes Show "This unfolds to..."	Describe shape properties Explain net relationships "The net shows..."	Analyze 3D properties Create precise nets Justify relationships
LT11	Sort measurements by type Label "Outside/Inside" Show "This measures __"	Compare attributes Explain differences "Surface area is different because..."	Analyze attribute relationships Justify distinctions Connect to real world
LT12	Follow measurement steps Use formulas with support Complete calculation guides	Apply formulas with reasoning Explain calculation process "I found __ by..."	Derive measurement formulas Justify calculation methods Connect multiple approaches
LT13	Use guided solutions Follow problem steps Show work with models	Solve with support Explain solution process "This makes sense because..."	Solve complex problems Create solution plans Justify approaches fully

Additional Sentence Frames and Stems

Section A

- I decomposed this figure into a _____ and _____ and rearranged them to make a _____.
- The area of the _____ is _____ the area of the _____ because ...
- The area of the _____ is _____ square units. I found this by ...

Section B

- I know the height of the parallelogram is _____ because it is perpendicular to the base which is _____.
- The lengths I will use to find the area of the parallelogram are _____ and _____ because...

Section C

- For a quadrilateral to be decomposed into two identical triangles, it must be (or must have) ...
- I decomposed the _____ to help me find the area of the triangle because...
- The area of the triangle is _____ because it is half the area of the parallelogram which has an area of _____.
- The height of the triangle is _____ because it is perpendicular to the base which is _____ .
- Figure _____ is/is not a polygon because ...

Section D

- The polyhedron is a pyramid/prism because ...
- The given net represents _____ because ...
- The faces of the net have individual areas of .... That means the total surface area of the polyhedron is _____.

Section E

- The area of the square is _____ , because ...
- The volume of the cube is _____ , because ...
- The surface area of the cube can be expressed like _____ because ...

Section F

- We chose our tent design because ...
- This tent design uses the least/most fabric because ...
- To find the surface area of each tent, I ...
- The amount of material needed for both tents is _____ because ...

Unit Outline

Lesson Sequence	Learning Target(s)	Success Criteria/Assessment
Section A Reasoning to Find Area (Lessons 1-3)	**Learning Target #1:** Understand what it means to find the area of shape. **Learning Target #2** Analyze a shape to identify a strategy to find its area.	**Lesson 1 Tiling the Plane** • I can explain the meaning of "area." **Lesson 2 Finding Area by Decomposing and Rearranging** • I can explain how to find the area of a figure that is composed of other shapes. • I know how to find the area of a figure by decomposing it and rearranging the parts. • I know what it means for two figures to have the same area. **Lesson 3 Reasoning to Find Area** • I can use different reasoning strategies to find the area of shapes.
Checkpoint A	*Responding to Student Thinking* More Chances: Students will have more opportunities to develop this understanding in later lessons. There is no need to slow down or add additional work to review this concept at this time.	

<table>
<tr><td>Section B
Parallelograms
(Lessons 4-6)</td><td>Learning Target #3
Understand the characteristics of a parallelogram.

Learning Target #4
Understand which measurements to use to calculate the area of parallelogram.

Learning Target #5
Generalize the process for finding the area of a parallelogram.</td><td>Lesson 4 Parallelograms
I can use reasoning strategies and what I know about the area of a rectangle to find the area of a parallelogram.
I know how to describe the characteristics of a parallelogram using mathematical vocabulary.
Lesson 5 Bases and Heights of Parallelograms
I can identify pairs of base and height of a parallelogram.
I can write and explain the formula for the area of a parallelogram.
I know what the terms "base" and "height" refer to in a parallelogram.
Lesson 6 Area of Parallelograms
I can use the area formula to find the area of any parallelogram.</td></tr>
<tr><td>Checkpoint B</td><td colspan="2">Responding to Student Thinking
Problem 1: Points to Emphasize If students struggle with finding base-height pairs for parallelograms, highlight these attributes when opportunities arise over the next several lessons. For example, urge students identify at least one base and a corresponding height for each parallelogram they compose in this activity: Grade 6, Unit 1, Lesson 7, Activity 3 A Tale of Two Triangles (Part 2)
Problem 2: Points to Emphasize If students struggle with finding the area of a parallelogram, discuss ways of doing so when opportunities arise over the next several lessons. For example, ask students to explain to a partner how to find the area of one of the parallelograms in this activity: Grade 6, Unit 1, Lesson 8, Activity 1 Composing Parallelograms
Problem 3: Points to Emphasize If students struggle with finding an unknown base or height of a parallelogram when the area is known, emphasize the relationship between those quantities when opportunities arise over the next several lessons. For example, highlight the connections between finding area and finding a missing base or height (given two other measurements) in this practice problem: Grade 6, Unit 1, Lesson 8, Practice Problem 5</td></tr>
<tr><td>Section C
Triangles and Other Polygons
(Lessons 7-11)</td><td>Learning Target #6
Understand the relationship between identical triangles and parallelograms.

Learning Target #7
Generalize the process for finding the area of a triangle.

Learning Target #8
Compare and contrast different strategies for finding areas of polygons.</td><td>Lesson 7 From Parallelograms to Triangles
I can explain the special relationship between a pair of identical triangles and a parallelogram.
Lesson 8 Area of Triangles
I can use what I know about parallelograms to reason about the area of triangles.
Lesson 9 Formula for the Area of a Triangle
I can use the area formula to find the area of any triangle.
I can write and explain the formula for the area of a triangle.
I know what the terms “base” and “height” refer to in a triangle.
Lesson 10 Bases and Heights of Triangles
I can identify pairs of base and corresponding height of any triangle.
When given information about a base of a triangle, I can identify and draw a corresponding height.
Lesson 11 Polygons
I can describe the characteristics of a polygon using mathematical vocabulary.
I can reason about the area of any polygon by decomposing and rearranging it, and by using what I know about rectangles and triangles.</td></tr>
<tr><td>Checkpoint C</td><td colspan="2">Responding to Student Thinking
Problem 1: Press Pause If most students struggle with identifying base-height pairs in triangles and parallelograms, make time to examine related work in the referenced section. The Course Guide provides additional ideas for revisiting earlier work. Grade 6, Unit 1, Section C Triangles and Other Polygons
Problem 2: Press Pause If most students struggle with finding the area of a triangle using given base-height pairs or by identifying those measurements first, make time to revisit these concepts. For example, plan to do the referenced optional activity about finding a base and a corresponding height that would facilitate area calculation. The Course Guide provides additional ideas for revisiting earlier work. Grade 6, Unit 1, Lesson 10, Activity 3 Some Bases Are Better Than Others
Problem 3: Points to Emphasize If students struggle to find the area of polygons, discuss ways to decompose polygons into triangles and parallelograms when opportunities arise in the next section. For example, provide access to colored pencils and ask students to color code the decomposed regions as they work on these practice problems: Grade 6, Unit 1, Lesson 13, Practice Problem 6</td></tr>
</table>

Mid-Unit Assessment		
Section D Surface Area (Lessons 12-16)	**Learning Target #9** Understand what it means to find the surface area of a three dimensional shape. **Learning Target #10** Understand the characteristics of polyhedra and their nets. **Learning Target #11** Comprehend that surface area and volume are two different attributes of three-dimensional objects.	**Lesson 12 What Is Surface Area?** ● I know what the surface area of a three-dimensional object means. **Lesson 13 Polyhedra** ● I can describe the features of a polyhedron using mathematical vocabulary. ● I can explain the difference between prisms and pyramids. ● I understand the relationship between a polyhedron and its net. **Lesson 14 Nets and Surface Area** ● I can match polyhedra to their nets and explain how I know. ● When given a net of a prism or a pyramid, I can calculate its surface area. **Lesson 15 More Nets, More Surface Area** ● I can calculate the surface area of prisms and pyramids. ● I can draw the nets of prisms and pyramids. **Lesson 16 Distinguishing Between Surface Area and Volume** ● I can explain how it is possible for two polyhedra to have the same surface area but different volumes, or to have different surface areas but the same volume. ● I know how one-, two-, and three-dimensional measurements and units are different.
Checkpoint D	*Responding to Student Thinking* ● Problem 1: Points to Emphasize If students struggle to connect polyhedra and their nets, discuss ways to relate the two representations. Do this when opportunities arise over the next several lessons. For example, instruct students to draw at least two nets: one net that would create a cube and one that would not create a cube. As they work on this activity, ask them to explain their reasoning: Grade 6, Unit 1, Lesson 18, Activity 2 The Net of a Cube ● Problem 2: Points to Emphasize If students struggle to find the surface area, emphasize ways to find the area of each face of a polyhedron and to organize the calculations systematically. For example, when students work on the indicated practice problem, ask them to identify all of its faces and then to determine (and, possibly, record) the shape and known measurements of each face. Next, ask them to come up with a strategy for finding the area of each face and the total surface area of the prism. Ask them to repeat those steps for Prism B. Grade 6, Unit 1, Lesson 17, Practice Problem 5	
Section E Squares and Cubes (Lessons 17-18)	**Learning Target #12** Generalize a process for finding the volume and surface area of a cube.	**Lesson 17 Squares and Cubes** ● I can write and explain the formula for the volume of a cube, including the meaning of the exponent. ● When I know the edge length of a cube, I can find the volume and express it using ● appropriate units. **Lesson 18 Surface Area of a Cube** ● I can write and explain the formula for the surface area of a cube. ● When I know the edge length of a cube, I can find its surface area and express it using appropriate units.
Checkpoint E	*Responding to Student Thinking* More Chances: Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.	
Section F Let's Put it To Work (Lesson 19)	**Learning Target #13** Use surface area to solve real world problems.	**Lesson 19 All about Tents** ● I can apply what I know about the area of polygons to find the surface area of three dimensional objects. ● I can use surface area to reason about real-world objects.
End of Unit Assessment		

Unit Title:

Unit 2: Introducing Ratios

Relevant Standards: Bold indicates priority

Lesson	Standards	Lesson	Standards
Lesson 1	6.RP.A.1	Lesson 10	6.RP.A.2 6.RP.A.3 6.RP.A.3.b
Lesson 2	6.RP.A.1	Lesson 11	6.RP.A.3 6.RP.A.3.a
Lesson 3	6.RP.A.1	Lesson 12	6.RP.A.3 6.RP.A.3.a
Lesson 4	6.RP.A.1	Lesson 13	6.RP.A.3 6.RP.A.3.a
Lesson 5	6.RP.A.1	Lesson 14	6.RP.A.3
Lesson 6	6.RP.A.3	Lesson 15	6.RP.A.3
Lesson 7	6.RP.A.3	Lesson 16	6.RP.A.3
Lesson 8	6.RP.A.3.b	Lesson 17	6.RP.A
Lesson 9	6.RP.A.3.b		

Essential Question(s):

- What does it mean for two quantities to be associated in a ratio, and how can we determine if ratios are equivalent?
- How do different representations (diagrams, tables, equations) help us understand and communicate ratio relationships?
- What role do unit rates play in comparing ratios and solving real-world problems?
- How do tables of equivalent ratios help us model and solve complex ratio problems?
- How does the context of a problem influence our choice of ratio solution strategy?

Enduring Understanding(s):

- Ratios can represent relationships in various contexts, such as part-to-part, part-to-whole, or comparisons between two sets of data.
- Generalizing strategies for equivalent ratios and using diagrams equips learners to solve a variety of real-world problems, such as scaling, budgeting, or comparing rates.
- The phrase "at this rate" describes a relationship, such as the cost per unit or the amount of an ingredient in a recipe per batch.
- Understanding the term “per” and how to calculate the price per item or unit enables problem-solving in various real-life situations, such as budgeting or shopping.
- Tables are a powerful tool for solving problems involving equivalent ratios, such as finding unit rates, unit prices, or quantities for consistent rates.
- Selecting the appropriate diagram (such as a tape diagram, ratio table, or double number line) is essential for organizing information and finding solutions efficiently.

Demonstration of Learning:

Checkpoint A is an opportunity for feedback
CFA 1: Checkpoint B (after lesson 5)
CFA 2 : Checkpoint C (after lesson 10)
Checkpoint D is an opportunity for feedback
CFA 3: Checkpoint E (after lesson 16)
EoU Assessment: A

Pacing for Unit

22 Days
Lesson Modifications:

- 6.2.1 and 6.2.2 combine to make sense of ratios.
- 6.2.3 and 6.2.4 have similar goals. 6.2.3 also requires physical manipulatives and could be combined with a lesson from the Accelerated course Acc6.2.2.
- 6.2.14 is optional and centers around using an additional strategy.
- 6.2.17 is optional. It is a culminating task that could be done outside of class.

Family Overview

Family Resources: Introducing Ratios
Recursos Familiares: Introducción a las razones

Integration of Technology:

- Desmos Online Graphing Calculator
- Pear Assessment (Edulastic)
- iM v.360 Digital Applets (see below)

Unit-specific Vocabulary:

	New Terminology	
Lesson	receptive	productive

Aligned Unit Materials, Resources, and Technology

Lessons 6-7: Double Number Line Diagrams
Interactive applets for:

- Creating and labeling double number lines

6.2.1	ratio, ___ to ___, ___ for every ___	
6.2.2	diagram	
6.2.3	recipe, batch, same taste, equivalent	ratio, ___ to ___, ___ for every ___
6.2.4	mixture, same color, check (an answer)	batch
6.2.5	equivalent ratios	
6.2.6	double number line diagram, tick marks, representation	diagram
6.2.7	per	
6.2.8	unit price, how much for 1, at this rate	double number line diagram
6.2.9	constant speed, meters per second	
6.2.10	same rate	equivalent ratios
6.2.11	table, row, column	
6.2.14	calculation	per, table
6.2.15	tape diagram, parts, suppose	
6.2.16	ratio, ___ to ___, ___ for every ___	tape diagram

- Visualizing equivalent ratios
- Practicing placement of tick marks and values
- Understanding the relationship between quantities

Lessons 11-13: Ratio Tables

Tools for:

- Building ratio tables from scratch
- Finding equivalent ratios
- Understanding the relationship between tables and double number lines
- Exploring patterns in ratio tables

Throughout Unit: Ratio Visualization Tools

Interactive tools for:

- Representing ratios with discrete diagrams
- Converting between different ratio representations
- Exploring equivalent ratios visually

These applets support key learning goals around understanding **ratio relationships, finding equivalent ratios**, and **using multiple representations.**

Opportunities for Interdisciplinary Connections:

Science

- Analyze genetic ratios
- Calculate body proportions

Art

- Use golden ratio in design
- Mix paint colors using ratios
- Create proportional drawings
- Study facial proportions in portraits

Music

- Analyze rhythm ratios
- Analyze tempo relationships
- Explore frequency ratios in scales

Physical Education

- Calculate winning ratios
- Study body proportions in athletics
- Balance workout routines
- Plane exercise-to-rest ratios

Anticipated misconceptions:

- Students add or subtract a value to both quantities thinking it makes an equivalent ratio
- Students reverse the order of the quantities thinking it makes an equivalent ratios
- Students deficient with math facts struggle to find a multiplier to use to find an equivalent ratio when using a ratio table
- Students try to compare ratios without finding a quantity that is in common to make a fair comparison of the ratio by analyzing the other quantity.

See teacher's guide for specific misconceptions aligned to each lesson.

Connections to Prior Units:

It builds on previous experiences students had with relating two quantities, such as converting measurements starting in grade 3, multiplicative comparison in grade 4, and interpreting multiplication as scaling in grade 5.

Essential prior concepts to engage with this unit:

- additive reasoning
- use of a number line
- dividing one whole number by another
- multiplication as scaling

Relevant Unit(s)/Lesson(s) to Review:

- IM Grade 5, Unit 3, Lessons 10 and 12 for division using algorithms.
- IM Grade 5, Unit 6, Lesson 17 for the use of a number line and multiplication by scaling.

Connections to Future Units:

The work prepares students to reason about unit rates and percentages in the next unit, proportional relationships in grade 7, and linear relationships in grade 8.

See Adaptation Pack in the IL Classroom (6.2 Plan) for more suggestions on connections, pacing modifications, modified plans based on Check Your Readiness Assessment, and a priority list of lessons.

Differentiation through *Universal Design for Learning*

Multiple Representations (Multiple Means of Representation)
- Discrete diagrams for ratio relationships
- Double number lines for equivalent ratios
- Tables for organizing ratio relationships
- Tape diagrams for ratio comparisons

Interpret, Explain, Compare (Action & Expression)
- Interpreting ratio statements and notations
- Explaining equivalence reasoning
- Comparing different ratio situations

Progressive Language Development (Engagement & Expression)
- Building from informal ratio language ("for every", "to")
- Developing precise mathematical vocabulary
- Connecting multiple ways to express ratios

The routines in this unit particularly support:
- Understanding ratios through concrete to abstract representations
- Building connections between different ratio models
- Developing precise mathematical language about ratios

Related *CELP standards* aligned to Learning Targets:

Progression of Disciplinary Language
In this unit, teachers can anticipate students using language for mathematical purposes, such as interpreting, explaining, and comparing. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Interpret
- Statements and notations describing ratios (Lesson 2).
- Different representations of ratios (Lessons 2 and 6).
- Situations involving equivalent ratios (Lesson 8).
- Situations with different rates (Lesson 9).
- Tables of equivalent ratios (Lessons 11 and 12).
- Questions about situations involving ratios (Lesson 17).

Explain
- Reasoning about equivalence (Lesson 4).
- Reasoning about equivalent rates (Lesson 10).
- Reasoning with reference to tables (Lesson 14).
- Reasoning with reference to tape diagrams (Lesson 15).

Compare
- Situations with and without equivalent ratios (Lesson 3).
- Representations of ratios (Lessons 6 and 13).
- Situations with different rates (Lessons 9 and 12).
- Situations with same rates and different rates (Lesson 10).
- Representations of ratio and rate situations (Lesson 16).

In addition, students are expected to describe and represent ratio associations, represent doubling and tripling of quantities in a ratio, represent equivalent ratios, justify whether ratios are or aren't equivalent and why information is needed to solve a ratio problem, generalize about equivalent ratios and about the usefulness of ratio representations, and critique representations of ratios.

