

Algebra III Content Standards Revisions 2022

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| Course Title: Algebra III | |
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| Course/Unit Credit: 1 | |
| Course Number: 439070 | |
| Teacher Licensure: Please refer to the Course Code Management System (<u>https://adedata.arkansas.gov/ccms/</u>) for the most curre | ent |
| licensure codes. | |
| Grades: 9-12 | |
| Prerequisite: Algebra I, Geometry, Algebra II | |

Course Description: This course will enhance the higher-level thinking skills developed in Algebra II through a more in-depth study of those concepts and exploration of some pre-calculus concepts. Students in Algebra III will be challenged to increase their understanding of algebraic, graphical, and numerical methods to analyze, translate and solve polynomial, rational, exponential, and logarithmic functions. Modeling real-world situations is an important part of this course. Sequences and series will be used to represent and analyze real-world problems and mathematical situations. Algebra III will also include an introductory study of matrices and conics. Algebra III is a transitional course and should prepare students for success in future courses. Algebra III does not require Arkansas Department of Education approval.

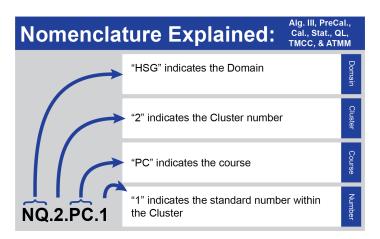
ACT 480: Please refer to the Mathematics Transitional Courses document (<u>https://rb.gy/ne4ul6</u>) for transitional math course options.

Introduction to Secondary Arkansas Mathematics Standards

When the Division of Elementary and Secondary Education (DESE) began the process of revising math standards, a diverse group of qualified educators from across the state came together to craft Arkansas standards specific to the schools and students in the state. The result of this work, the Arkansas Mathematics Standards, is contained in this document. These standards reflect what educators across the state know to be best for Arkansas students.

Standards Organization: The revision committee maintained the organizational structure and nomenclature of the previous standards. Secondary Arkansas Mathematics Standards are categorized into domains, clusters, and standards.

- **Domains** represent the big ideas to be studied in each course and sometimes across grade bands. These big ideas support educators in determining the proper amount of focus and instructional time to be given to each of these topics.
- **Clusters** represent collections of standards grouped to help educators understand the building blocks of rich and meaningful instructional units. These units help students make connections within clusters and avoid seeing mathematics as a discrete list of skills they must master.
- **Standards** represent the foundational building blocks of math instruction. The standards outlined in this document work together to ensure that students are college and career ready and on track for success.



Standards Support: The revision of the Arkansas Mathematics Standards represent the work of the committee to provide greater clarity, strength, and support of the standards. Additionally, the revised mathematics standards are designed to help educators better understand the areas of emphasis and the focus within the standards. Educators should address the bulleted content as more than a checklist of items that they must teach individually. Content is bulleted to provide specificity of learning expectations included within some extensive standards. In some instances, the standard document includes Arkansas examples, teacher notes, specifications, and italicized words to assist educators with planning, teaching, and student learning.

- Examples included in the original standards were either changed for clarity or separated from the body of the actual standard. The examples included in the body of the standards document in no way reflect all of the possible examples. Likewise, these examples do not mandate curriculum or problem types. Local districts are free to select the high-quality curricula and instructional methods that best meet the needs of their students.
- **Teacher notes** offer clarification of the standards. These notes are intended to clarify, for teachers, what the expectations are for the learner. Likewise, these notes provide instructional guidance and limitations so that educators can better understand the scope of the standard. This will help with determining what is developmentally appropriate for students when working with specific standards.
- Standard specifications are to strengthen standards. The specifications are precise statements highlighting the need for mastery or function-type parameters for specific standards. This will assist educators in pinpointing the best opportunities for students to gain and master the knowledge and skills needed to succeed in a progression.
- Asterisks (*) are denoted to represent the modeling component of the standards. These standards should be presented in a modeling context which allows students to engage in the modeling process that is outlined in the Standards for Mathematical Process. (See Appendix A)
- Italicized words are defined in the glossary.

Finally, the Arkansas Mathematics Standards will be a living document. As these standards are implemented across schools in the state, DESE welcomes further suggestions related to notes of clarification, examples, professional development needs, and future revisions of the standards.

