Bristol Public Schools

Office of Teaching & Learning

Department	Mathematics
Department Philosophy	*Students learn by doing math, solving problems in mathematical and real-world contexts, and constructing arguments using precise language.* The Bristol mathematics curricula embeds this *learn-by-doing* philosophy by focusing on high expectations for all students and providing students with opportunities that build conceptual understanding, computational and procedural fluency, and problem solving through the use of a variety of strategies, tools, and technologies. The mathematics curriculum is responsive to the individual needs of students, while providing a structure tied to the Common Core State Standards in Connecticut. The *learn-by-doing* philosophy develops mathematically literate and productive students who can effectively and efficiently apply mathematics in their lives to make informed decisions about the world around them by doing math. To be mathematically literate, one must understand major mathematics concepts, possess computational facility, and have the ability to apply these understandings to situations in daily life. Making connections between mathematics and other disciplines is key to the appropriate application of mathematics skills and concepts to solve problems. The ability to read, discuss, and write within the discipline of mathematics is an integral skill that supports mathematical understanding, reasoning and communication. The opportunity to think critically and creatively to solve problems is important to deepen mathematical knowledge and foster innovation. A rich hands-on mathematical experience is essential to provide the foundational knowledge and skills that prepare students to be mathematically literate, productive citizens.
Course	Grade 5 Mathematics
Grade Level	Grade 5
Pre-requisites	Grade 4

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M-Major Cluster, S-Supporting Cluster, A-Additional Cluster

District Learning Expectations and Standards	Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6	Unit 7	Unit 8 (optional)
Operations and Algebraic Thinking								
Write and interpret numerical expressions.								
5.OA.A.1 Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.	A			A	A			
5.OA.A.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.	A	A		A	A		A	
Analyze patterns and relationships.								
5.OA.B.3 Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.							A	
Number and Operations in Base Ten								
Understand the Place Value System								
5.NBT.A.1 Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place					M	M		

to its right and 1/10 of what it represents in the place to its left.								
5.NBT.A.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.						M		
5.NBT.A.3 Read, write, and compare decimals to thousandths.					M			
5.NBT.A.3.A Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000).					M			
5.NBT.A.3.B Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.					M			
5.NBT.A.4 Use place value understanding to round decimals to any place.					M			
Perform operations with multi-digit whole numbers and with decimals to hundredths.								
5.NBT.B.5 Fluently multiply multi-digit whole numbers using the standard algorithm.				M				
5.NBT.B.6 Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.				M				M
5.NBT.B.7: Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the					M			

relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.								
Number and Operations - Fractions								
Use equivalent fractions as a strategy to add and subtract fractions.								
5.NF.A.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.						M		M
5.NF.A.2 Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.						M		
Apply and extend previous understandings of multiplication and division.								
5.NF.B.3 Interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?		M						

5.NF.B.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.		M	M					M
5.NF.B.4.A Interpret the product (a/b) × q as a part of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a × q ÷ b. For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = (ac)/(bd).		M	M					
5.NF.B.4.B Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.		M	M					
5.NF.B.5 Interpret multiplication as scaling (resizing), by:						M		
5.NF.B.5.A Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.						M		
5.NF.B.5.B Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence a/b = (n × a)/(n × b) to the effect of multiplying a/b by 1.						M		
5.NF.B.6 Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.			M			M		

5.NF.B.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.			M					
5.NF.B.7.A Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.			M					
5.NF.B.7.B Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.			M					
5.NF.B.7.C Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?			M					
Measurement and Data								
Convert like measurement units within a given measurement system.								
5.MD.A.1 Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.						S		
Represent and interpret data.								
5.MD.B.2 Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use						S		

operations on fractions for this grade to solve problems involving information presented in line plots. For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.								
Geometric measurement: understand concepts of volume.								
5.MD.C.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement.	M			M				M
5.MD.C.3.A A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.	M							
5.MD.C.3.B A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.	M							
5.MD.C.4 Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.	M							
5.MD.C.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.	M			M				M
5.MD.C.5.A Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.	M							
5.MD.C.5.B Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.	M			M				

5.MD.C.5.C Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.	M							
Geometry								
Graph points on the coordinate plane to solve real-world and mathematical problems.								
5.G.A.1 Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).							A	
5.G.A.2 Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.							A	
Classify two-dimensional figures into categories based on their properties.								
5.G.B.3 Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.							A	A
5.G.B.4 Classify two-dimensional figures in a hierarchy based on properties.							A	A

UNIT 1: FINDING VOLUME

Illustrative Mathematics Unit Focus: Students find the volume of right rectangular prisms and solid figures composed of two right rectangular prisms.

Essential Questions:
What does volume measure in a solid figure?
How can we represent mathematical situations?