	Emerging	Expanding	Bridging
LT1	Match quantities in pairs Use visual models Complete "For every __, there are __"	Describe ratio relationships Explain connections "This is a ratio because..."	Analyze quantitative relationships Justify ratio definitions Use precise ratio language
LT2	Draw ratio pictures Use given diagrams Label ratio parts	Create representations Explain diagram choices "I showed the ratio by..."	Select effective representations Connect multiple forms Justify representation choices
LT3	Test if ratios match Use multiplication tables Show "These are equal when..."	Find equivalent ratios Explain testing method "I know they're equal because..."	Create general strategies Prove equivalence Use mathematical reasoning
LT4	Copy ratio diagrams Fill in missing values	Create ratio diagrams Explain diagram features	Design effective diagrams Justify diagram choices

	Follow diagram patterns	"The diagram shows..."	Connect to problem context
LT5	Use given models Match equivalent forms Show "Same ratio as..."	Choose representations Explain relationships "This shows they're equal by..."	Select optimal representations Justify method choices Connect multiple approaches
LT6	Identify same/different rates Use rate comparison guides Complete "These are __ rate"	Compare rates with support Explain rate relationships "They're the same rate because..."	Analyze rate relationships Justify comparisons Use rate reasoning
LT7	Find patterns in tables Complete missing values Show "The pattern is..."	Describe table relationships Explain number patterns "In the table, when __ then __"	Analyze table structures Justify relationships Connect to ratio concepts
LT8	Follow solution steps Point to table values Show "I used __ to find __"	Explain solution process Describe table use "The table helped me by..."	Justify solution methods Evaluate table effectiveness Connect to other strategies
LT9	Use guided approaches Follow solution steps Complete solution frames	Solve with support Explain strategy choice "I solved it by..."	Solve complex problems Create solution plans Justify approaches
LT10	Match similar methods List solution steps Show "These are alike/different"	Compare approaches Explain similarities "This way is better because..."	Evaluate methods critically Justify method preferences Analyze effectiveness

Additional Sentence Frames and Stems

Section A

- There is/are _____ for every _____.
- For every _____ there is/are _____.
- The ratio of _____ to _____ is _____ to _____ or _____:_____.

Section B

- The ratio of _____ to _____ is equivalent to the ratio _____ to _____ because ...
- To make _____ the amount of the recipe/mixture, we can _____ each ingredient by _____. Therefore, an equivalent recipe/mixture would be _____ to _____.
- I can make a ratio equivalent to _____ to _____ by _____ each value by _____.

Section C

- The ratio _____ to _____ is equivalent to _____ to _____ because the double number line _____.
- To create my double number line, I chose to make the intervals _____ and _____ because ...
- Using a _____, I know the amount of _____ per one _____ is _____ because ...
- The speed of _____ is _____ per _____ because ...

Section D

- For every _____ there is/are _____.
- I _____ to find a ratio equivalent to the ratio _____ to _____.
- I _____ both values in the ratio by _____ to get the equivalent ratio of _____ to _____.

Section E

- Each part in the tape diagram represents _____ because the ratio of _____ to _____ is _____ to _____ and the total number of items is _____.
- I chose to use a _____ to solve this problem because it helped me see the relationship between the number of _____ and the number of _____.

Section F

- If I know how many _____ are in one _____, I can _____ to find out how many _____ are in _____.
- To solve this problem, I have to assume _____. From there, I can _____ to find a solution.

Unit Outline

Lesson Sequence	Learning Target(s)	Success Criteria/Assessment
Section A What are Ratios? (Lessons 1-2)	**Learning Target #1:** Understanding that an association between two quantities describes a ratio **Learning Target #2** Use multiple representations to describe ratio situations	**Lesson 1 Introducing Ratios and Ratio Language** • I can write or say a sentence that describes a ratio. • I know how to say words and numbers in the correct order to accurately describe the ratio. **Lesson 2 Representing Ratios with Diagrams** • I can draw a diagram that represents a ratio and explain what the diagram means. • I include labels when I draw a diagram that represents a ratio, so that the meaning of the diagram is clear.
Checkpoint A	*Responding to Student Thinking*	

	More Chances: Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.	
Section B Equivalent Ratios (Lessons 3-5)	**Learning Target #3** Generalize a strategy for determining if ratios are equivalent. **Learning Target #4** Create and use diagrams to represent equivalent ratios when solving problems in context	**Lesson 3 Recipes** • I can explain what it means for two ratios to be equivalent using a recipe as an example. • I can use a diagram to represent a recipe and to represent a double batch and a triple batch of the recipe. • I know what it means to double or triple a recipe. **Lesson 4 Color Mixtures** • I can explain the meaning of equivalent ratios using a color mixture as an example. • I can use a diagram to represent a single batch, a double batch, and a triple batch of color mixture. • I know what it means to double or triple a color mixture. **Lesson 5 Defining Equivalent Ratios** • If I have a ratio, I can create a new ratio that is equivalent to it. • If I have two ratios, I can decide whether they are equivalent to each other.
Checkpoint B	*Responding to Student Thinking* Points to Emphasize: If students struggle to describe whether two ratios in context are equivalent, integrate discussions about this when opportunities arise over the next several lessons. For example, ask students to describe to a partner some ways to tell that the ratios 4 : 1 and 12 : 3 are equivalent before discussing double number line diagrams in this activity: Grade 6, Unit 2, Lesson 6, Activity 1 Drink Mix on a Double Number Line	
Section C Representing Equivalent Ratios (Lessons 6-10)	**Learning Target #5** Use multiple representations to describe situations involving equivalent ratios **Learning Target #6** Analyze situations to determine if they are happening at the same rate.	**Lesson 6 Introducing Double Number Line Diagrams** • I can label a double number line diagram to represent batches of a recipe or color mixture. • When I have a double number line that represents a situation, I can explain what it means. **Lesson 7 Creating Double Number Line Diagrams** • I can create a double number line diagram and correctly place and label tick marks to represent equivalent ratios. • I can explain what the word "per" means • I can choose and create diagrams to help me reason about prices. • I can explain what the phrase “at this rate” means, using prices as an example. • If I know the price of multiple things, I can find the price per thing. **Lesson 8 How Much for One?** • I can choose and create diagrams to help me reason about prices. • I can explain what the phrase “at this rate” means, using prices as an example. · • If I know the price of multiple things, I can find the price per thing. **Lesson 9 Constant Speed** • I can choose and create diagrams to help me reason about constant speed. • If I know that an object is moving at a constant speed, and I know two of these things: the distance it travels, the amount of time it takes, and its speed, I can find the other thing. **Lesson 10 Comparing Situations by Examining Ratios** • I can decide if two situations are happening at the same rate. • I can explain what it means if two situations happen at the same rate. • I know some examples of situations where things can happen at the same rate.
Checkpoint C	*Responding to Student Thinking* • Problem 1: Press Pause: By this point in the unit, there should be some student mastery of finding equivalent ratios. If students struggle with the concepts in this Checkpoint, make time to examine related work in the section referred to here. The Course Guide provides additional ideas for revisiting earlier work. Grade 6, Unit 2, Section C Representing Equivalent Ratios • Problem 2: More Chances: Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.	
Section D	**Learning Target #7**	**Lesson 11 Representing Ratios with Tables**

Solving Ratio and Rate Problems (Lessons 11-14)	Understand the characteristics of a table involving equivalent ratios. **Learning Target #8** Describe how a table of equivalent ratios was used to solve a problem. **Learning Target #9** Analyze real world situations to solve problems involving equivalent ratios.	• If I am looking at a table of values, I know where the rows are and where the columns are. • When I see a table representing a set of equivalent ratios, I can come up with numbers to make a new row. • When I see a table representing a set of equivalent ratios, I can explain what the numbers mean. **Lesson 12 Navigating a Table of Equivalent Ratios** • I can solve problems about situations happening at the same rate by using a table and finding a "1" row. • I can use a table of equivalent ratios to solve problems about unit price. **Lesson 13 Tables and Double Number Line Diagrams** • I can create a table that represents a set of equivalent ratios. • I can explain why sometimes a table is easier to use than a double number line to solve problems involving equivalent ratios. • I include column labels when I create a table, so that the meaning of the numbers is clear. **Lesson 14 Solving Equivalent Ratio Problems** • I can decide what information I need to know to be able to solve problems about situations happening at the same rate. • I can explain my reasoning using diagrams that I choose.
Checkpoint D	*Responding to Student Thinking* • Problem 1: Points to Emphasize: If students struggle with finding and applying unit rates, integrate these ideas when opportunities arise over the next several lessons. For example, prompt students to identify the unit rate and to explain how it can help with problem-solving in these practice problems: Grade 6, Unit 2, Lesson 15, Practice Problem 4 Grade 6, Unit 2, Lesson 15, Practice Problem 6 • Problem 2: Points to Emphasize: If students struggle to choose multipliers strategically when solving rate problems, integrate discussions about this when opportunities arise over the next several lessons. For example, invite students to identify one or more multipliers that are helpful and some that are less so when solving this practice problem: Grade 6, Unit 2, Lesson 16, Practice Problem 4	
Section E Part-Part-Whole Ratios (Lessons 15-16)	**Learning Target #10** Compare and contrast different representations and solution methods for the same problem.	**Lesson 15 Part-Part-Whole Ratios** • I can create tape diagrams to help me reason about problems that involve both a ratio and a total amount. • I can solve problems when I know a ratio and a total amount. **Lesson 16 Solving More Ratio Problems** • I can choose and create diagrams to help think through my solution. • I can solve all kinds of problems about equivalent ratios. • I can use diagrams to help someone else understand why my solution makes sense.
Checkpoint E	*Responding to Student Thinking* • Problem 1: Points to Emphasize: If students struggle to solve problems that involve the sum of the quantities in a ratio, highlight strategies for reasoning about such problems as opportunities arise in upcoming sections. For example, ask students to explain or show how to tell which two numbers that add up to 35 cups are possible amounts of ingredients in the recipe in this practice problem: Grade 6, Unit 3, Lesson 1, Practice Problem 4 • Problem 2: Press Pause: If most students struggle to find and apply equivalent ratios to solve rate problems in context, make time to revisit these ideas. For example, do the optional activity referred to here, and discuss how to use equivalent ratios and diagrams to solve the problems. The Course Guide provides additional ideas for revisiting earlier work. Grade 6, Unit 2, Lesson 16, Activity 3 Cleaning Fluid and Moving Boxes	
Section F Let's Put it To Work (Lesson 17)		**Lesson 17 A Fermi Problem** • I can apply what I have learned about ratios and rates to solve a more complicated problem. • I can decide what information I need to know to be able to solve a real-world problem about ratios and rates.
End of Unit Assessment		

Unit Title:

Unit 3: Unit Rates and Percentages

Relevant Standards: Bold indicates priority

Lesson	Standards	Lesson	Standards
Lesson 1		Lesson 10	6.RP.A.3.c
Lesson 2	6.RP.A.3.d	Lesson 11	6.RP.A.3.c
Lesson 3	6.RP.A.3.d	Lesson 12	6.RP.A.3.c
Lesson 4	6.RP.A.2 6.RP.A.3.b	Lesson 13	6.RP.A.3.c
Lesson 5	6.RP.A.2 6.RP.A.3 6.RP.A.3.b	Lesson 14	6.RP.A.3.c
Lesson 6	6.RP.A.2 6.RP.A.3 6.RP.A.3.b	Lesson 15	6.RP.A.3 6.RP.A.3.c
Lesson 7	6.RP.A.3 6.RP.A.3.b	Lesson 16	6.RP.A.3.c
Lesson 8	6.RP.A.2 6.RP.A.3.b	Lesson 17	6.RP.A 6.RP.A.3
Lesson 9	6.RP.A.3 6.RP.A.3.b		

Essential Question(s):

- What strategies can we use to determine if two units of measure are equivalent?
- What is a unit rate, and how can it be applied in different contexts?
- How are ratios, rates, and percentages related, and how do they help us solve real-world problems?
- What tools and strategies (e.g., tables, double number lines) are most useful for solving problems with unit rates and percentages?
- How can we use unit rates and percentages to compare quantities and solve practical problems that represent proportional relationships in everyday life?
- How do we use multiplication and division of fractions and decimals to solve percentage problems?

Enduring Understanding(s):

- Using conversion factors and proportional reasoning helps us to determine if two units of measure are equivalent.
- Unit rates represent a simplified ratio of two quantities and can be applied to solve problems in contexts like speed, cost, and efficiency.
- Ratios, rates, and percentages are different ways to express proportional relationships and help solve real-world problems involving comparisons and scaling.
- Mathematical tools like tables and double number lines help organize data and visually represent relationships between quantities to solve problems.
- Unit rates and percentages are essential for comparing quantities and solving practical problems that involve proportional relationships in everyday life.
- Multiplication and division of fractions and decimals are necessary for calculating and interpreting percentages in various problem-solving situations.

Demonstration of Learning:

CFA 1: Checkpoint A (after lesson 3)
CFA 2: Checkpoint B (after lesson 9)
CFA 3: Checkpoint C (after lesson 16)
EOU Assessment: A

Pacing for Unit

21 Days
Lesson Modifications:

- Lesson 2: This lesson is designed to anchor students' perception of standard units. Could remove.
- Lesson 15: This lesson presents a more efficient way for finding A% of B. Could remove.
- Lesson 17: This culminating lesson provides an application of the material learned in the unit. It could be moved to outside of class if the additional time is needed.

Family Overview

Family Resources: Unit Rates and Percentages
Recursos Familiares: Tasas unitarias y porcentajes

Integration of Technology:

- Desmos Online Graphing Calculator
- Pear Assessment (Edulastic)
- iM v.360 Digital Applets (see below)

Unit-specific Vocabulary:

	New Terminology	
Lesson	receptive	productive
6.3.2	order	
6.3.4	(good / better / best) deal rate per 1	unit price, same speed

Aligned Unit Materials, Resources, and Technology

Lesson 2: Measurement Tools
Digital scale simulation applet for:

- Measuring mass and weight of objects
- Comparing different units of measurement
- Understanding relationships between units

Lessons 11-12: Percentage Visualization Tools
Interactive applets for:

6.3.5	unit rate	
6.3.6	result	unit rate
6.3.7		meters per second, (good / better / best) deal
6.3.8		at this rate
6.3.9	pace	speed
6.3.10	percentage ___% of	
6.3.11		tick marks
6.3.12	___% as much as	___% of
6.3.14		
6.3.15		percentage

- Creating and manipulating double number lines for percentages
- Exploring tape diagrams to represent percentages
- Converting between different percentage representations
- Understanding the relationship between percentages and fractions

Throughout Unit: Rate and Unit Rate Tools

Interactive tools for:

- Comparing rates and unit rates
- Visualizing speed and pricing scenarios
- Converting between different rate representations
- Understanding the relationship between rates and ratios

These applets support key learning goals around understanding **unit rates, converting measurements**, and **working with percentages.**

Opportunities for Interdisciplinary Connections:

Science

- Calculate rates of change in experiments
- Convert units of measurements
- Study speed and velocity rates
- Study resource consumption rates
- Analyze population growth

Social Studies

- Study voting rates
- Calculate immigration rates
- Calculate population density rates

Physical Education and Wellness

- Analyze calories per serving
- Study heart rate zones
- Study metabolic rates

Anticipated Misconceptions:

- Using the inverse of the unit rate instead of the unit rate itself.
- Not using appropriate labels from a context.
- Not multiplying the unit rate to get to a percent.
- Confusing decimals and fractions (such as ⅛ equals 1.6).
- Rounding and estimation errors, especially with decimals.
- Uneven scaling or nonproportional reasoning.

See teacher's guide for specific misconceptions aligned to each lesson.

Connections to Prior Units:

Students build on their experience with equivalent ratios and constant rates earlier in the course. They also build on knowledge of measurement and unit conversion in earlier grades. When learning about percentages, they draw on ideas about multiplicative comparison and equivalent fractions from grade 4 and multiplication of fractions from grade 5. Students begin by recalling what they know about standard units of measurement.

Essential prior concepts to engage with this unit:

- Converting units of the same scale
- Ratio reasoning
- Use of a double number line

Relevant Unit(s)/Lesson(s) to Review:

Grade 4 Unit 5: Multiplicative Comparison and Measurement

Connections to Future Units:

In grade 7, students will rely on their knowledge of equivalent ratios and unit rates to make sense of proportional relationships and constants of proportionality. Their understanding of percentages will support them in reasoning about percent increase and decrease.

See Adaptation Pack in the IL Classroom (6.3 Plan) for more suggestions on connections, pacing modifications, modified plans based on Check Your Readiness Assessment, and a priority list of lessons.

Differentiation through *Universal Design for Learning*

Multiple Representations (Multiple Means of Representation)

- Tables for equivalent ratios
- Double number line diagrams for rates
- Tape diagrams for percentages
- Visual models for unit rates

Interpret, Explain, Justify (Action & Expression)

- Interpreting unit rates in context
- Explaining measurement relationships
- Justifying comparisons of rates and percentages

Progressive Language Development (Engagement & Expression)

- Building rate language (e.g., "per", "for each")
- Connecting everyday speed language to mathematical concepts

- Developing precise percentage vocabulary

The routines in this unit particularly support:

- Understanding rates through multiple representations
- Making connections between ratios and percentages
- Building real-world context understanding

Related *CELP standards* aligned to Learning Targets:

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes, such as interpreting, explaining, and justifying. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Interpret

- Unit rates in different contexts (Lesson 5).
- A context in which identifying a unit rate is helpful (Lesson 8).
- Situations involving constant speed (Lesson 9).
- Diagrams used to represent percentages (Lessons 11 and 12).
- Situations involving measurement, rate, and cost (Lesson 17).

Explain

- Reasoning for estimating and sorting measurements (Lesson 1).
- Reasoning about relative sizes of units of measurement (Lesson 2).
- Reasoning for comparing rates (Lessons 4 and 7).
- Reasoning about percentages (Lesson 11).
- Strategies for finding missing information involving percentages (Lesson 14).

Justify

- Reasoning about equivalent ratios and unit rates (Lesson 6).
- Reasoning about finding percentages (Lessons 15 and 16).
- Reasoning about costs and time (Lesson 17).