| K - 12 Standards for Mathematical Practices | | |
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| Make sense of problems and persevere in solving them. Reason abstractly and quantitatively. Construct viable arguments and critique the reasoning of others. Model with mathematics.* | Use appropriate tools strategically. Attend to precision. Look for and make use of structure. Look for and express regularity in repeated reasoning. | |

| Algebra III Standards: Overview Abbreviations: The following abbreviations are for the conceptual categories and domains for the Arkansas Mathematics Standards. | | |
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| | 1. Students will perform operations with matrices and use them to solve systems of equations with or without the appropriate technology. | |
| Conic Sections - CS | | |
| | 2. Students will identify, analyze, and sketch the graphs of the conic sections and relate the equations and graphs. | |
| Function Operations and Properties - FOP | | |
| | 3. Students will be able to describe properties of functions and build new functions from existing functions. | |
| Interpreting Functions - IF | | |
| | 4. Students will be able to interpret different types of functions and key characteristics including polynomial, exponential, logarithmic, and rational functions. | |
| Sequences and Series - SS | | |
| | 5. Students will use sequences and series to represent and analyze mathematical situations. | |

| | Matrix Operations | |
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| Students will p | perform operations with matrices and use them to solve systems of equations with or without the appropriate technology. | |
| MO.1.AIII.1 | Use matrices to represent and manipulate data. | |
| | Teacher Note: Use matrices to describe, list, and manipulate data in many different situations. | |
| | Example: Represent payoffs or incidence relationships in a network. | |
| MO.1.AIII.2 | Multiply matrices by scalars to produce new matrices | |
| | Teacher Note: To explore the relationship of scalar multiplication then solve different contextual situations. | |
| | Example: Increase the cost of merchandise at a store to fit a desired profit margin. | |
| MO.1.AIII.3 | Add, subtract, and multiply matrices of appropriate dimensions. | |
| MO.1.AIII.4 | Understand that, unlike multiplication of numbers, <i>matrix multiplication</i> for square matrices is not a commutative operation but still satisfies the associative and distributive properties. | |
| MO.1.AIII.5 | Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers; the <i>determinant</i> of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. | |
| MO.1. AIII.8 | Represent and solve a system of linear equations represented by an augmented matrix. | |

| Conic Sections | |
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| Students will identify, analyze, and sketch the graphs of the conic sections and relate the equations and graphs. | |
| CS.2. AIII.1 | Find the conjugate of a complex number and use conjugates to find quotients of complex numbers. |
| CS.2. AIII.3 | Complete the square in order to generate an equivalent form of an equation for a <i>conic section;</i> use that equivalent form to identify key characteristics of the <i>conic section</i> . |
| CS.2. AIII.4 | Identify, graph, write, and analyze equations of each type of <i>conic section</i> , using properties such as <i>symmetry</i> , intercepts, <i>foci</i> , <i>asymptotes</i> , and <i>eccentricity</i> and using technology when appropriate. |

| | Function Operations and Properties | |
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| Students will b | Students will be able to describe properties of functions and build new functions from existing functions. | |
| FOP.3.AIII.1 * | Compose functions and interpret the meaning in a real-world context. | |
| FOP.3. AIII.2 | Verify, by <i>composition,</i> that one function is the inverse of another. | |
| FOP.3. AIII.3 | Read values of an <i>inverse function</i> from a graph or a table, given that the function has an inverse. | |
| FOP.3. AIII.4 | Produce an <i>invertible function</i> from a <i>non-invertible function</i> by restricting the domain. | |
| FOP.3. AIII.5 | Combine standard function types using arithmetic operations (e.g., Given that $f(x)$ and $g(x)$ are functions developed from a context, find $(f + g)(x)$, $(f - g)(x)$, $(fg)(x)$, $(f/g)(x)$, and any combination thereof, given $g(x) \neq 0$). | |
| FOP.3. AIII.6 | Understand the inverse relationship between exponents and <i>logarithms;</i> use this relationship to solve problems involving exponents and <i>logarithms</i> . | |
| FOP.3.AIII.7 | Graph <i>transformations</i> of functions including quadratic, absolute value, square root, cube root, cubic, and step functions; graph <i>piecewise-defined</i> functions including these <i>transformations</i> . | |
| FOP.3. AIII.8 | Determine if a function is even, odd, or neither. | |

| | Interpreting Functions | |
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| Students will be able to interpret different types of functions and key characteristics including polynomial, exponential, logarithmic, and rational functions. | | |
| IF.4. AIII.1 | Graph rational functions identifying zeros and asymptotes when suitable factorizations are available; show end behavior. | |
| IF.4. AIII.2 | Analyze and interpret polynomial functions numerically, graphically, and algebraically, identifying key characteristics such as intercepts, end behavior, domain and range, relative and absolute <i>maximum</i> and <i>minimum</i> , and intervals over which the function increases and decreases. | |
| IF.4. AIII.3 | Analyze and interpret rational functions numerically, graphically, and algebraically; identify key characteristics such as <i>asymptotes</i> (<i>vertical, horizontal</i> , and slant), end behavior, point discontinuities, intercepts, and domain and range. | |
| IF.4. AIII.4 | Analyze and interpret <i>exponential functions</i> numerically, graphically, and algebraically; identify key characteristics such as <i>asymptotes</i> , end behavior, intercepts, and domain and range. | |

| IF.4. AIII.5 | Analyze and interpret <i>logarithmic functions</i> numerically, graphically, and algebraically; identify key characteristics such as <i>asymptotes</i> , end behavior, intercepts, and domain and range. |
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| IF.4. AIII.6* | Build and analyze functions to model real-world applications using algebraic operations on functions and <i>composition of functions</i> , with and without appropriate technology. |
| | Teacher Note: This standard emphasizes the change in characteristics when the new function is formed. This standard connects to FOP.3.AIII.1. |
| | Example: Profit functions, volume, and surface area (optimization subject to constraints). |