In addition, students have opportunities to generalize about equivalent ratios, unit rates, and percentages from multiple contexts and with reference to benchmark percentages, tape diagrams, and other mathematical representations. Students can also be expected to describe measurements and observations, describe and compare situations involving percentages, compare speeds, compare prices, and critique reasoning about costs and time.

	Emerging	Expanding	Bridging
LT1	Match units to objects Order units by size Complete "A ___ is longer than a ___"	Compare units with reasoning Explain size relationships "I know ___ is larger because..."	Analyze unit relationships Justify comparisons fully Make precise connections
LT2	Follow conversion steps Use conversion charts Show "__ = __ because..."	Create conversion strategies Explain method choices "To convert, I first..."	Develop efficient methods Justify conversion approaches Connect multiple strategies
LT3	Match equivalent ratios Use visual models Complete "These are equal because..."	Find equivalent ratios Explain relationships "The ratios are equal when..."	Analyze ratio relationships Prove equivalence Use proportional reasoning
LT4	Find unit prices Use rate tables Show "One ___ costs ___"	Calculate unit rates Explain rate meaning "The rate means..."	Apply rates flexibly Justify rate calculations Connect to proportions
LT5	Identify related rates Use rate pairs Complete "If __, then __"	Explain inverse relationships Show connections "When one rate is __, the other..."	Analyze rate relationships Justify inverse connections Use precise language
LT6	Show parts per 100 Use percent models Label "__ out of 100"	Connect ratios to percents Explain percent meaning "50% means..."	Apply percent concepts Justify representations Make connections fluently
LT7	Match equivalent forms Use visual models Show "This means the same as..."	Compare representations Explain similarities "This shows ___ because..."	Analyze representations Evaluate effectiveness Justify preferences
LT8	Follow percent steps Use calculation guides Show "To find __, I..."	Select solution methods Explain calculations "I solved by..."	Choose efficient strategies Justify solution methods Apply flexibly
LT 9	Use guided solutions Follow problem steps Complete solution frames	Solve with support Explain reasoning "The solution makes sense because..."	Solve complex problems Create solution plans Justify approaches fully

Additional Sentence Frames and Stems

Section A

- The ratio of _____ to _____ is equivalent to _____ per _____, and I can use that rate to find ...
- I will use _____ to measure _____ because ...
- I know there are _____ in one _____, and I used that to find how many _____ are in _____.

Section B

- Given there are _____ for every _____ in this situation, _____ will happen because ...
- The two unit rates for this scenario are _____ per _____ and _____ per _____. I chose the unit rate _____ to solve this problem because ...
- It takes more/fewer _____ than _____ to measure _____ because ...
- The better deal is _____ because ...

Section C

- I chose _____ to solve this percentage problem because ...
- _____% of _____ is _____. I know this because ...
- _____ is _____% of _____. I know this because ...
- I calculated the percent of _____ by _____. I chose this strategy because ...

Section D

- Before I purchase the materials needed to paint this room, I need to know _____.
- We should buy _____ containers to paint _____ square feet because...
- It takes each person _____ to paint _____. I can save _____ by inviting a friend to paint with me, which would save _____ because ...

Unit Outline

Lesson Sequence	Learning Target(s)	Success Criteria/Assessment
Section A Units of Measurement and Unit Conversion (Lessons 1-3)	**Learning Target #1** Understand and compare different units of measure, including their approximate size. **Learning Target #2** Represent a strategy to solve unit conversions, including unit rate.	**Lesson 1 Anchoring Units of Measurement** • I can name common objects that are about as long as 1 inch, foot, yard, mile, millimeter, centimeter, meter, or kilometer. • I can name common objects that weigh about 1 ounce, pound, ton, gram, or kilogram, or that hold about 1 cup, quart, gallon, milliliter, or liter. • When I read or hear a unit of measurement, I know whether it is used to measure length, weight, or volume. **Lesson 2 Measuring with Different-Size Units** • When I know a measurement in one unit, I can decide whether it takes more or less of a different unit to measure the same quantity. **Lesson 3 Converting Units** • I can convert measurements from one unit to another, using double number lines, tables, or by thinking about "how much for 1." • I know that when we measure things in two different units, the pairs of measurements are equivalent ratios.
Checkpoint A	*Responding to Student Thinking* • Problem 1: Points To Emphasize: If most students struggle to recognize a pair of measurements (of the same thing) in different units as a ratio, reinforce this idea throughout the next section. For example, in the practice problem referred to here, emphasize that the given information—10 kilograms being approximately 22 pounds—forms a ratio. This means that we can convert a measurement in one unit into the other unit by finding ratios that are equivalent to the given ratio. Grade 6, Unit 3, Lesson 6, Practice Problem 7 • Problem 2: Points To Emphasize: If most students struggle to perform unit conversion when the numbers in the given rate and in the known measurement are not related by whole-number multiples, over the next section emphasize the usefulness of finding a rate per 1. For example, when discussing the practice problem referred to here, ask students to explain why finding the weight of 1 slice of cheese might be an effective way to find the weight of 18 slices. Grade 6, Unit 3, Lesson 7, Practice Problem 1	
Section B Rates (Lessons 4-9)	**Learning Target #3** Compare ratios and determine if they are equivalent. **Learning Target #4** Understand "how much for 1" to solve ratio	**Lesson 4 Comparing Speeds and Prices** • I understand that if two ratios have the same rate per 1, they are equivalent ratios. • When measurements are expressed in different units, I can decide who is traveling faster or which item is the better deal by comparing "how much for 1" of the same unit. **Lesson 5 Interpreting Rates**

	problems in different contexts. **Learning Target #5** Recognize the inverse relationship between the two unit rates in a context.	• I can choose which unit rate to use based on how I plan to solve the problem. • When I have a ratio, I can calculate its two unit rates and explain what each of them means in the situation. **Lesson 6 Equivalent Ratios Have the Same Unit Rates** • I can give an example of two equivalent ratios and show that they have the same unit rates. I can multiply or divide by the unit rate to calculate missing values in a table of equivalent ratios. **Lesson 7 More Rate Comparisons** • I can choose how to use unit rates to solve problems. **Lesson 8 Solving Rate Problems** • I can see that thinking about "how much for 1" is useful for solving different types of problems. **Lesson 9 More about Constant Speed** • I can solve more complicated problems about constant speed situations.
Checkpoint B	*Responding to Student Thinking* • Problem 1: Points to Emphasize: If most students struggle with recognizing both unit rates given a ratio or rate, find opportunities for students to identify and interpret unit rates in upcoming sections. For example, make time to find the unit rates in each situation when discussing the practice problem or activity referred to here. Grade 6, Unit 3, Lesson 10, Practice Problem 6 Grade 6, Unit 3, Lesson 7, Practice Problem 1 • Problem 2: Press Pause: If most students struggle to use unit rates or equivalent ratios to solve problems involving rates, make time to revisit different strategies for reasoning about such problems. For example, spend time discussing ways to solve the first several practice problems of the lesson referred to here. Invite multiple students to share their thinking. Consider recording their strategies on a chart to use for future reference. Grade 6, Unit 3, Lesson 8, Practice Problems	
Section C Percentages (Lessons 10-16)	**Learning Target #6** Understand the meaning of percentage as the rate per 100. **Learning Target #7** Compare and contrast multiple representations involving percentages. **Learning Target #8** Solve for the percentage of a given value using multiplication and division.	**Lesson 10 What Are Percentages?** • I can create a double number line diagram with percentages on one line and dollar amounts on the other line. • I can explain the meaning of percentages using dollars and cents as an example. **Lesson 11 Representing Percentages with Double Number Line Diagrams** • I can use double number line diagrams to solve different problems like "What is 40% of 60?" or "60 is 40% of what number?" **Lesson 12 Representing Percentages in Different Ways** • I can use tape diagrams and tables to solve different problems like "What is 40% of 60?" or "60 is 40% of what number?" **Lesson 13 Benchmark Percentages** • When I read or hear that something is 10%, 25%, 50%, or 75% of an amount, I know what fraction of that amount they are referring to. **Lesson 14 Solving Percentage Problems** • I can choose and create diagrams to help me solve problems about percentages. **Lesson 15 Finding This Percent of That** • I can solve different problems like "What is 40% of 60?" by dividing and multiplying. **Lesson 16 Finding the Percentage** • I can solve different problems like "60 is what percentage of 40?" by dividing and multiplying.
Checkpoint C	*Responding to Student Thinking* • Problem 1: Press Pause: If most students struggle to find percentages of a number or to identify values corresponding to 100%, make time to revisit strategies for making sense of situations involving percentages. For example, ask students to create a story or a diagram that represents each statement in these practice problems before solving them. Encourage students to explain their reasoning and to look for structure in the problems and representations. Grade 6, Unit 3, Lesson 15, Practice Problem 5	

	• Problem 2: Points to Emphasize: If students struggle with finding percentages greater than 100, plan multiple ways of reasoning about such percentages when opportunities arise over the next several lessons. For example, urge students to use double number line diagrams or tables to reason about 140% and 150% in these practice problems: Grade 6, Unit 4, Lesson 2, Practice Problem 6 Grade 6, Unit 4, Lesson 4, Practice Problem 7	
Section D Let's Put It to Work (Lesson 17)	**Learning Target #9** Use unit rates and percentages to solve real world problems.	**Lesson 17 Painting a Room** • I can apply what I have learned about unit rates and percentages to predict how long it will take and how much it will cost to paint all the walls in a room.
End of Unit Assessment		

Unit Title:

Unit 4: Dividing Fractions

Relevant Standards: Bold indicates priority

Lesson	Standards	Lesson	Standards
Lesson 1	6.EE.A.2.b	Lesson 10	6.NS.A.1
Lesson 2	Building towards 6.NS.A	Lesson 11	6.NS.A.1
Lesson 3	Building towards 6.NS.A.1	Lesson 12	6.NS.A.1
Lesson 4	6.NS.A.1	Lesson 13	6.NS.A.1
Lesson 5	6.NS.A.1	Lesson 14	6.G.A.1 6.G.A.2 6.NS.A.1
Lesson 6	6.NS.A.1	Lesson 15	6.G.A.2
Lesson 7	6.NS.A.1	Lesson 16	6.NS.A.1
Lesson 8	6.NS.A.1	Lesson 17	6.G.A.2 6.NS.A.1
Lesson 9	6.NS.A.1		

Essential Question(s):

- How can division be understood as finding an unknown factor in a multiplication equation?
- What strategies can we use to solve division problems when the size of the group is known but the number of groups is not?
- How does dividing a number by a fraction relate to multiplying the number by the reciprocal of that fraction?
- In what ways can tape diagrams and other visual models help us understand division of fractions?
- How can we apply division of fractions to solve multi-step problems that involve fractions in different contexts involving measurements?
- How does a deeper understanding of division and multiplication of fractions prepare us for operations with decimals and variable equations?

Enduring Understanding(s):

- Division can be understood as finding an unknown factor in a multiplication equation, connecting division to multiplication and making it easier to solve fraction problems.
- Dividing a number by a fraction is the same as multiplying that number by the reciprocal of the fraction, providing a consistent method for solving fraction division problems.
- Visual models like tape diagrams help represent and clarify the division of fractions, providing a concrete way to understand how fractions relate to each other in division situations.
- Division of fractions can be applied to solve multi-step problems in real-world contexts, such as measurements of length, area, or volume, by interpreting and calculating with fractional values.
- A deeper understanding of fraction division and multiplication supports the development of skills needed for operations with decimals and for writing and solving variable equations in future math concepts.

Demonstration of Learning:

CFA 1: Checkpoint A (after lesson 3)
CFA 2: Checkpoint B (after lesson 9)
MOU: Assessment A (after lesson 9)
CFA 3: Checkpoint C (after lesson 11)
CFA 4: Checkpoint D (after lesson 15)
EoU: Assessment A

Pacing for Unit

22 Days
Lesson Modifications:

- Remove 6.4.17: This lesson is an application of the concepts from the unit. It can be moved outside of class.
- Combine Lessons 6.4.12 and 6.4.13: Focus on the application of division to lengths of objects
- Combine Lessons 6.4.14 and 6.4.15: Focus on the application of division to the volume of objects

Family Overview

Family Resources: Dividing Fractions
Recursos Familiares: División de fracciones

Integration of Technology:

- Desmos Online Graphing Calculator
- Pear Assessment (Edulastic)
- iM v.360 Digital Applets (see below)

Unit-specific Vocabulary:

	New Terminology	
Lesson	receptive	productive
6.4.1	divisor, dividend	quotient
6.4.2	equation, interpretation, equal-size	How many groups of ___?,

Aligned Unit Materials, Resources, and Technology

Lessons 4-6: Division Visualization Tools
Interactive applets for:

- Creating and manipulating tape diagrams
- Visualizing division with fractions
- Understanding "how many groups" questions
- Exploring relationships between multiplication and division

Lessons 13-15: Area and Volume Tools

		How many ___ in each group?
6.4.3	unknown	
6.4.4	whole	
6.4.5		whole
6.4.6		equal-size
6.4.7	times as ___, fraction of ___	
6.4.8	container, section	unknown fraction of ___
6.4.10	observations	times as ___, numerator, denominator
6.4.11	reciprocal	
6.4.13		gaps
6.4.14	packed	
6.4.17	assumption	packed

Interactive tools for:

- Visualizing rectangles with fractional side lengths
- Exploring area of rectangles with fractions
- Understanding volume of rectangular prisms
- Manipulating 3D shapes with fractional edge lengths

Throughout Unit: Fraction Modeling Tools

Tools for:

- Representing division situations with diagrams
- Converting between different fraction representations
- Understanding relationships between fractions
- Visualizing fraction operations

These applets support key learning goals around understanding **division of fractions, working with fractional measurements**, and **calculating area and volume.**

Opportunities for Interdisciplinary Connections:

Science

- Calculate portions of mixtures and compounds
- Convert between measurement units using fractions

Culinary Arts

- Adjust recipes quantities up/down
- Calculate serving sizes
- Planning events
- Analyze nutritional information per serving size

Music

- Understand time signatures (dividing whole notes)
- Calculate note durations
- Divide measures into beats

Anticipated Misconceptions:

- Converting between fractions and mixed numbers
- Completing diagrams - not recognizing the places that the information is shown in the diagrams
- Confusing which denominator to reference when finding the value of remainders
- Errors when simplifying

See teacher's guide for specific misconceptions aligned to each lesson.

Connections to Prior Units:

This work draws on students' prior knowledge of multiplication, division, and the relationship between the two. It also builds on concepts from grades 3 to 5 about multiplicative situations–equal-size groups, multiplicative comparison, and the area of a rectangle–and about fractions. Students begin by exploring meanings of division and the relationship between the quantities in division situations. They recall that we can think of dividing as finding an unknown factor in a multiplication equation. In situations involving equal-size groups, division can be used to answer two questions: "How many groups?" and "How much in each group?"

Essential prior concepts to engage with this unit:

- Understand division as an unknown-factor problem.
- Understand and represent "How many groups?" and "How many in each group?" problems and understand division as an unknown factor problem.
- Interpret division as a whole number.

Relevant Unit(s)/Lesson(s) to Review:

- Grade 5, Unit 3, Lesson 11: This lesson focuses on students determining the size of the piece when a

Connections to Future Units:

A deeper understanding of multiplication, division, and ways to represent them will support students in reasoning about decimal operations as well as in writing and solving variable equations later in the course.

See Adaptation Pack in the IL Classroom (6.4 Plan) for more suggestions on connections, pacing modifications, modified plans based on Check Your Readiness Assessment, and a priority list of lessons.

unit fraction is divided into equally sized pieces. This supports students to be able to interpret a fraction divided into equal pieces and, in this context, and to connect multiplication with division.
- If the Check Your Readiness assessment shows that students need additional familiarity with interpreting division, consider also referring to Grade 3 Unit 3 where students interpret whole number division using grouping.

Differentiation through *Universal Design for Learning*

Visual Models & Representations (Multiple Means of Representation)
- Tape diagrams for division situations
- Area models for fractional dimensions
- Multiple representations of division scenarios

Interpret, Represent, Justify (Action & Expression)
- Interpreting division situations
- Representing division with diagrams and equations
- Justifying division strategies

Progressive Language Development (Engagement & Expression)
- Building from informal to formal division language
- Connecting visual models to mathematical notation
- Moving from concrete to abstract representations

The routines in this unit particularly support:
- Understanding division concepts through multiple representations
- Making connections between visual models and algorithms
- Developing precise mathematical language about division

A note about diagrams:
Because tape diagrams are a flexible tool for illustrating and reasoning about division of fractions, they are the primary representation used in this unit. Students may, however, create other representations to support their reasoning.

Related *CELP standards* aligned to Learning Targets:

Progression of Disciplinary Language
In this unit, teachers can anticipate students using language for mathematical purposes, such as interpreting, representing, justifying, and explaining. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Interpret and Represent
- Situations involving division (Lessons 2, 3, 9, 12, and 16).
- Situations involving measurement constraints (Lesson 17).

Justify
- Reasoning about division and diagrams (Lessons 4 and 5).
- Strategies for dividing numbers (Lesson 11).
- Reasoning about volume (Lesson 15).

Explain
- How to create and make sense of division diagrams (Lesson 6).
- How to represent division situations (Lesson 9).
- How to find unknown lengths (Lesson 14).
- A plan for optimizing costs (Lesson 17).

In addition, students are expected to critique the reasoning of others about division situations and representations, and to make generalizations about division by comparing and connecting across division situations and across the representations used in reasoning about these situations. The Lesson Syntheses in Lessons 2 and 12 offer specific disciplinary language that may be especially helpful for supporting students in navigating the language of important ideas in this unit.

	Emerging	Expanding	Bridging
LT1	Match related facts Use visual models Complete "__ × __ = __, so __ ÷ __ = __"	Explain relationships with frames Show connections with diagrams "I know __ ÷ __ = __ because..."	Analyze relationships deeply Justify connections Use mathematical reasoning

LT2	Draw equal groups Label parts of diagrams Show "__ groups of __"	Create diagrams with support Explain grouping strategy "I made __ groups because..."	Model division situations Explain reasoning clearly Connect multiple representations
LT3	Follow solution steps Use strategy guides Show work with models	Choose strategies with support Explain method selection "This strategy works because..."	Select efficient strategies Justify method choices Evaluate effectiveness
LT4	Measure with fraction tools Label fractional parts Complete "The length is __"	Solve problems with guidance Explain solution process "To find the length, I..."	Solve complex problems Justify solution methods Connect to geometric concepts
LT5	Use visual models Follow solution guides Complete word problems	Solve problems with support Explain reasoning steps "First I __, then I __"	Solve contextual problems Create solution plans Justify approaches fully

Additional Sentence Frames and Stems

Section A

- I know this question is asking for the quotient between ___ and ___ because ...
- I notice that the quotient is larger/smaller than the dividend when the divisor is _____.
- I can interpret the division expression _____ as equal groups of _____ or as having _____ equal size groups.

Section B

- I can create a tape diagram with _____ sections that are each _____. This represents the division expression _____ because ...
- The division expression _____ can mean how many groups of _____ are in _____.
- I know _____ divided by _____ equals _____ because I used the multiplication expression _____ to check and got _____.
- I think the quotient for _____ divided by _____ will be less/greater than 1 because ...

Section C

- I can simplify the expression _____ by multiplying _____ by ____ because dividing by the unit fraction _____ results in the same value as multiplying by _____.
- I simplified the fraction _____ by ...

Section D

- The area of the rectangle is _____ because ...
- The area of the rectangle is _____ and the length of one side is _____. This helped me find the missing side length of the rectangle to be _____ because ...
- The volume of the prism is _____. I found this by ...
- Given the base area of the prism is _____ and the volume is _____, I know the height of the prism is _____ because ...

Section E

- I used the expression _____ to answer this question because ...
- I used _____ to help me solve this problem because ...
- The answer _____ makes sense/does not make sense because ...

Unit Outline

Lesson Sequence	Learning Target(s)	Success Criteria/Assessment
Section A Making Sense of Division (Lessons 1-3)	**Learning Target #1** Identify and interpret the relationship between multiplication and division.	**Lesson 1 Size of Divisor and Size of Quotient** • When dividing, I know how the size of a divisor affects the quotient. **Lesson 2 Meanings of Division** • I can explain how multiplication and division are related. • I can explain two ways of interpreting a division expression such as $27 \div 3$. • When given a division equation, I can write a multiplication equation that represents the same situation. **Lesson 3 Interpreting Division Situations** • I can create a diagram or write an equation that represents division and multiplication questions. • I can decide whether a division question is asking "How many groups?" or "How many in each group?".
Checkpoint A	*Responding to Student Thinking* • Problem 1: Points to Emphasize If most students struggle to interpret the division expression in two ways or to write a corresponding multiplication equation, reiterate the relationship between quantities in equal-group situations throughout the next section. For example, emphasize that multiplying the number of groups and the size of a group gives the total amount, so dividing the total amount by a number could mean finding either the number of groups or the size of one group. Consider creating a classroom display that summarizes this idea: (number of groups) · (size of one group)=total	

	total÷size of one group=number of groups total÷number of groups=size of one group • Problem 2: Points to Emphasize If most students struggle to interpret the multiplication equation in terms of equal-size groups, practice using words to describe equations with known and unknown values. For example, when discussing the diagram in the Warm-up referred to here, express $5 \cdot 1/5=1$ verbally as "5 times 1/5 is 1" and "5 groups of 1/5 make 1." Follow that by describing an equation such as $5 \cdot ?=1$ as "5 times what number is 1?" and "5 groups of what number make 1?" Grade 6, Unit 4, Lesson 4, Activity 1 Equal-size Groups	
Section B Meanings of Fraction Division (Lessons 4-9)	**Learning Target #2** Reason about division and division diagrams in terms of groups.	**Lesson 4 How Many Groups? (Part 1)** • I can find how many groups there are when the amount in each group is not a whole number. • I can use diagrams and multiplication and division equations to represent "How many groups?" questions. **Lesson 5 How Many Groups? (Part 2)** • I can find how many groups there are when the number of groups and the amount in each group are not whole numbers. **Lesson 6 Using Diagrams to Find the Number of Groups** • I can use a tape diagram to represent equal-size groups and to find the number of groups. **Lesson 7 What Fraction of a Group?** • I can tell when a question is asking for the number of groups and that number is less than 1. • I can use diagrams and multiplication and division equations to represent and answer "What fraction of a group?" questions. **Lesson 8 How Much in Each Group? (Part 1)** • I can tell when a question is asking for the amount in one group. • I can use diagrams and multiplication and division equations to represent and answer "How much in each group?" questions. **Lesson 9 How Much in Each Group? (Part 2)** • I can find the amount in one group in different real-world situations.
Checkpoint B	*Responding to Student Thinking* • Problem 1: Points to Emphasize: If most students struggle to represent and reason about the size of one group given the amount in a fraction of a group, revisit ways to use a diagram to make sense of the quantities in such a situation. For example, when doing the practice problem referred to here, ask students to create a diagram to show the relationship between 1 batch, ⅔ batch, 8/5 kg, and the unknown quantity. Grade 6, Unit 4, Lesson 10, Practice Problem 5 • Problem 2: Press Pause: If most students struggle to write equations to represent the situation and to explain the answer of 15, revisit the idea that "How many of these are in that?" questions can be understood in terms of equal-size groups. For example, complete one or both of the optional activities referred to here. Ask students to think about how each situation can be framed in terms of number of groups $\cdot$ size of group = total amount and expressed with a multiplication equation and a corresponding division equation. Also discuss the values that would make each equation true. Grade 6, Unit 4, Lesson 5, Activity 3 Drawing Diagrams to Show Equal-size Groups Grade 6, Unit 4, Lesson 7, Activity 3 Fractions of Ropes	
Mid Unit Assessment		
Section C Algorithm for Fraction Division (Lessons 10-11)	**Learning Target #3** Justify strategies for dividing numbers.	**Lesson 10 Dividing by Unit and Non-Unit Fractions** • I can divide a number by a non-unit fraction by reasoning with the numerator and denominator, which are whole numbers. • I can divide a number by a unit fraction $\frac{1}{b}$ by reasoning with the denominator, which is a whole number. **Lesson 11 Using an Algorithm to Divide Fractions** • I can describe and apply a rule to divide numbers by any fraction.
Checkpoint C	*Responding to Student Thinking* • Problem 1: Points to Emphasize: If most students struggle to identify at least two expressions that involve multiplying by 2 and dividing by 3 (or their equivalents), use a tape diagram to reinforce the idea that dividing a number n by a unit fraction 1/b gives $n \cdot b$ parts, and dividing by a/b means putting those pieces into groups of a parts, which means dividing by a. For example, when discussing $12 \div 4/3$, ask students to show how dividing 12 by ⅓ creates $12 \cdot 3$ parts and dividing 12 by 4/3 means putting those pieces into groups of 4 or dividing them by 4. Grade 6, Unit 4, Lesson 13, Practice Problem 2 • Problem 2: More Chances: Students will have more opportunities to apply the division algorithm in	

	future sections. There is no need to slow down or add additional work to review this concept at this time.	
Section D Fractions in Lengths, Areas, and Volumes (Lessons 12-15)	**Learning Target #4** Reason about geometric figures involving fractional lengths using division and multiplication.	**Lesson 12 Fractional Lengths** • I can use division and multiplication to solve problems involving fractional lengths. **Lesson 13 Rectangles with Fractional Side Lengths** • I can use division and multiplication to solve problems involving areas of rectangles with fractional side lengths. **Lesson 14 Fractional Lengths in Triangles and Prisms** • I can explain how to find the volume of a rectangular prism using cubes that have a unit fraction as their edge length. • I can use division and multiplication to solve problems involving areas of triangles with fractional bases and heights. • I know how to find the volume of a rectangular prism even when the edge lengths are not whole numbers. **Lesson 15 Volume of Prisms** • I can solve volume problems that involve fractions.
Checkpoint D	*Responding to Student Thinking* • Problem 1: Points to Emphasize: If most students struggle to find an unknown side length of a rectangle, reinforce the relationship between the side lengths and the area of a rectangle as opportunities arise over the next several lessons. For example, when discussing problems D1 and D2 in the activity referred to here, emphasize that the equation $l \cdot w=A$ still represents that relationship even when the measurements are fractional. Grade 6, Unit 4, Lesson 16, Activity 3 Pairs of Problems Grade 6, Unit 4, Lesson 13, Practice Problem 2 • Problem 2: Points to Emphasize: If most students identify choice D but do not identify B or E, revisit the idea that the volume of a rectangular prism can also be found by using the number of unit cubes that can be packed in the prism and the volume of each cube. For example, when discussing the practice problem referred to here, ask students to find the volume of the ice storage box in two ways: by using the volume of each ice block and the number of the blocks that can fit in the box, and by using the edge lengths of the storage box. Grade 6, Unit 4, Lesson 16, Practice Problem 4 • Problem 3: Points to Emphasize: If most students struggle to see the question in terms of finding an unknown factor in the multiplication equation (area of base) $\cdot$ (height of prism)=volume, use this Checkpoint problem to highlight the relationship between the edge lengths, area of the base, and volume of a prism. Reinforce this idea as opportunities arise over the next several lessons.	
Section E Let's Put It to Work (Lessons 16-17)	**Learning Target #5** Reason about real world situations involving fractional lengths using division and multiplication.	**Lesson 16 Solving Problems Involving Fractions** • I can use mathematical expressions to represent and solve word problems that involve fractions. **Lesson 17 Fitting Boxes into Boxes** • I can use multiplication and division of fractions to reason about real-world volume problems.
End of Unit Assessment		

Unit Title:

Unit 5: Arithmetic in Base Ten

Relevant Standards: Bold indicates priority

Lesson	Standards	Lesson	Standards
Lesson 1		Lesson 9	6.NS.B.2 6.NS.B.3
Lesson 2	6.NS.B.3	Lesson 10	6.NS.B.2
Lesson 3	6.NS.B.3	Lesson 11	6.NS.B.2
Lesson 4	6.NS.B.3	Lesson 12	6.NS.B.2
Lesson 5	6.EE.A 6.NS.B	Lesson 13	6.NS.B.2 6.NS.B.3
Lesson 6	6.NS.B	Lesson 14	6.NS.B.2 6.NS.B.3
Lesson 7	6.NS.B.3	Lesson 15	6.NS.B.3
Lesson 8	6.NS.B.3		

Essential Question(s):

- How does the base-ten number system help us understand and perform operations with decimals?
- What strategies can we use to add, subtract, and multiply decimals beyond tenths and hundredths?
- How do different methods for multiplying decimals (such as using decimal fractions and partial products) relate to the standard multiplication algorithm?
- In what ways can visual models like base-ten blocks and diagrams support access to decimal division?
- How do we divide decimals by whole numbers and decimals, and what patterns emerge from different division methods, such as long division and partial quotients?
- How can we apply decimal operations to solve real-world problems, and why is it important to consider the appropriate level of precision in these contexts?
- How does our understanding of place value influence the way we perform operations with decimals and apply them to various mathematical situations?

Enduring Understanding(s):

- The base-ten number system helps us understand and perform operations with decimals by organizing numbers into place values based on powers of ten, which allows for consistent calculations.
- We can use strategies like estimation, place value alignment, and visual representations to add, subtract, and multiply decimals beyond tenths and hundredths.
- Different methods for multiplying decimals, such as using decimal fractions or partial products, are connected to the standard multiplication algorithm by showing how values are grouped and calculated based on place value.
- Visual models like base-ten blocks and diagrams make decimal division more accessible by illustrating how numbers are split and how place value affects the quotient.
- Dividing decimals by whole numbers or decimals involves recognizing patterns in the division process, such as adjusting the decimal point, and using methods like long division and partial quotients.
- Decimal operations help solve real-world problems, and understanding the importance of precision ensures that calculations are accurate and meaningful in context.
- A strong understanding of place value is crucial when performing operations with decimals because it determines how we manipulate numbers and interpret results in different mathematical situations.

Demonstration of Learning:

CFA 1: Checkpoint A (after lesson 4)
CFA 2: Checkpoint B (after lesson 8)
Mid Unit: Assessment A (after lesson 8)
CFA 3: Checkpoint C (after lesson 13)
EoU: Assessment A

Pacing for Unit

20 Days
Lesson Modifications:
Student performance on the 6.5 Check Your Readiness assessment will determine whether to skip optional lessons/activities (*). Remove only if performance is strong on corresponding items (see list).

- Remove 6.5.2*- optional lesson
- Remove 6.5.3.2*- optional activity
- Remove 6.5.5.2*- optional activity
- Remove 6.5.7.2*- optional activity
- Remove 6.5.15 - culminating lesson and is tagged as optional

Family Overview

[Family Resources: Arithmetic in Base Ten](#)
[Recursos Familiares: Aritmética en base diez](#)

Integration of Technology:

- Desmos Online Graphing Calculator
- Pear Assessment (Edulastic)
- iM v.360 Digital Applets (see below)

Unit-specific Vocabulary:

	New Terminology	
Lesson	receptive	productive
6.5.1	digits, budget, at least	
6.5.2	base-ten diagram, compose, vertical calculation	place value, digits
6.5.3	decompose	
6.5.4	method	Compose, decompose
6.5.5	powers of 10	Product, decimal point
6.5.7	partial products	method
6.5.9	remainder	
6.5.10	partial quotients	divisor
6.5.11	long division	
6.5.12		remainder
6.5.13		long division
6.5.14	precision, accuracy, operation	

Aligned Unit Materials, Resources, and Technology

Lessons 1-4: Base Ten Visualization Tools

Interactive applets for:

- Representing decimals with base-ten blocks
- Adding and subtracting decimals
- Understanding place value relationships
- Visualizing decimal operations

Lessons 5-8: Multiplication Area Model Tools

Interactive tools for:

- Creating area models for decimal multiplication
- Exploring partial products
- Understanding decimal multiplication strategies
- Connecting area models to standard algorithms

Lessons 9-13: Division Visualization Tools

Tools for:

- Representing division with base-ten diagrams
- Understanding decimal division strategies
- Exploring relationships between multiplication and division
- Visualizing division of decimals by decimals

These applets support key learning goals around **understanding decimal operations, using multiple representations**, and **developing fluency with standard algorithms**.

Base-ten blocks and paper versions of them will be useful throughout the unit. Consider preparing commercially produced base-ten blocks, if available, or printing representations of base-ten units on card stock, cutting them out, and organizing them for easy reuse.

Opportunities for Interdisciplinary Connections:

Science

- Calculate precise measurements in scientific experiments
- Record and analyze decimal measurements (mass, volume, length)
- Calculate changes in temperature

Social Studies

- Create and manage budgets
- Compare prices and discounts
- Analyze population statistics

Physical Education and Wellness

- Calculate sports averages
- Track running times and distances
- Measure performance improvements
- Calculate serving sizes
- Track daily intake values

Technology

- Use precise measurements in designs
- Calculate dimension and scale
- Work with computer measurements (pixels, bytes)

Anticipated Misconceptions:

- Misaligning the decimal points with addition and subtraction
- Misplacing the decimal point within all four operations
- Regrouping 10s while subtracting. 0 - 5 \= 5
- Misrepresenting repeating decimals, such as without notation

See teacher's guide for specific misconceptions aligned to each lesson.

Connections to Prior Units:

In grade 5, students also calculate sums, differences, products, and quotients of decimals to hundredths, using concrete representations or drawings, and strategies

Connections to Future Units:

In grade 7, students will study addition and subtraction of signed numbers and apply those concepts to simplify expressions and solve equations.

based on place value, properties of operations, and the relationship between addition and subtraction. They connect their strategies to written methods and explain their reasoning.

In this unit, students learn an efficient algorithm for division and extend their use of other base-ten algorithms to decimals of arbitrary length. Because these algorithms rely on the structure of the base-ten system, students build on the understanding of place value and the properties of operations developed during earlier grades.

Essential prior concepts to engage with this unit:

- By the end of grade 5, students learn to use efficient algorithms to fluently calculate sums, differences, and products of multi-digit whole numbers. They calculate quotients of multi-digit whole numbers with up to four-digit dividends and two-digit divisors. Students also calculate sums, differences, products, and quotients of decimals to hundredths.
- Strategies are based on place value understanding, properties of operations, and the relationship between multiplication and division, as well as the relationship between addition and subtraction.
- Students use concrete representations or drawings, such as rectangular arrays and area diagrams, as well as equations and other written methods.

Relevant Unit(s)/Lesson(s) to Review:

- Grade 5 Unit 4: Wrapping Up Multiplication and Division with Multi-Digit Numbers
- Grade 5 Unit 5: Place Value Patterns and Decimal Operations
- Student performance on the 6.5 Check Your Readiness assessment will determine if these lessons need to be added.
    - Visuals in synthesis of 5.5.1.2 to support discussion of Lesson 2 as needed
    - Add 5.4.11

See Adaptation Pack in the IL Classroom (6.5 Plan) for more suggestions on connections, pacing modifications, modified plans based on Check Your Readiness Assessment, and a priority list of lessons.

Differentiation through *Universal Design for Learning*

Concrete to Representational to Abstract (Multiple Means of Representation)

- Base-ten blocks for decimal operations
- Area diagrams for decimal multiplication
- Transition to standard algorithms

Explain, Interpret, Compare (Action & Expression)

- Explaining estimation strategies
- Interpreting base-ten diagrams
- Comparing different calculation methods

Progressive Language Development (Engagement & Expression)

- Building place value vocabulary
- Connecting visual models to mathematical terms
- Moving from informal to precise mathematical language

The routines in this unit particularly support:

- Building conceptual understanding through concrete models
- Developing multiple solution strategies
- Making connections between representations and algorithms

Related *CELP standards* aligned to Learning Targets:

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes, such as explaining, interpreting, and comparing. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers

can formatively assess how students are using language in these ways, particularly when students are using language to:

Explain

- Processes of estimating and finding costs (Lesson 1).
- Approaches to adding and subtracting decimals (Lesson 4).
- Reasoning about products and quotients involving powers of 10 (Lesson 5).
- Methods for multiplying decimals (Lesson 8).
- Reasoning about relationships among measurements (Lesson 15).