| Sequences and Series | |
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| Students will use sequences and series to represent and analyze mathematical situations. | |
| SS.5. AIII.1 | Write arithmetic and geometric sequences both recursively and with an explicit formula; translate between the two forms. |
| SS.5. AIII.2 | Use arithmetic and geometric sequences both recursively and with an explicit formula to model situations. |
| SS.5. AIII.3 | Recognize that sequences are functions, sometimes defined recursively, whose domains are subsets of the integers. |
| SS.5. AIII.4* | Use sequences and series to solve real-world problems, with and without appropriate technology. |

Glossary

| Arithmetic Sequence | A sequence in which each term after the first is found by adding a constant, called the common difference <i>d</i> , to the previous term. |
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| Asymptote(s) | Line(s) to which a graph becomes arbitrarily close as the value of x or y increases or decreases without bound (vertical, horizontal, slant) |
| Complex number(s) | Number(s) that can be written as the sum or difference of a real number and an imaginary number (e.g., $5 + 2i$) |
| Composition of Functions | Suppose <i>f</i> and <i>g</i> are functions such that the range of <i>g</i> is a subset of the domain of <i>f</i> , then the composite function <i>f</i> of <i>g</i> can be described by the equation. For Example: $[f \circ g](x) = f[g(x)]$ |
| Conic section | Any figure that can be formed by slicing a double cone with a plane |
| Conjugate(s) | The result of writing the sum of two terms as a difference or vice-versa |
| Eccentricity | A number that indicates how drawn out or attenuated a conic section is; eccentricity is represented by the letter e (no relation to $e = 2.718$) |
| Exponential Function | A function in which the variable(s) occurs in the exponent [e.g., $f(x) = ab^x$, $b > 0$] |
| Foci | Two fixed points on the interior of an ellipse used in the formal definition of the curve |
| Geometric Sequence | A sequence in which each term after the first is found by multiplying the previous term by a constant, called the common ratio, r . |
| Horizontal Asymptote | A horizontal line to which a graph becomes arbitrarily close as the value of x increases or decreases without bound. |
| Inverse Function | Two functions <i>f</i> and <i>g</i> are inverse functions, if and only if, both of their <i>composition</i> s yield the identity function; <i>p</i> for example, $[f \circ g](x) = x$ and $[g \circ f](x) = x$ |
| Invertible function | A function that has an inverse, and one-to-one. |
| Logarithmic Function | A function of the form $y = log_b x$, where $b > 0$, $x > 0$ and $b \neq 1$ |

| Logarithm(s) | The power to which a base <i>b</i> must be raised to yield a given number <i>x</i> . The logarithmic form is represented by $\frac{1}{2}$ |
|---------------------------|---|
| | $log_{b}a = x$, where $b^{x} = a$. For example: $log_{3}81 = x$ then $x = 4$ because $3^{4} = 81$ |
| Matrix multiplication | Given Matrix <i>A</i> with dimensions $[g X h]$ and Matrix <i>B</i> with dimensions $[y X z]$; the matrix multiplication of <i>AB</i> results in a matrix with dimensions of $[g X z]$ and is only possible if the number of columns in <i>A</i> is equal to the number of rows in <i>B</i> (if and only if $h = y$); matrices are multiplied as shown below: |
| | $[a b c d] \cdot [e f g h i j] = [ae + bh af + bi ag + bj ce + dh cf + di cg + dj]$ |
| | Example: $[1 2 3 4] \cdot [5 6 7 8 9 10] = [1 \cdot 5 + 2 \cdot 8 1 \cdot 6 + 2 \cdot 9 1 \cdot 7 + 2 \cdot 10 3 \cdot 5 + 4 \cdot 8 3 \cdot 6 + 4 \cdot 9 3 \cdot 7 + 4 \cdot 10]$ |
| Maximum | The greatest value of a function if the function has such an extreme value. |
| Minimum | The least value of a function if the function has such an extreme value. |
| Non-invertible function | A function that does not have an inverse. |
| Optimization | The process by which one seeks to minimize or maximize a real function by systematically choosing the values of real or integer variables from within a domain. |
| Piecewise Function | A function defined by different rules for different parts of the domain. |
| Symmetry | A figure has symmetry if the figure and its image coincide after a transformation. |
| Transformations of graphs | Operations that alter the form of a figure (e.g., horizontal stretches, vertical stretches, reflections) |
| Vertical Asymptote | A vertical line to which a graph becomes arbitrarily close as the value of $f(x)$ increases or decreases without bound |

Appendix A.

Mathematical Modeling Cycle

The basic modeling cycle is summarized in this diagram. It involves: (1) identifying variables in the situation and selecting those that represent essential features; (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables; (3) analyzing and performing operations on these relationships to draw conclusions; (4) interpreting the results of the mathematics in terms of the original situation; (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable; (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.