Interpret

- Representations of decimals (Lesson 2).
- Base-ten diagrams showing addition or subtraction of decimals (Lesson 3).
- Area diagrams showing products of decimals (Lesson 7).
- Base-ten diagrams representing division of a whole number or a decimal by a whole number (Lessons 9, 12).
- Calculations showing partial quotients or steps in long division (Lessons 10, 11, 12).

Compare

- Base-ten diagrams with numerical calculations (Lesson 4).
- Methods for multiplying decimals (Lesson 6).
- Methods for finding quotients (Lessons 10, 11, 12).
- Measurements of two- and three-dimensional objects (Lesson 15).

In addition, students are expected to describe decimal values to hundredths, generalize about multiplication by powers of 10 and about decimal measurements, critique approaches to operations on decimals, and justify strategies for finding sums, differences, products, and quotients.

	Emerging	Expanding	Bridging
LT1	Use price lists with visuals Round to nearest dollar Complete "This costs ___"	Estimate totals with support Explain estimation strategy "I rounded ___ to ___ because..."	Calculate costs efficiently Justify estimation choices Use mental math strategies
LT2	Line up decimal points Use place value charts Show regrouping with colors	Explain steps with frames Show work systematically "I lined up ___ because..."	Solve problems independently Explain method selection Use efficient strategies
LT3	Use base-ten blocks Follow multiplication steps Show "First multiply ___"	Connect to whole numbers Explain decimal placement "The product is ___ because..."	Apply properties flexibly Justify solution methods Use multiple strategies
LT4	Draw area models Use partial products guide Label parts of solution	Choose methods with support Explain solution steps "This method works because..."	Select efficient methods Explain strategy choices Connect representations
LT5	Follow division models Group base-ten blocks Show equal sharing	Create diagrams with support Explain division process "I divided by grouping..."	Model division independently Justify method selection Connect to algorithms
LT6	Follow division steps Use guided practice Record partial quotients	Apply methods with support Explain solution process "First I estimate __, then..."	Choose methods strategically Explain efficient approaches Solve complex problems
LT7	Solve with visual support Use operation guides Complete solution frames	Select operations with support Explain solution steps "To solve this, I need to..."	Solve complex problems Justify method choices Create solution plans
LT8	Use price lists with visuals Round to nearest dollar Complete "This costs ___"	Estimate totals with support Explain estimation strategy "I rounded ___ to ___ because..."	Calculate costs efficiently Justify estimation choices Use mental math strategies

Additional Sentence Frames and Stems

Section A

- I estimated the answer to be _____ because ...
- When dealing with money, it is/is not reasonable to round when _____.
- I used _____ to visualize the decimal expression _____ because ...
- After finding the sum/difference between the decimal values_____ and _____, I can check my solution by _____.
- The value of the expression is _____. This makes sense/does not make sense because ...

Section B

- I wrote the expression _____ to model this situation because ...
- I used the whole numbers _____ and _____ when multiplying _____ by _____ because ...
- The fraction _____ is equivalent to the decimal _____. This helps me multiply decimal values because ...
- I used an area diagram to multiply _____ by _____. I labeled each section _____ and found the product _____. This makes sense because ...

Section C
- I wrote the expression _____ to model this situation because ...
- To find the quotient, I used _____. I chose this strategy because...
- When using long division, I think about _____ to help keep me organized.
- I divided _____ by ____ and the quotient resulted in a value greater than/less than 1. This makes sense because ...

Section D
- I wrote the expression _____ to model this situation because ...
- The value of the expression is _____. This makes sense/does not make sense because ...
- I used _____ to visualize the decimal expression _____ because ...

Unit Outline

Lesson Sequence	Learning Target(s)	Success Criteria/Assessment
Section A Exploring, Adding, and Subtracting Decimals (Lessons 1-4)	**Learning Target #1** Use decimals to estimate and calculate costs in real-world situations, such as shopping. **Learning Target #2** Represent and solve addition and subtraction of decimals using diagrams, place value, and vertical calculations.	**Lesson 1 Using Decimals in a Shopping Context** • I can use decimals to make estimates and calculations about money. **Lesson 2 Using Diagrams to Represent Addition and Subtraction** • I can use diagrams to represent and reason about addition and subtraction of decimals. • I can use place value to explain addition and subtraction of decimals. • I can use vertical calculations to represent and reason about addition and subtraction of decimals. **Lesson 3 Adding and Subtracting Decimals with Few Non-Zero Digits** • I can tell whether writing or removing a zero in a decimal will change its value. • I know how to solve subtraction problems with decimals that require decomposing. **Lesson 4 Adding and Subtracting Decimals with Many Non-Zero Digits** • I can solve problems that involve addition and subtraction of decimals.
Checkpoint A	*Responding to Student Thinking* • Problem 1: Press Pause: If most students struggle to recognize that adding decimals involves combining like base-ten units (which is easier to do by aligning the decimal points of the addends), make time to revisit the work of the activity referred to here. Ask students to explain to a partner why the first method is most conducive to adding and subtracting decimals correctly. Emphasize the value of each place. Encourage students to represent $5 as $5.00 if they are not sure. Grade 6, Unit 5, Lesson 4, Activity 1 The Cost of a Photo Print • Problem 2: Press Pause: If most students struggle to subtract a decimal by decomposing larger units (or regrouping) as needed, make time to revisit the work in the referred-to activities. Before starting any subtraction problem, ask students to examine the numbers involved, determine whether it would be necessary to decompose one or more digits in order to subtract a number, and if so, which base-ten unit(s) may need to be decomposed. Grade 6, Unit 5, Lesson 3, Activity 3 Subtracting Decimals of Different Lengths Grade 6, Unit 5, Lesson 4, Activity 3 Missing Numbers	
Section B Multiplying Decimals (Lessons 5-8)	**Learning Target #4** Use place value and fractions to reason about multiplying decimals. **Learning Target #5** Represent and solve decimal multiplication using area diagrams, partial products, and other various methods.	**Lesson 5 Using Fractions to Multiply Decimals** • I can use place value and fractions to reason about the multiplication of decimals. **Lesson 6 Methods for Multiplying Decimals** • I can use area diagrams to represent and reason about multiplication of decimals. • I know and can explain more than one way to multiply decimals using fractions and place value. **Lesson 7 Using Diagrams to Represent Multiplication** • I can use area diagrams and partial products to represent and find products of decimals. **Lesson 8 Calculating Products of Decimals** • I can describe and apply a method for multiplying decimals. • I know how to use a product of whole numbers to find a product of decimals.
Checkpoint B	*Responding to Student Thinking*	

	• Problem 1: Press Pause: If most students struggle to find the product of the two decimals using a strategy of their choice, revisit the ideas in the first lesson of the section, and do one of the optional activities referred to here. Grade 6, Unit 5, Lesson 7, Activity 2 Connecting Area Diagrams to Calculations with Whole Numbers Grade 6, Unit 5, Lesson 7, Activity 4 Using Partial Products Grade 6, Unit 5, Lesson 5 Using Fractions to Multiply Decimals Grade 6, Unit 4, Lesson 13, Practice Problem 2 • Problem 2: Points to Emphasize: If most students struggle to explain how the product of two whole numbers could be used to find the product of two decimals with the same significant digits, focus on identifying a power of 10 that can relate a decimal to a whole number. For example, when finding the area of parallelograms with decimal side lengths in the practice problem referred to here, ask students whether to multiply each side length by 10, 100, or 1,000 so that the side lengths could be whole numbers when they are multiplied. Then discuss what needs to be done to the whole-number product afterward, in order to reflect the actual area of each parallelogram. Grade 6, Unit 5, Lesson 10, Practice Problem 6	
Mid Unit Assessment		
Section C Dividing Decimals (Lessons 9-13)	**Learning Target #6** Use base-ten diagrams to represent the division of whole numbers and decimals by whole numbers. **Learning Target #7** Apply partial quotients and long division to find the quotient of whole numbers and decimals.	**Lesson 9 Using Base-Ten Diagrams to Divide** • I can use base-ten diagrams to represent division of whole numbers and division of a decimal by a whole number. **Lesson 10 Using Partial Quotients** • I can use partial quotients to find a quotient of two whole numbers. **Lesson 11 Using Long Division** • I can use long division to find a quotient of two whole numbers when the quotient is a whole number. **Lesson 12 Dividing Numbers that Result in a Decimal** • I can use long division to divide two whole numbers when the quotient is not a whole number, or to divide a decimal by a whole number. **Lesson 13 Dividing a Decimal by a Decimal** • I can explain how multiplying the dividend and the divisor by the same power of 10 can help me find a quotient of two decimals. • I can find the quotient of two decimals.
Checkpoint C	*Responding to Student Thinking* • Problem 1: Points to Emphasize: If most students struggle to use long division to divide a decimal by a whole number, look for opportunities to walk through the division process as a class and invite students to explain what is happening in each step of the process. For example, when discussing the Warm-up referred to here, consider using long division to calculate the first quotient and see how close the actual value is to students' estimates. Grade 6, Unit 5, Lesson 14, Activity 1 Math Talk: Close Estimates • Problem 2: Points to Emphasize: If most students struggle to identify a division expression that uses whole numbers and that is equivalent to the given decimal expression, look for opportunities to reinforce this idea during the next several lessons. For example, when students work on the practice problem referred to here, direct them to find at least one of the quotients by identifying an equivalent expression using whole numbers. Grade 6, Unit 5, Lesson 14, Practice Problem 6	
Section D Let's Put It to Work (Lessons 14-15)	**Learning Target #8** Apply addition, subtraction, multiplication, and division with decimals to solve real-world problems, including finding surface areas.	**Lesson 14 Solving Problems Involving Decimals** • I can use addition, subtraction, multiplication, and division on decimals to solve problems. **Lesson 15 Making and Measuring Boxes** • I can use the four operations on decimals to find surface areas and reason about real-world problems.
End of Unit Assessment		

Unit Title:

Unit 6: Expressions and Equations

Relevant Standards: Bold indicates priority

Lesson	Standards	Lesson	Standards
Lesson 1		Lesson 11	6.EE.A.2 6.EE.A.3 6.EE.A.4
Lesson 2	6.EE.B.5 6.EE.B.6	Lesson 12	6.EE.A.1
Lesson 3	6.EE.B.5 6.EE.B.6 6.EE.B.7	Lesson 13	6.EE.A.1
Lesson 4	6.EE.B 6.EE.B.5 6.EE.B.7 6.NS.B.3	Lesson 14	6.EE.A.1 6.EE.A.2.c
Lesson 5	6.EE.B.5 6.EE.B.6 6.EE.B.7	Lesson 15	6.EE.A.1 6.EE.A.2.c 6.EE.B.5
Lesson 6	6.EE.B.6 6.EE.B.7 6.RP.A.3.c	Lesson 16	6.EE.C.9 6.RP.A.3.a
Lesson 7	6.EE.A.2 6.EE.A.2.a 6.EE.B.6	Lesson 17	6.EE.C.9 6.RP.A.3.a 6.RP.A.3.b
Lesson 8	6.EE.A.4	Lesson 18	6.EE.C.9
Lesson 9		Lesson 19	6.EE.A.2 6.EE.B.7 6.EE.C.9
Lesson 10	6.EE.A.2 6.EE.A.3 6.EE.A.4		

Essential Question(s):

- What does it mean to solve an equation, and why is this important in both mathematical and real-world contexts?
- How do properties of operations help us manipulate and simplify algebraic expressions?
- How can different representations (equations, tables, graphs) reveal different aspects of mathematical relationships?
- Why is a systematic approach important when solving equations, and how does order of operations ensure accuracy?
- How can we determine and justify when two expressions are equivalent?

Enduring Understanding(s):

- Solving an equation means finding a value that makes the equation true, which allows us to answer both mathematical and real-world questions.
- The properties of operations (commutative, associative, distributive) provide a foundation for manipulating and simplifying algebraic expressions efficiently.
- Different representations (equations, tables, graphs) can describe the same relationship, each revealing unique insights about the mathematical situation.
- Using a systematic approach and following order of operations ensures accurate solutions when solving equations and simplifying expressions.
- Equivalent expressions represent the same value in different forms, and we can prove their equivalence using properties of operations.

Demonstration of Learning:

CFA 1: Checkpoint A (after lesson 6)
CFA 2: Checkpoint B (after lesson 11)
Mid Unit: Assessment A (after lesson 11)
CFA 3: Checkpoint C (after lesson 15)
Checkpoint D is an opportunity for feedback
EoU: Assessment A

Pacing for Unit

26 Days
Lesson Modifications:

- Lesson 11: optional lesson where students practice identifying and writing equivalent expressions using the distributive property.
- Lesson 18: optional lesson that offers opportunities to look at multiple representations (equations, graphs, and tables) for some different contexts.
- Lesson 19: culminating lesson where students look at several examples of equations that represent important relationships from real-world situations. Can remove.

Family Overview

Family Resources: Expressions and Equations
Recursos Familiares: Expresiones y ecuaciones

Integration of Technology:

- Desmos Online Graphing Calculator
- Pear Assessment (Edulastic)
- iM v.360 Digital Applets (see below)

Unit-specific Vocabulary:

	New Terminology	
Lesson	receptive	productive
6.6.1	value (of a variable)	operation
6.6.2	Variable, coefficient	value (of a variable)

Aligned Unit Materials, Resources, and Technology

Lessons 1-5: Equation Visualization Tools
Interactive applets for:

- Visualizing balanced equations
- Manipulating hanger diagrams
- Understanding equation solving
- Exploring equivalent expressions

Lessons 9-11: Distributive Property Tools

	solution to an equation, true equation, false equation	
6.6.3	each side (of an equal sign), balanced hanger diagram	
6.6.4	solve (an equation)	each side (of an equal sign)
6.6.6		true equation false equation
6.6.8	equivalent expressions, commutative property	
6.6.9	distributive property, area as a product, area as a sum	
6.6.10	term	equivalent expressions
6.6.12	to the power	
6.6.13		to the power exponent
6.6.15		solution to an equation
6.6.16	independent variable dependent variable horizontal axis vertical axis	variable relationship
6.6.17	coordinates	
6.6.18	plot	

Tools for:
- Creating area models for the distributive property
- Visualizing equivalent expressions
- Understanding factoring and expanding
- Connecting diagrams to expressions

Lessons 12-15: Exponent Tools

Interactive applets for:
- Visualizing exponential growth
- Understanding repeated multiplication
- Exploring patterns with exponents
- Representing powers

Lessons 16-19: Relationship Tools

Tools for:
- Graphing relationships between quantities
- Creating tables of values
- Understanding dependent and independent variables
- Exploring multiple representations

These applets support key learning goals around **understanding equations and expressions, working with properties of operations**, and **representing relationships between quantities.**

Opportunities for Interdisciplinary Connections:

Science
- Use equations to model physical relationships (speed, distance, time)

Social Studies
- Substitute into various formulas to answer real world questions
- Model real world economic situations using equations and expressions

Anticipated Misconceptions:
- Understanding a variable represents a number
- Reading 3x as “3 and x” rather than “3 times x”
- Thinking 2(x+3) and 2x + 3 are equivalent
- Not recognizing x + x + x as 3x
- Confusing equations and expressions
- Show repeated multiplication as $2^3 = 2 \times 2 \times 2$

See teacher's guide for specific misconceptions aligned to each lesson.

Connections to Prior Units:

This work draws on students’ prior knowledge from grade 5 of writing simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them.

Essential prior concepts to engage with this unit:
- Relationship between operations (addition and subtraction, multiplication and division)
- Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them.

Relevant Unit(s)/Lesson(s) to Review:
- Grade 5 Unit 8: Putting it All Together

Connections to Future Units:

The work here prepares students to represent quantities and relationships involving all rational numbers in a later unit, as well as to solve equations that are more complex and work with proportional relationships in grade 7.

See Adaptation Pack in the IL Classroom (6.6 Plan) for more suggestions on connections, pacing modifications, modified plans based on Check Your Readiness Assessment, and a priority list of lessons.

Differentiation through Universal Design for Learning

Interpret, Describe, Explain (All UDL Principles)
- Interpreting tape and hanger diagrams for equations

- Describing relationships between variables
- Explaining solution strategies for equations

Visual Representations (Representation & Expression)

- Tape diagrams for linear equations
- Hanger diagrams for balanced equations
- Area models for distributive property
- Graphs for relationships between variables

Progressive Language Development (Engagement & Expression)

- Moving from informal to formal algebraic notation
- Building understanding of variables and coefficients
- Connecting multiple representations (tables, graphs, equations)

The routines in this unit particularly support:

- Building conceptual understanding through concrete models
- Developing algebraic reasoning through multiple representations
- Making connections between visual and symbolic representations

Related *CELP standards* aligned to Learning Targets:

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes, such as interpreting, describing, and explaining. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Interpret

- Tape diagrams involving letters that stand for numbers (Lesson 1).
- The parts of an equation (Lesson 5).
- Numerical expressions involving exponents (Lesson 12).
- Different representations of the same relationship between quantities (Lesson 17).

Describe

- Solutions to equations (Lesson 2).
- Stories represented by given equations (Lesson 5).
- Patterns of growth that can be represented using exponents (Lesson 12).
- Relationships between independent and dependent variables using tables, graphs, and equations (Lesson 16).

Explain

- The meaning of a solution using hanger diagrams (Lesson 3).
- How to solve an equation (Lesson 4).
- How to use equations to solve problems involving percentages (Lesson 6).
- How to determine whether two expressions are equivalent, including with reference to diagrams (Lesson 8).
- Strategies for determining whether expressions are equivalent (Lesson 14).

In addition, students are expected to compare descriptions of situations, expressions, equations, diagrams, tables, and graphs. They generalize about properties of operations and strategies for solving equations. Students also justify claims about equivalent expressions and justify reasoning when evaluating expressions.

	Emerging	Expanding	Bridging
LT1	Follow step-by-step solutions Use visual models to solve Complete "First I ___"	Explain steps with sentence frames Show work with verbal support "To solve, I first ___ then ___"	Solve equations independently Explain solution methods clearly Use precise mathematical language
LT2	Check solutions with guidance Circle true/false statements Complete "__ is a solution"	Test values using sentence frames Explain why solutions work "When x is __, the equation is __"	Verify solutions independently Explain solution process fully Evaluate complex equations
LT3	Match equivalent expressions Use number substitution guide Show "Same answer when x is __"	Test multiple values with support Explain why expressions match "These are equal because..."	Prove equivalence independently Use algebraic reasoning Justify with mathematical logic
LT4	Use visual models to distribute Follow pattern examples Complete partial expressions	Apply property with guidance Explain steps in distribution "I multiply ___ by each ___"	Generate equivalent expressions Explain property application Create complex expressions
LT5	Calculate with number substitution Use guided evaluation steps Complete "When x=__, answer is __"	Evaluate using order of operations Show work with explanations "First calculate __, then __"	Evaluate complex expressions Explain evaluation process Use efficient strategies
LT6	Compare using given values Circle equal/not equal	Evaluate and compare with support Explain calculation steps	Compare expressions efficiently Justify conclusions fully

	Show numerical work	"These are equal/different because..."	Use mathematical reasoning
LT7	Complete partial tables Plot given points Follow equation patterns	Create representations with support Connect different forms "The table shows __ so the graph..."	Create all representations Explain connections clearly Use precise mathematical terms
LT8	Match variables to descriptions Label input/output Use "If this changes, that changes"	Identify variables with reasoning Explain relationships "___ depends on ___ because..."	Analyze variable relationships Explain dependencies clearly Use contextual reasoning
LT9	Match graphs to descriptions Complete partial descriptions Label key features	Create representations with support Describe relationships "As x increases, y ___"	Create detailed representations Explain relationships fully Use precise vocabulary
LT10	Connect matching representations Verify with given values Show "Same numbers in both"	Explain matches with support Compare values systematically "These match because..."	Justify matches independently Analyze relationships deeply Provide mathematical proof
LT11	Identify key features Point out similarities Use comparison words	Compare representations with frames Explain advantages/limitations "This shows ___ better because..."	Analyze representations deeply Critique effectiveness Justify preferences fully

Additional Sentence Frames and Stems

Section A

- The value of the variable in the equation must be _____ because ...
- The equation _____ represents this situation because ...
- To solve the equation, I _____ because ...
- The solution to the equation is _____ which means _____.

Section B

- I know the expressions are equal/not equal because ...
- The expression _____ is/is not equivalent to the expression _____ because ...
- The expression _____ represents this situation because ...
- I can _____ to prove that the expression _____ is equal to _____.

Section C

- The value of the expression is _____ because ...
- When I substitute _____ into the expression _____ for _____, I get _____ because ...
- To find the value of the expression, first I _____, then I _____ ...

Section D

- In this scenario, as _____ increases/decreases, _____ increases/decreases.
- The equation _____ represents the quantities of _____ because ...
- I created a _____ to relate the quantities of _____ and _____ because ...
- The independent variable in this situation is _____ because ...
- The dependent variable in this situation is _____ because ...

Section E

- In the equation _____, _____ represents the number of _____, and _____ represents the number of _____.
- The equation _____ means _____.
- When creating a graph of the scenario, first I _____, then I _____ ...

Unit Outline

Lesson Sequence	Learning Target(s)	Success Criteria/Assessment
Section A Equations in One Variable (Lessons 1-6)	**Learning Target #1** Solve equations of the form x+p=q or px=q and explain the solution method. **Learning Target #2** Understand that solving an equation with a variable means finding a value for the variable that makes the equation true, and use substitution to determine whether a number is a solution to the equation.	**Lesson 1 Tape Diagrams and Equations** • I can tell whether or not an equation could represent a tape diagram. • I can use a tape diagram to represent an equation. **Lesson 2 Truth and Equations** • I can replace a variable in an equation with a number that makes the equation true, and know that this number is called a “solution” to the equation. **Lesson 3 Staying in Balance** • I can compare the process of removing or grouping weights to keep a hanger diagram balanced and the process of subtracting or dividing numbers to solve an equation. • I can explain what a balanced hanger diagram and a true equation have in common. • I can write equations that could represent the weights on a balanced hanger diagram. **Lesson 4 Practice Solving Equations** • I can solve addition and multiplication equations with one variable.

		Lesson 5 Represent Situations with Equations • I can explain how an equation with a variable represents a real-world problem. • I can use equations with variables to solve real-world problems. **Lesson 6 Percentages and Equations** • I can solve percent problems by writing and solving equations.
Checkpoint A	*Responding to Student Thinking* • Problem 1: Points to Emphasize: If most students struggle with substituting a value for a variable to decide whether it's a solution to an equation, reinforce the idea throughout the next section. For example, when discussing whether expressions are equal in the referenced activity, emphasize the ways students substituted values into each expression to create their tape diagrams. Grade 6, Unit 6, Lesson 8, Activity 2 Using Tape Diagrams to Show That Expressions Are Equivalent • Problem 2: Points to Emphasize: If most students struggle with solving an equation by performing the same operations to each side, reinforce the idea throughout the next section. For example, explicitly demonstrate solution methods when discussing the referenced practice problem. Grade 6, Unit 6, Lesson 8, Practice Problem 5	
Section B Equal and Equivalent (Lessons 6-11)	**Learning Target #3** Justify whether two expressions are "equivalent," or equal to each other for every value of their variable. **Learning Target #4** Use the distributive property to write equivalent algebraic expressions.	**Lesson 7 Write Expressions with Variables** • I can use an expression that represents a situation to find an amount in a story. • I can write an expression with a variable to represent a calculation where I do not know one of the numbers. **Lesson 8 Equal and Equivalent** • I can explain what it means for two expressions to be equivalent. • I can use what I know about operations to decide whether two expressions are equivalent. **Lesson 9 The Distributive Property, Part 1** • I can use a diagram of a rectangle split into two smaller rectangles to write different expressions representing its area • I can use the distributive property to explain how two expressions with numbers are equivalent. **Lesson 10 The Distributive Property, Part 2** • I can use a diagram of a split rectangle to write different expressions with variables representing its area. **Lesson 11 The Distributive Property, Part 3** • I can use the distributive property to write equivalent expressions with variables
Checkpoint B	*Responding to Student Thinking* • Problem 1: Press Pause: By this point in the unit, there should be some student mastery of identifying equivalent expressions with variables and using substitution or properties of operations to justify that identification. If students struggle, make time to revisit related work in the Section referred to here. See the Course Guide for ideas to help students re-engage with earlier work. Grade 6, Unit 6, Section B Equal and Equivalent • Problem 2: Press Pause: By this point in the unit, there should be some student mastery of identifying equivalent expressions with variables and using substitution or properties of operations to justify that identification. If students struggle, make time to revisit related work in the Section referred to here. See the Course Guide for ideas to help students re-engage with earlier work. Grade 6, Unit 6, Section B Equal and Equivalent	
Mid Unit Assessment		
Section C Expressions with Exponents (Lessons 12-15)	**Learning Target #5** Evaluate expressions with whole-number exponents at specific values of their variables. **Learning Target #6** Justify whether numerical expressions involving whole-number exponents are equal by evaluating the expressions and	**Lesson 12 Meaning of Exponents** • I can find the value of expressions with exponents and write expressions with exponents that are equal to a given number. • I understand the meaning of an expression with an exponent like 3^5. **Lesson 13 Expressions with Exponents** • I can decide if expressions with exponents are equal by finding the values of the expressions or by understanding what exponents mean. **Lesson 14 Evaluating Expressions with Exponents** • I know how to find the value of expressions that have both an exponent and addition or subtraction. • I know how to find the value of expressions that have both an exponent and multiplication or division. **Lesson 15 Equivalent Exponential Expressions** • I can find solutions to equations with exponents in a list of numbers.

	performing operations in the conventional order.	• I can replace a variable with a number in an expression with exponents and use the correct order of operations to find the value of the expression.
Checkpoint C	*Responding to Student Thinking* • Problem 1: Press Pause: By this point in the unit, there should be some student mastery of evaluating numerical expressions with exponents and using the results to justify whether or not two expressions are equivalent. If students struggle, make time to revisit related work in the referenced lesson. See the Course Guide for ideas to help students re-engage with earlier work. Grade 6, Unit 6, Lesson 13 Expressions with Exponents • Problem 2: Press Pause: By this point in the unit, there should be some student mastery of evaluating exponential expressions with variables for given values of the variable. If students struggle, make time to revisit related work in the referenced lesson. See the Course Guide for ideas to help students re-engage with earlier work. Grade 6, Unit 6, Lesson 14 Evaluating Expressions with Exponents	
Section D Relationships Between Quantities (Lessons 16-18)	**Learning Target #7** Create a table, graph, and equation to represent the relationship between two quantities. **Learning Target #8** Identify the independent and dependent variable in a situation.	**Lesson 16 Two Related Quantities, Part 1** • I can create tables and graphs that show the relationship between two amounts. • I can write an equation with two variables that shows the relationship between two amounts. **Lesson 17 Two Related Quantities, Part 2** • I can create tables and graphs to represent the relationship between distance and time for something moving at a constant speed. • I can write an equation with variables to represent the relationship between distance and time for something moving at a constant speed **Lesson 18 More Relationships** • I can create tables and graphs that show different kinds of relationships between amounts. • I can write equations that describe relationships with area and volume.
Checkpoint D	*Responding to Student Thinking* More Chances: Students will have more opportunities to develop this understanding in later courses. There is no need to slow down or add additional work to review this concept at this time.	
Section E Let's Put It to Work (Lesson 19)	**Learning Target #9** Create a verbal description and a graph to represent the relationship shown in an equation and table. **Learning Target #10** Identify tables and equations that represent the same relationship and justify (orally) the match. **Learning Target #11** Interpret and critique (orally) different representations of the same relationship, i.e. table, equation, graph, and verbal description.	**Lesson 19 Tables, Equations, and Graphs, Oh My!** • I can create a table and a graph that represent the relationship in a given equation. • I can explain what an equation tells us about the situation.
End of Unit Assessment		

Unit Title:

Unit 7: Rational Numbers

Relevant Standards: Bold indicates priority

Lesson	Standards	Lesson	Standards
Lesson 1	6.NS.C.5 6.NS.C.6	Lesson 11	6.NS.C.6.b 6.NS.C.6.c 6.NS.C.8
Lesson 2	6.NS.C.6 6.NS.C.6.a 6.NS.C.6.c	Lesson 12	6.NS.C. 6.b 6.NS.C.6.c
Lesson 3	6.NS.C.7.a 6.NS.C.7.b	Lesson 13	6.NS.C.6.c 6.NS.C.7.c 6.NS.C.8
Lesson 4	6.NS.C 6.NS.C.6 6.NS.C.6.a 6.NS.C.7	Lesson 14	6.NS.C.6 6.NS.C.6.b 6.NS.C.8
Lesson 5	6.NS.C.5	Lesson 15	6.G.A.3 6.NS.C.6.c 6.NS.C.8
Lesson 6	6.NS.C.7 6.NS.C.7.c 6.NS.C.7.d	Lesson 16	6.NS.B.4
Lesson 7	6.NS.C.6 6.NS.C.7 6.NS.C.7.a 6.NS.C.7.d	Lesson 17	6.NS.B.4
Lesson 8	6.EE.B.6 6.EE.B.8 6.NS.C.7.b	Lesson 18	6.NS.B.4
Lesson 9	6.EE.B.5 6.EE.B.8 6.NS.C.7.a	Lesson 19	6.G.A.36.NS.C.8
Lesson 10	6.EE.A.2.b 6.EE.B.5 6.EE.B.6 6.EE.B.8		

Essential Question(s):

- How do positive and negative numbers, their opposites, and absolute value help us describe and compare real-world situations?
- How can we use words, symbols, and absolute value to compare rational numbers in real-world situations?
- How can we use inequalities and number lines to represent and make sense of real-world situations?
- How do the signs of coordinates and reflections across axes help us understand the location and movement of points in the coordinate plane?
- How can we use coordinates and absolute value to find distances between points and describe their relationships?
- How can understanding factors and multiples help us solve real-world problems?
- How can we use coordinates and patterns of points to create and understand shapes and images in the coordinate plane?

Enduring Understanding(s):

- Positive and negative numbers, along with their opposites and absolute values, help us describe and interpret real-world quantities and changes.
- Number lines and inequalities allow us to visualize, compare, and reason about rational numbers in meaningful ways.
- The coordinate plane helps us understand relationships between points, including distance, reflection, and symmetry.
- Signs of coordinates reveal important information about location and movement in all four quadrants of the coordinate plane.
- Understanding factors, multiples, and absolute value helps us solve real-world problems involving rational numbers.

Demonstration of Learning:

CFA 1: Checkpoint A (after lesson 7)
CFA 2: Checkpoint B (after lesson 10)
CFA 3: Checkpoint C (after lesson 15)
CFA 4: Checkpoint D (after lesson 18)
EoU: Assessment A

Pacing for Unit

22 Days
Lesson Modifications:

- Remove 6.7.7: This lesson synthesizes the learning of the first six lessons and features an optional activity.
- Remove 6.7.15: This lesson has students plot vertices of polygons in the coordinate plane. It is an application of prior learned concepts to related standards.
- Remove 6.7.18: The majority of this lesson asks students to think about the greatest common factor and least common multiple for sets of 3 whole numbers, where the standards only call for students to analyze pairs of whole numbers.
- Remove 6.7.19: This lesson is optional.

Family Overview

Family Resources: Rational Numbers
Recursos Familiares: Números racionales

Integration of Technology:

- Desmos Online Graphing Calculator
- Pear Assessment (Edulastic)
- iM v.360 Digital Applets (see below)

Unit-specific Vocabulary:

Lesson	New Terminology: receptive	New Terminology: productive
6.7.1	positive number, negative number, temperature, degrees Celsius, elevation, sea level	number line, below zero
6.7.2	opposite (numbers), rational number, location, distance (away) from zero	
6.7.3	sign, inequality, closer to 0, farther from 0	greater than, less than
6.7.4	from least to greatest	temperature, elevation sea level
6.7.5	positive change, negative change, context	
6.7.6	absolute value	positive number, negative number, distance (away) from zero
6.7.7		closer to 0 farther from 0
6.7.8	maximum, minimum	
6.7.9	requirement, solution to an inequality	
6.7.10	unbalanced hanger	inequality
6.7.11	quadrant, coordinate plane, *x*-coordinate, *y*-coordinate	
6.7.12	(line) segment	axis
6.7.13	degrees Fahrenheit	degrees Celsius
6.7.14		absolute value, *x*-coordinate, *y*-coordinate
6.7.16	common factor, greatest common factor (GCF)	factor
6.7.17	common multiple, least common multiple (LCM)	multiple

Aligned Unit Materials, Resources, and Technology

Lessons 1-7: Number Line Tools

Interactive applets for:

- Plotting positive and negative numbers
- Understanding opposites
- Comparing rational numbers
- Exploring absolute value
- Visualizing distance from zero

Lessons 8-10: Inequality Tools

Tools for:

- Graphing inequalities on number lines
- Understanding solution sets
- Exploring open and closed endpoints
- Representing real-world constraints

Lessons 11-15: Coordinate Plane Tools

Interactive applets for:

- Plotting points in all quadrants
- Creating polygons in the coordinate plane
- Measuring distances between points
- Understanding coordinate relationships
- Exploring geometric shapes

Throughout Unit: Rational Number Visualization Tools

Tools for:

- Representing rational numbers in different forms
- Converting between decimals and fractions
- Understanding positive and negative numbers
- Exploring number relationships

These applets support key learning goals around understanding **rational numbers, working with inequalities**, and **using the coordinate plane**.

Opportunities for Interdisciplinary Connections:

Science

- Analyze positive and negative temperatures in weather data
- Study temperature changes across seasons
- Measure elevations above and below sea level
- Study depth measurements in oceanography
- Plot coordinates in geological features

Social Studies

- Understand profit and loss
- Compare positive/negative changes in world

Anticipated Misconceptions:

- Thinking larger negative numbers are “greater than” smaller “negative numbers”
- Confusing absolute value with order
- Reversing x and y coordinates when plotting points
- Thinking 0 has an opposite
- Not recognizing the opposite of negative is positive
- Confusing increase/decrease with positive/negative

See teacher's guide for specific misconceptions aligned to each lesson.

markets
- Study elevation changes in landforms over time

Physical Education and Wellness
- Analyze sports analytics
- Plot players positions using coordinates
- Monitor weight changes
- Study heart rate variations

Connections to Prior Units:	**Connections to Future Units:**
In previous units, students worked exclusively with non-negative numbers, where magnitude and order were indistinguishable: if one number was greater than another, it always appeared to the right on the number line and was always farther from zero. In this unit, as students begin working with signed numbers, they learn to distinguish between magnitude (the absolute value of a number) and order (its relative position on the number line). They come to understand the difference between comparisons such as "greater than" versus "greater absolute value," and "less than" versus "smaller absolute value." **Essential prior concepts to engage with this unit:** • locating fractions on number lines • graphing in quadrant I on the coordinate plane • factors and multiples **Relevant Unit(s)/Lesson(s) to Review:** • Grade 5 Unit 7: Shapes on the Coordinate Plane • These lessons incorporate grade 5 work with graphing ordered pairs in quadrant I. ○ 5.7.2 and 5.7.11	In grade 7, students will perform arithmetic operations with signed numbers and write and solve more complex inequalities. *See Adaptation Pack in the IL Classroom (6.7 Plan) for more suggestions on connections, pacing modifications, modified plans based on Check Your Readiness Assessment, and a priority list of lessons.*

Differentiation through *Universal Design for Learning*

Notice and Wonder (Engagement & Representation)
- Exploring number line representations of negative numbers
- Discovering patterns in coordinate plane quadrants
- Making sense of absolute value in real-world contexts

Mathematical Language Routines (All UDL Principles)
- Progressive vocabulary development from receptive to productive use
- Connecting everyday language (above/below zero) to mathematical concepts
- Building understanding through real-world contexts (temperature, elevation)

Describe and Interpret (Action/Expression & Representation)
- Describing situations involving negative numbers
- Interpreting points on coordinate planes
- Explaining relationships between numbers and their opposites

The routines in this unit particularly support students in:
- Building conceptual understanding through multiple representations
- Developing precise mathematical language
- Making connections to real-world contexts

Related *CELP standards* aligned to Learning Targets:

Progression of Disciplinary Language
In this unit, teachers can anticipate students using language for mathematical purposes, such as describing, interpreting, justifying, and generalizing. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Describe and Interpret
- Situations involving negative numbers (Lesson 1).
- Features of a number line (Lessons 2, 4 and 6).
- Situations involving elevation (Lesson 7).
- Situations involving minimums and maximums (Lesson 8).
- Points on a coordinate plane (Lessons 11 and 14).

- Situations involving factors and multiples (Lesson 18).

Justify

- Reasoning about magnitude (Lesson 3).
- Reasoning about a situation involving negative numbers (Lesson 5).
- Reasoning about solutions to inequalities (Lesson 9).
- That all possible pairs of factors have been identified (Lesson 16).

Generalize

- The meaning of integers for a specific context (Lesson 5).
- Understanding of solutions to inequalities (Lesson 9).
- About the relationships between shapes (Lesson 10).
- About greatest common factors (Lesson 16).
- About least common multiples (Lesson 17).

In addition, students are expected to critique the reasoning of others, represent inequalities symbolically and in words, and explain how to order rational numbers and how to determine distances on the coordinate plane. Students also have opportunities to use language to compare magnitudes of positive and negative numbers, compare features of ordered pairs, and compare appropriate axes for different sets of coordinates.

	Emerging	Expanding	Bridging
LT1	Use gestures to show above/below zero Match numbers to real contexts Complete "The temperature is ___"	Explain meaning using sentence frames Connect numbers to real situations "This means ___ below/above zero"	Interpret numbers independently Use precise mathematical language Make contextual connections
LT2	Point to locations on number line Mark positive/negative numbers State "Same distance from zero"	Plot numbers using guided steps Explain opposite number locations "The opposite of ___ is ___"	Plot numbers independently Explain relationships fluently Connect concepts mathematically
LT3	Use greater/less than symbols Circle larger/smaller numbers Complete "__ is more than __"	Write comparison statements Explain reasoning with support "I know ___ is greater because..."	Compare numbers fluently Use precise comparison language Justify comparisons fully
LT4	Test given values Circle yes/no for solutions Use number line to check	Explain why values work/don't work Use guided solution checking "This works because..."	Determine solutions independently Justify solution reasoning Evaluate complex cases
LT5	Mark points on number line Use open/closed circles Follow visual models	Draw diagrams with guidance Explain representation choices "The circle shows..."	Create diagrams independently Explain representations fully Use precise notation
LT6	Select from inequality options Match situations to symbols Complete "x ___ number"	Write inequalities with support Explain meaning using frames "We use > because..."	Write inequalities independently Justify symbol choices Create complex constraints
LT7	Identify quadrant numbers Point to reflected points Label positive/negative coordinates	Describe point locations Explain reflection patterns "When x changes sign..."	Analyze coordinate relationships Explain reflections completely Use mathematical precision
LT8	Count grid spaces Use number line to measure State "The distance is ___"	Calculate with support Explain method using frames "I found distance by..."	Calculate distances independently Explain multiple methods Apply to complex situations
LT9	Follow step-by-step plotting Match points to coordinates Use coordinate template	Plot points with some support Explain location process "First I go ___, then ___"	Plot points independently Explain efficient strategies Navigate all quadrants fluently
LT10	List factors using visual aids Circle common numbers Complete factor pairs	Find factors systematically Explain common factors "These share factors because..."	Find all factors efficiently Explain factor relationships Apply to complex problems
LT11	List multiples using patterns Circle shared multiples Complete number patterns	Generate multiples systematically Explain common multiples "Both numbers go into..."	Find multiples efficiently Explain relationships fully Solve complex problems
LT12	Plot given ordered pairs Copy coordinate patterns Label points from model	Create simple shapes Explain process with frames "The points make ___ shape"	Design complex images Explain strategy clearly Justify point selection

Additional Sentence Frames and Stems

Section A

- _____ is greater/less than _____ because ...
- The opposite of _____ is _____ because ...
- The absolute value of _____ is _____.

- The value _____ is located _____ spaces to the right/left of zero on a number line which means its opposite, _____ is located _____ spaces to the right/left of zero.
- The value _____ makes sense in this situation because ...

Section B

- The phrase _____ means _____ and can be represented by the inequality _____.
- The value _____ is a solution to the inequality _____ because ...
- The solution to the inequality _____ can be represented on a number line diagram by ...
- The inequality _____ represents this situation because ...

Section C

- Point _____ is in quadrant _____ because ...
- The distance between the point _____ and the point _____ is _____ because ...
- To plot the point _____, first I _____, then I _____ and place the point.

Section D

- The greatest common factor between _____ and _____ is _____.
- The least common multiple between _____ and _____ is _____.
- For this situation, I need to first find the greatest common factor/least common multiple because ...

Section E

- I chose to design a _____ on the coordinate plane and used the ordered pairs _____.
- I began my design in the _____ quadrant because ...

Unit Outline

Lesson Sequence	Learning Target(s)	Success Criteria/Assessment
Section A Negative Numbers and Absolute Value (Lessons 1-7)	**Learning Target #1** Interpret a rational number and the absolute value of a number in context. **Learning Target #2** Plot rational numbers and their opposites on a number line; know that a number and its opposite have the same absolute value. **Learning Target #3** Use words and symbols to compare rational numbers, where a rational number could also be the absolute value of a number.	**Lesson 1 Positive and Negative Numbers** • I can explain what 0, positive numbers, and negative numbers mean in the context of temperature and elevation. • I can use positive and negative numbers to describe temperature and elevation. • I know what positive and negative numbers are. **Lesson 2 Points on the Number Line** • I can represent negative numbers on a number line. • I can tell or approximate the value of any point on a number line. • I understand what it means for numbers to be opposites. **Lesson 3 Comparing Positive and Negative Numbers** • I can explain how to use the positions of numbers on a number line to compare them. • I can use inequalities to compare positive and negative numbers. **Lesson 4 Ordering Rational Numbers** • I can compare and order rational numbers. • I can use phrases like “greater than,” “less than,” and “opposite” to compare rational numbers. **Lesson 5 Using Negative Numbers to Make Sense of Contexts** • I can explain and use negative numbers in situations involving money. • I can interpret and use negative numbers in different contexts. **Lesson 6 Absolute Value of Numbers** • I can explain what the absolute value of a number is. • I can find the absolute values of rational numbers. • I can recognize and use the notation for absolute value. **Lesson 7 Comparing Numbers and Distance from Zero** • I can explain what absolute value means in situations involving elevation. • I can use absolute values to describe elevations. • I can use inequalities to compare rational numbers and the absolute values of rational numbers.
Checkpoint A	*Responding to Student Thinking* • Problem 1: Point to Emphasize: If most students struggle with locating points on the number line, including opposites and absolute values, reinforce the idea in the next section. For example, when students notice and wonder about points on the number line in the Warm-up referred to here, ask students to determine possible values for each letter based on its location. Grade 6, Unit 7, Lesson 9, Activity 1 Notice and Wonder: Unknowns on a Number Line • Problem 2: Points to Emphasize: If most students struggle with using inequality signs to compare values, especially absolute values, reinforce the idea in the next section. For example, when finding	

	the distances between points in the coordinate plane in the activity referred to here, relate the idea of finding the distance of each point to the x - or y - axis with absolute value. Grade 6, Unit 7, Lesson 14, Activity 2 Signs of Numbers in Coordinates	
Section B Inequalities (Lessons 8-10)	**Learning Target #4** Determine whether a given value is a solution to a given inequality. **Learning Target #5** Draw and label a number line diagram to represent the solutions to an inequality. **Learning Target #6** Write an inequality statement to represent a constraint.	**Lesson 8 Writing and Graphing Inequalities** • I can graph inequalities on a number line. • I can write an inequality to represent a situation. **Lesson 9 Solutions of Inequalities** • I can explain what it means for a number to be a solution to an inequality. • I can graph the solutions to an inequality on a number line. • I can tell if a particular number is a solution to an inequality. **Lesson 10 Interpreting Inequalities** • I can explain what the solution to an inequality means in a situation. • I can write inequalities that involve more than one variable.
Checkpoint B	*Responding to Student Thinking* • Problem 1: Points to Emphasize: If most students struggle with using inequality symbols to write statements that compare values in a situation, reinforce this idea in the next section. For example, ask students to practice writing inequalities when comparing high and low temperatures in Noma, Alaska in the activity referred to here. Grade 6, Unit 7, Lesson 13, Activity 3 High and Low Temperatures • Problem 2: Points to Emphasize: If most students struggle with determining if a number is a solution to an inequality, reinforce this idea in the next section. For example, when students find the coordinates of points in all four quadrants, ask students to identify points that have a greater or lesser x - or y -coordinate. Grade 6, Unit 7, Lesson 11, Activity 2 The Coordinate Plane	
Section C The Coordinate Plane (Lessons 11-15)	**Learning Target #7** Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. **Learning Target #8** Use coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate. **Learning Target #9** Working in all four quadrants, plot a point given its coordinates, or identify the coordinates of a given point in the coordinate plane.	**Lesson 11 Points in the Coordinate Plane** • I can plot points with negative coordinates in the coordinate plane. • I know what a coordinate plane is and can describe the four quadrants. • I know what negative numbers in coordinates tell us. **Lesson 12 Constructing the Coordinate Plane** • When given points to plot, I can construct a coordinate plane with an appropriate scale and pair of axes. **Lesson 13 Interpreting Points in a Coordinate Plane** • I can explain how rational numbers represent balances in a money context. • I can explain what points in a four-quadrant coordinate plane represent in a situation. • I can plot points in a four-quadrant coordinate plane to represent situations and solve problems. **Lesson 14 Distances in the Coordinate Plane** • I can find horizontal and vertical distances between points on the coordinate plane. **Lesson 15 Shapes in the Coordinate Plane** • I can find the lengths of horizontal and vertical segments in the coordinate plane. • I can plot polygons on the coordinate plane when I have the coordinates for the vertices.
Checkpoint C	*Responding to Student Thinking* • Problem 1: Press Pause: By this point in the unit, there should be some student mastery of plotting and labeling the coordinates of points in the coordinate plane. If most students struggle, make time to revisit related work in the section referred to here. See the Course Guide for ideas to help students re-engage with earlier work. Grade 6, Unit 7, Section C The Coordinate Plane • Problem 2: Point to Emphasize: If most students struggle to recognize that when points are related by reflections, their coordinates only differ by signs, revisit this idea in the next section. For example, in the practice problem referred to here, sketch the possible locations of points C and D on a coordinate plane, and notice how the coordinates are opposites. Grade 6, Unit 7, Lesson 17, Practice Problem 5 • Problem 3: Points to Emphasize: If most students struggle to find the distance between two points	

	that both lie on the same vertical or horizontal line, revisit this idea in the next section. For example, ask students to plot the points in the practice problem referred to here, and ask them if they notice any connections between the length of the side and the numbers in the coordinates. Grade 6, Unit 7, Lesson 17, Practice Problem 4	
Section D Common Factors and Common Multiples (Lessons 16-18)	**Learning Target #10** List the factors of a number and identify common factors for two numbers in a real-world situation. **Learning Target #11** List the multiples of a number and identify common multiples for two numbers in a real-world situation.	**Lesson 16 Common Factors** • I can explain what a common factor is. • I can explain what the greatest common factor is. • I can find the greatest common factor of two whole numbers. **Lesson 17 Common Multiples** • I can explain what a common multiple is. • I can explain what the least common multiple is. • I can find the least common multiple of two whole numbers. **Lesson 18 Using Common Multiples and Common Factors** • I can solve problems using common factors and multiples.
Checkpoint D	*Responding to Student Thinking* • Problem 1: Points to Emphasize: If most students struggle to solve problems involving common factors, revisit this idea in the next unit. For example, when students answer survey questions in the activity referred to here, choose two numbers, and ask students to find common factors of both numbers. Grade 6, Unit 8, Lesson 1, Activity 2 Surveying the Class • Problem 2: Point to Emphasize: If most students struggle to solve problems involving common multiples, revisit this idea in the next unit. For example, when students answer survey questions in the activity referred to here, choose two numbers, and ask students to find common multiples of both numbers. Grade 6, Unit 8, Lesson 1, Activity 2 Surveying the Class	
Section E Let's Put It to Work (Lesson 19)	**Learning Target #12** Generate a list of ordered pairs to create an image in the coordinate plane, and explain (orally) the reasoning.	**Lesson 19 Drawing in the Coordinate Plane** • I can use ordered pairs to draw a picture.
End of Unit Assessment		

Unit Title:

Unit 8: Data Sets and Distributions

Relevant Standards: Bold indicates priority

Lesson	Standards	Lesson	Standards
Lesson 1	6.SP.B	Lesson 10	6.SP.A.3 6.SP.B.5.c
Lesson 2	6.SP.A 6.SP.A.1 6.SP.B 6.SP.B.5.b	Lesson 11	6.SP.A.2 6.SP.A.3 6.SP.B.5.c
Lesson 3	6.SP.A.1 6.SP.B.4 6.SP.B.5.a 6.SP.B.5.b	Lesson 12	6.NS.B.3 6.SP.B.5.c 6.SP.B.5.d
Lesson 4	6.SP.A.2 6.SP.B 6.SP.B.4 6.SP.B.5.a	Lesson 13	6.SP.B 6.SP.B.5.c
Lesson 5	6.SP.A.2 6.SP.B 6.SP.B.4 6.SP.B.5.b	Lesson 14	6.SP.B.5.b 6.SP.B.5.c 6.SP.B.5.d
Lesson 6	6.SP.A.1 6.SP.A.3 6.SP.B.4 6.SP.B.5.b	Lesson 15	6.SP.B.5.c 6.SP.B.5.d
Lesson 7	6.SP.A.1 6.SP.A.2 6.SP.B 6.SP.B.4 6.SP.B.5.b	Lesson 16	6.SP.B.4 6.SP.B.5.c 6.SP.B.5.d
Lesson 8	6.SP.A.2 6.SP.B.4	Lesson 17	6.SP.A.1 6.SP.B.4 6.SP.B.5
Lesson 9	6.SP.A.3 6.SP.B 6.SP.B.5.c	Lesson 18	6.SP.A.2 6.SP.B 6.SP.B.5.c 6.SP.B.5.d

Essential Question(s):

- How can we interpret data to answer statistical questions and make sense of real-world situations?
- How do we decide which data displays and statistical measures best represent and explain what we observe?
- What can data tell us, and how do we choose the best way to represent and describe it?
- How do different types of data, displays, and measures help us understand variability and compare information?

Enduring Understanding(s):

- Statistical questions involve variability and are answered by collecting and analyzing data.
- Data can be numerical or categorical, and the type of data affects how it should be organized and represented.
- A data distribution can be described by its center, spread, and overall shape.
- The mean and median describe the center of a data set, while the mean absolute deviation (MAD) and interquartile range (IQR) describe the variability.
- Different data displays such as dot plots, histograms, and box plots highlight different features of a distribution and can be used to draw conclusions.
- Choosing appropriate representations and measures depends on the question being asked and the nature of the data.
- Analyzing and interpreting data helps us make sense of real-world contexts, identify patterns, and support reasoning with evidence.

Demonstration of Learning:

If entire Unit is taught:
Checkpoint A is an opportunity for feedback
CFA 1: Checkpoint B (after lesson 8)
CFA 2: Checkpoint C (after lesson 12)
Mid Unit: Assessment A (after lesson 12)
CFA 3: Checkpoint D (after lesson 17)
EoU: Assessment A

Pacing for Unit

4+ Days

- Pacing currently allows for Lessons 1-4 of the unit to be taught., In that case, there would be no common assessments.

Lesson Modifications:

- Combine Lessons: 6.8.1 and 6.8.2. Lesson 6.8.1 contains a class survey that may need to be significantly adjusted to reflect students' adjusted school and social experiences. The survey can be removed to allow Lessons 6.8.1 and 6.8.2 to be combined.
- Remove Lesson: 6.8.12. This lesson is the application of concepts learned in prior lessons.
- Remove Lesson: 6.8.18. This lesson is the application of concepts learned in prior lessons.

Family Overview

Family Resources: Data Sets and Distributions
Recursos Familiares: Conjuntos de datos y distribuciones

Integration of Technology:

- Desmos Online Graphing Calculator
- Pear Assessment (Edulastic)
- iM v.360 Digital Applets (see below)

Unit-specific Vocabulary:

	New Terminology	
Lesson	receptive	productive

Aligned Unit Materials, Resources, and Technology

Lessons 1-5: Data Visualization Tools
Interactive applets for:

6.8.1	numerical data, categorical data, dot plot	
6.8.2	statistical question, variability	
6.8.3	distribution, frequency	bar graph
6.8.4	typical	
6.8.5	center, spread	variability
6.8.6	histogram, bins	distribution, center
6.8.7		statistical question, spread
6.8.8	symmetrical, peak, cluster, unusual value	numerical data, categorical data, gap
6.8.9	average, mean, fair share	
6.8.10	measure of center balance point	
6.8.11	mean absolute deviation, (MAD) measure of spread	symmetrical, mean
6.8.12		mean absolute deviation (MAD), typical
6.8.13	median	measure of center
6.8.14		peak, cluster, unusual value
6.8.15	range, quartile, interquartile range, (IQR), five-number summary	measure of spread, minimum, maximum,
6.8.16	box plot, whisker	median, interquartile range (IQR)
6.8.17		range, quartile
6.8.18		dot plot, histogram, box plot

- Creating dot plots
- Organizing data sets
- Understanding distributions
- Exploring statistical questions
- Analyzing data displays

Lessons 6-8: Histogram Tools

Tools for:

- Creating histograms
- Understanding frequency
- Comparing data displays
- Analyzing distributions
- Exploring data grouping

Lessons 9-12: Mean and Variability Tools

Interactive applets for:

- Visualizing the mean
- Understanding balance point
- Exploring variability
- Calculating MAD
- Making comparisons

Lessons 13-18: Box Plot Tools

Tools for:

- Creating box plots
- Understanding quartiles
- Comparing distributions
- Analyzing spread
- Exploring data summaries

These applets support key learning goals around **understanding data distributions, creating statistical displays**, and **analyzing measures of center and spread.**

Opportunities for Interdisciplinary Connections:

Science

- Use statistical questions to form hypothesis
- Create data displays to communicate findings
- Analyze variability in experimental results

Social Studies

- Analyze historical populations
- Use graphs to analyze migration patterns
- Use data displays to create and answer questions about elections and polling

Physical Education and Wellness

- Track physical activity metrics
- Analyze nutrition data
- Study grow patterns in adolescents

Anticipated Misconceptions:

- Thinking that any question about numbers is a statistical question.
- Confusing numerical and categorical data.
- Confusing histograms and bar graphs.
- Always using mean, even with skewed data.
- Thinking mean and median are always close.
- Confusing range with IQR
- Not understanding with MAD represents

See teacher's guide for specific misconceptions aligned to each lesson.

Connections to Prior Units:

In previous units, students used percentages to describe real-world situations. In this unit, they continue to apply percentages to analyze data. Students will also use skills from Unit 5, such as computing with decimals, to calculate measures

Connections to Future Units:

In later courses, when student understanding of variability and their exposure to additional distributions is expanded, students will learn about standard deviation and evolve their understanding away from mean absolute deviation.

of central tendency. **Essential prior concepts to engage with this unit:** • This unit begins the study of statistics. Prior to grade 6, students have had experience creating and interpreting bar graphs and line plots. They construct and interpret line plots with fractions in grades 4 and 5, though that work is not a formal prerequisite to the grade 6 standards on dot plots, histograms, or box plots. **Relevant Unit(s)/Lesson(s) to Review:** • Grade 5, Unit 6: More Decimal and Fraction Operations	*See Adaptation Pack in the IL Classroom (6.8 Plan) for more suggestions on connections, pacing modifications, modified plans based on Check Your Readiness Assessment, and a priority list of lessons.*

Differentiation through *Universal Design for Learning*

Notice and Wonder (Engagement & Representation)

Use data displays to:

- Make statistical concepts accessible to all learners
- Build curiosity through open-ended observation
- Support entry into complex data analysis tasks

Collect and Display (Action/Expression & Representation)

- Capturing student language about statistical concepts
- Creating visual references for key vocabulary (mean, median, distribution)
- Building bridges between informal and formal statistical language

Mathematical Language Routines (All UDL Principles)

- Creating word walls for statistical terms
- Using multiple representations of data
- Connecting everyday language to statistical concepts

Related *CELP standards* aligned to Learning Targets:

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes, such as justifying, representing, and interpreting. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Justify

- Reasoning for matching data sets to questions (Lesson 2).
- Reasoning about dot plots (Lesson 3).
- Reasoning about mean and median (Lesson 13).
- Reasoning about changes in mean and median (Lesson 14).
- Reasoning about which information is needed (Lesson 17).
- Which summaries and graphs best represent given data sets (Lesson 18).

Represent

- Data using dot plots (Lessons 3 and 4).
- Data using histograms (Lesson 7).
- Mean using bar graphs (Lesson 9).
- Data with five number summaries (Lesson 15).
- Data using box plots (Lesson 16).

Interpret

- Dot plots (Lessons 4 and 11).
- Histograms (Lessons 6 and 18).
- Mean of a data set (Lesson 9).
- Five-number summaries (Lesson 15).
- Box plots (Lesson 16).

In addition, students are expected to critique the reasoning of others, describe how quantities are measured, describe and compare features and distributions of data sets, generalize about means and distances in data sets, generalize categories for sorting data sets, and generalize about statistical questions. Students are also expected to use language to compare questions that produce numerical and categorical data, compare dot plots and histograms, and compare histograms and bar graphs.

	Emerging	**Expanding**	**Bridging**

LT1	Match data types to examples using visuals Use word bank to identify numerical/categorical "This is ___ data"	Sort data using sentence frames Explain why data is numerical/categorical "The data is ___ because..."	Independently classify data types Use academic vocabulary fluently Complete explanations
LT2	Use yes/no to identify if answers will vary Point to examples of statistical questions "Same/Different answers"	Explain variability using guided structures Identify statistical questions with reasons "This is statistical because..."	Justify statistical nature of questions Analyze variability independently Complex reasoning
LT3	Point to where most data is located Use "few/many" to describe groups Label basic features	Describe center and spread with frames Use "clusters" and "gaps" in descriptions "The data shows..."	Give detailed distribution descriptions Use statistical vocabulary fluently Make connections between features
LT4	Identify bars with most/least data Count frequencies Use "high/low" descriptions	Answer questions about data using frames Compare heights of bars "Most values are between..."	Interpret histograms comprehensively Answer complex questions Make statistical inferences
LT5	Use "greater/less than" for means Identify larger/smaller MAD Basic numerical comparisons	Compare means with sentence frames Describe differences in spread "The mean is ___ more than..."	Compare distributions comprehensively Use precise statistical language Make meaningful connections
LT6	Match measures to simple contexts Use basic vocabulary with support Fill in single-word responses	Explain measures using frames Connect numbers to context "This shows the typical..."	Interpret measures in context Explain significance of values Make statistical arguments
LT7	Label parts of box plots Make simple comparisons Use "higher/lower"	Compare features using frames Describe differences in spread "The median shows..."	Compare distributions fully Use statistical vocabulary Make supported conclusions
LT8	Identify IQR from given values Match quartiles to visual marks Basic numerical recognition	Explain IQR with sentence frames Describe spread in context "The middle 50% is..."	Interpret IQR meaningfully Connect to context Make statistical arguments
LT9	Choose from limited options Match displays to purposes Use yes/no responses	Explain choices with frames Compare display features "I chose ___ because..."	Justify choices independently Evaluate effectiveness Make statistical arguments
LT10	Select measures from options Match measures to basic uses Use provided vocabulary	Compare measures using frames Explain advantages "This works better for..."	Select and justify measures Compare advantages/limitations Complete statistical reasoning

Additional Sentence Frames and Stems

Section A

- _____ is an example of categorical/numerical data because ...
- In this context, the data point _____ means ...
- The question _____ is statistical /not statistical because ...

Section B

- The frequency of _____ is _____.
- The histogram/dot plot _____ represents/does not represent _____ because....
- I would use a _____ rather than a _____ to represent the data because ...
- I think that the spread of data represented in this dot plot is _____ because ...
- The data point ____ is the center of the distribution because ...

Section C

- The data set that represents _____ has a mean of _____ because ...
- The mean of the first data set is _____ and the mean for the second data set is _____. This tells me that ...
- The data set that represents _____ has a mean absolute deviation of ____. This tells me the variability is _____ because ...
- The mean absolute deviation of the first data set is _____ and the mean absolute deviation for the second data set is _____. This tells me that ...

Section D

- The data set that represents _____ has a median of _____ because ...
- The data set that represents _____ has an interquartile range of ____ because ...
- The interquartile range for the first data set is _____ and the interquartile range for the second data set is _____. This tells me that ...
- Comparing the box plots for the two data sets, I can see that ...

Section E

- The median for this data set is _____ and the mean is _____. The _____ is the best measure of center to describe the data

because ...
- The mean of _____ is less/greater than the median because...
- The shape of the distribution appears _____ because ...
- I chose the _____ to represent the data because ...

Unit Outline

Lesson Sequence	Learning Target(s)	Success Criteria/Assessment
Section A Data, Variability, and Statistical Questions (Lessons 1-2)	**Learning Target #1** Comprehend and use the terms “numerical” and “categorical” to describe data sets. **Learning Target #2** Justify whether a question is “statistical” based on whether variability is expected in the data that could be collected.	**Lesson 1 Got Data?** • I can collect the correct data to answer a question and use the correct units. • I can explain the difference between categorical and numerical data. **Lesson 2 Statistical Questions** • I can tell statistical questions from non-statistical questions and explain the difference. • I can tell when data has variability.
Checkpoint A	*Responding to Student Thinking* • Problem 1: Points to Emphasize: If students struggle to classify questions as categorical or numerical, plan to reinforce the idea in the following sections by asking students to classify questions as they arise. For example, when students create questions about collected bottle caps, have students classify their suggested questions or specifically create a question of each type. Grade 6, Unit 8, Lesson 3, Activity 1 Curious about Caps • Problem 2: Points to Emphasize: If students struggle to understand what statistical questions are, reinforce the idea by asking about the questions in upcoming sections. For example, as students consider computer upgrades, ask why a dot plot is necessary to answer the question and whether students can think of a non-statistical question they could ask about the situation. Grade 6, Unit 8, Lesson 4, Activity 2 Computer Upgrades	
Section B Dot Plots and Histograms (Lessons 3-8)	**Learning Target #3** Describe a distribution represented by a dot plot, including informal observations about its center and spread. **Learning Target #4** Interpret a histogram to answer statistical questions about a data set.	**Lesson 3 Representing Data Graphically** • I can describe the information presented in tables, dot plots, and bar graphs. • I can use tables, dot plots, and bar graphs to represent distributions of data. **Lesson 4 Dot Plots** • I can describe the center and spread of data from a dot plot. **Lesson 5 Using Dot Plots to Answer Statistical Questions** • I can use a dot plot to represent the distribution of a data set and answer questions about the real-world situation. • I can use center and spread to describe data sets, including what is typical in a data set. **Lesson 6 Interpreting Histograms** • I can recognize when a histogram is an appropriate graphical display of a data set. • I can use a histogram to get information about the distribution of data and explain what it means in a real-world situation. **Lesson 7 Using Histograms to Answer Statistical Questions** • I can draw a histogram from a table of data. • I can use a histogram to describe the distribution of data and determine a typical value for the data. **Lesson 8 Describing Distributions on Histograms** • I can describe the shape and features of a histogram and explain what they mean in the context of the data. • I can distinguish histograms and bar graphs.
Checkpoint B	*Responding to Student Thinking* • Problem 1: Points to Emphasize: If students struggle to describe the distribution, emphasize center and spread throughout the next section by focusing on mean and mean absolute deviation. For example, as students examine dot plots and calculate measures of center and variability, ask students to summarize their findings by describing the distribution. Grade 6, Unit 8, Lesson 11, Activity 2 Shooting Hoops (Part 2) • Problem 2: Points to Emphasize: If students struggle to interpret a histogram, revisit histograms as	

	they come up in future sections. For example, after students examine distributions shown using histograms, ask how many individuals fall into a particular interval or group of intervals. Grade 6, Unit 8, Lesson 14, Activity 3 Card Sort: Mean or Median?	
Section C Measures of Center and Variability (Lessons 9-12)	**Learning Target #5** Compare the means and mean absolute deviations of different distributions. **Learning Target #6** Interpret the mean and mean absolute deviation (MAD) in the context of the data.	**Lesson 9 Mean** • I can explain how the mean for a data set represents a "fair share." • I can find the mean for a numerical data set. **Lesson 10 Finding and Interpreting the Mean as the Balance Point** • I can describe what the mean tells us in the context of the data. • I can explain how the mean represents a balance point for the data on a dot plot. **Lesson 11 Variability and MAD** • I can find the MAD for a set of data. • I know what the mean absolute deviation (MAD) measures and what information it provides. **Lesson 12 Using Mean and MAD to Make Comparisons** • I can say what the MAD tells us in a given context. • I can use means and MADs to compare groups.
Checkpoint C	*Responding to Student Thinking* Points to Emphasize: If students struggle to use mean and MAD to compare groups, emphasize the role of both center and variability in comparing groups (as those measures arise in the next section). For example, as students compare paper-airplane data using box plots, ask how the measures of center compare as well as how the measures of variability compare. Grade 6, Unit 8, Lesson 17, Activity 3 Paper Planes	
Section D Median and IQR (Lessons 13-17)	**Learning Target #7** Compare and contrast distributions that are represented with box plots. **Learning Target #8** Interpret the median and interquartile range (IQR) in the context of the data.	**Lesson 13 Median** • I can find the median for a set of data. • I can say what the median represents and what it tells us in a given context. **Lesson 14 Comparing Mean and Median** • I can determine when the mean or the median is more appropriate to describe the center of data. • I can explain how the distribution of data affects the mean and the median. **Lesson 15 Quartiles and Interquartile Range** • I can use the IQR to describe the spread of data. • I know what quartiles and the interquartile range (IQR) measure and what they tell us about the data. • When given a list of data values or a dot plot, I can find the quartiles and interquartile range (IQR) for the data. **Lesson 16 Box Plots** • I can use the five-number summary to draw a box plot. • I know what information a box plot shows and how it is constructed. **Lesson 17 Using Box Plots** • I can use a box plot to answer questions about a data set. • I can use medians and IQRs to compare groups.
Checkpoint D	*Responding to Student Thinking* • Problem 1: Press Pause: If students struggle to interpret median and interquartile range in a situation, make time to offer additional situations for students to interpret. For example, use practice problems to provide examples, or ask students to bring in their own real-world situations in which median and interquartile range are used to describe a situation, and ask them to interpret the measures. Grade 6, Unit 8, Lesson 17, Practice Problem 4 • Problem 2: Press Pause: If students struggle to compare two groups using a box plot, make time to revisit the idea in different situations. For example, use practice problems or other activities to provide contexts with one or more box plots. Ask students what another box plot would look like for another group in that situation if the box plots had the same median, the same IQR, both, or neither. Then ask students to interpret what those different situations would mean when comparing the groups. Grade 6, Unit 8, Lesson 16, Practice Problem 4	
Section E Let's Put It to Work (Lesson 18)	**Learning Target #9** Recognize that different graphical displays offer different insights into a distribution. Choose an appropriate graphical display to represent a data set, and	**Lesson 18 Using Data to Solve Problems** • I can decide whether the mean and MAD, or the median and IQR, would be more appropriate • for describing the center and spread of a data set. • I can draw an appropriate graphical representation for a set of data. • I can explain what the mean and MAD, or the median and IQR, tell us in the context of a situation, and I can use them to answer questions.

	justify the choice (orally and in writing). **Learning Target #10** Recognize that different measures of center and variability offer different insights into a data set. Choose an appropriate measure of center and variability to describe a data set, and justify the choice (orally and in writing).	
End of Unit Assessment		

Course Assessment Map

Unit	Assessment 1	Assessment 2	Assessment 3	Assessment 4	Assessment 5	Assessment 6	Assessment 7
Unit 1 *Area and Surface Area*	CFA 1: Checkpoint B (after L6)	CFA 2: Checkpoint C (after L11)	MOU (A)	CFA 3 Checkpoint E (after L18)	EOU(A)		
	G6 U1 CFA 1 (CP-B)	G6 U1 CFA 2 (CP-C)	G6 U1 MOU	G6 U1 CFA 3 (CP-E)	G6 U1 EOU		
Unit 2 *Introducing Ratios*	Fall IAB	CFA 1: Checkpoint B (after L5)	CFA 2: Checkpoint C (after L10)	CFA 3: Checkpoint E (after L16)	EOU (A)		
	Grade 6 IAB Geometry (pg. 33)	G6 U2 CFA 1 (CP-B)	G6 U2 CFA 2 (CP-C)	G6 U2 CFA 3 (CP-E)	G6 U2 EOU		
Unit 3 *Unit Rates and Percentages*	CFA 1: Checkpoint A (after L3)	CFA 2: Checkpoint B (after L9)	CFA 3: Checkpoint C (after L16)	EOU (A)			
	G6 U3 CFA 1 (CP-A)	G6 U3 CFA 2 (CP-B)	G6 U3 CFA 3 (CP-C)	G6 U3 EOU			
Unit 4 *Dividing Fractions*	CFA 1: Checkpoint A (after L3)	Winter IAB	CFA 2: Checkpoint B (after L9)	MOU (A)	CFA 3: Checkpoint C (after L11)	CFA 4: Checkpoint D (after L15)	EOU (A)
	G6 U4 CFA 1 (CP-A)	Grade 6 IAB Ratios and Proportional Relationships (pg. 32)	G6 U4 CFA 2 (CP-B)	G6 U4 MOU	G6 U4 CFA 3 (CP-C)	G6 U4 CFA 4 (CP-D)	G6 U4 EOU
Unit 5 *Arithmetic In Base Ten*	CFA 1: Checkpoint A (after L4)	Spring IAB	CFA : Checkpoint B (after L8)	MOU (A)	CFA 3: Checkpoint C (after L13)	EOU(A)	
	G6 U5 CFA 1 (CP-A)	Grade 6 IAB Divide Fractions by Fractions (pg. 30)	G6 U5 CFA 2 (CP-B)	G6 U5 MOU	G6 U5 CFA 3 (CP-C)	G6 U5 EOU	
Unit 6 *Expressions and Equations*	CFA 1: Checkpoint A (after L6)	CFA 2: Checkpoint B (after L11)	MOU (A)	CFA 2: Checkpoint C (after L15)	EOU(A)		
	G6 U6 CFA 1 (CP-A)	G6 U6 CFA 2 (CP-B)	G6 U6 MOU	G6 U6 CFA 3 (CP-C)	G6 U6 EOU		
Unit 7 *Rational Numbers*	CFA 1: Checkpoint A (after L7)	SBA	CFA 2: Checkpoint B (after L10)	CFA 3: Checkpoint C (after L15)	CFA 4: Checkpoint D (after L18)	EOU(A)	
	G6 U7 CFA 1 (CP-A)	Grade 6 SBA Sample Items	G6 U7 CFA 2 (CP-B)	G6 U7 CFA 3 (CP-C)	G6 U7 CFA 4 (CP-D)	G6 U7 EOU	
Unit 8 *Data Sets and Distributions*	Cover first 4 lessons, no common assessments						
	G6 U8 (CP-A)	G6 U8 CFA 1 (CP-B)	G6 U8 MOU	G6 U8 CFA 2 (CP-C)	G6 U8 CFA 3 (CP-D)	G6U8 EOU	

*Pear Assessment links to be completed throughout 2025-26

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