Unit Pacing: 20 days (11 required lessons, 7 flex, 2 assessment and reaction)

UNWRAPPED STANDARDS

Grade Level Standard	Standard Progression	Concepts (Big Ideas/ Understandings)	Academic Vocabulary (Standard Based)
5.OA.A.1 Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.	Students write expressions to express a calculation, e.g. writing 2 x (8 + 7) to express the calculation “add 8 and 7, then multiply by 2.”	Parentheses and brackets can be used in writing expressions to group numbers and operations together. There are multiple ways to write equivalent expressions to represent a given situation. There is a specific convention used to interpret and evaluate expressions. The information within parentheses and brackets are evaluated first.	Expression Parentheses Brackets Braces Evaluate
5.OA.A.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them.	Students evaluate and interpret expressions, e.g., using their conceptual understanding of multiplication to interpret 3 x (18932 + 921) as being three times as large as 18932 + 921, without having to calculate the indicated sum or product.	Mathematical situations can be translated and represented abstractly using variables, expressions, and equations. You can make observations about the relative magnitude of the value of an expression without having to perform any calculations.	Expression Parentheses Variable Evaluate Interpret
5.MD.C.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement.	Students learn about a unit of volume, such as a cube with a side length of 1 unit, called a unit cube. They pack cubes (without gaps) into right rectangular prisms and count the cubes to	Volume is an attribute of a three-dimensional solid figure that is measured in cubic units.	Measurement Attribute Volume Solid figure

[5.MD.C.3.A](#) A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.	determine the volume or build right rectangular prisms from cubes and see the layers as they build.	Volume can be measured (or determined) by finding the total number of cubic units required to fill the space without gaps or overlaps	Unit cube Gap Overlap Cubic units Three-dimensional Space Volume Solid figure Rectangular prism Length Width Height Base area
[5.MD.C.3.B](#) A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.			
[5.MD.C.4](#) Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.			
[5.MD.C.5](#) Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.	Students understand that multiplying the length times the width of a right rectangular prism can be viewed as determining how many cubes would be in each layer if the prism were packed with or built up from unit cubes. They also learn that the height of the prism tells how many layers would fit in the prism.	The area of a base of a rectangular prism is found by multiplying the length by width ($b = \ell \times w$). In a right rectangular prism, any two parallel faces can be the bases. The volume of a rectangular prism can be found by multiplying the length by width by height ($\ell \times w \times h$) or by multiplying the area of the base by height ($b \times h$).	Volume Cubic units Multiplication Addition Rectangular prism Length Width Height Base area Formula Associative Property Distributive Property Commutative Property Decompose
[5.MD.C.5.A](#) Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.			
[5.MD.C.5.B](#) Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.			
[5.MD.C.5.C](#) Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems	Students rely on their understanding of the additive nature of area to generalize that this same idea applies to volume. By composing and decomposing right rectangular prisms and finding the volume of each, students can prove that volume is additive and therefore enhance their understanding of the concept.	A figure composed of rectangular prisms may be decomposed into two non-overlapping rectangular prisms whose volumes may be added to find the volume of the figure.	

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<tr><th colspan="5">Section A: Unit Cubes and Volume</th></tr>
<tr><td>5.MD.C.3
5.MD.C.3.b
5.MD.C.4
5.MD.C.5.a
5.OA.A.2</td><td>I can find the volume of a rectangular prism by counting unit cubes.</td><td rowspan="2">X Selected Response
X Constructed Response
    Performance
X Observation</td><td>Lesson Progression:
Students use unit cubes to develop the understanding that volume is a measurement of any three-dimensional figure. They build figures with unit cubes and view images of cubes configured in a variety of ways. The activities in this section encourage students to count the number of unit cubes to find the volume of three-dimensional figures in cubic units. Students then build and examine images of right rectangular prisms constructed from unit cubes. The structure of these prisms allows students to use their own methods of counting unit cubes. As the right rectangular prisms are oriented in different ways, students begin to think about the layers and dimensions of the prisms and make connections to the commutative and associative properties of multiplication.</td><td>Mandatory Lessons/Activities:
iM Lessons 1, 2, 3, 4</td></tr>
<tr><td>Pacing:</td><td>4 days</td><td>Math Practices:
SMP 3, 5, 6, 7, 8</td><td>Assessments:
Cool-downs: 2, 4
Checkpoint A</td></tr>
<tr><th colspan="5">Section B: Volume Formulas</th></tr>
<tr><td>5.MD.C.4
5.MD.C.5.a
5.MD.C.5.b
5.OA.A.1</td><td>I can apply a formula to find the volume of a rectangular prism.</td><td>X Selected Response</td><td>Lesson Progression:
Continuing the work with rectangular prisms, this section provides opportunities for students to conceptualize volume as the measurement of an</td><td>Mandatory Lessons/Activities:
iM Lessons 5, 6, 7</td></tr>
</table>

5.OA.A.2		X Constructed Response Performance X Observation	amount of space inside a three-dimensional figure. They use what they know about the area of a two-dimensional figure to make sense of the grid structure at the base of a rectangular prism made from interlocking cubes. As students make connections between area and volume, they begin to see that each layer is composed of rows and columns that can be decomposed into unit cubes. They are confronted with the complexities of measuring volume as they learn that they can measure volume using cubic centimeters, cubic inches, and cubic feet. The activities in this section allow students to discover the formulas for finding the volume of a rectangular prism. They realize that the number of cubes in one layer of a rectangular prism is equal to its length times width, and that the height represents the number of layers in the rectangular prism. They generalize this understanding to see that volume is derived by multiplying length x width x height *(V = l x w x h)* and area of the base x height *(V = B x h)*, and understand the relationship between these two formulas. They use these formulas to solve real world and mathematical problems about rectangular prisms with whole number edge lengths.	
Pacing:	3 days		**Math Practices:** SMP 1, 2, 3, 4, 5, 6, 7, 8	**Assessments:** Cool-down 6, 7 Checkpoint B
Section C: Volume of Solid Figures				
5.MD.C.5 5.MD.C.5.c 5.OA.A.1 5.OA.A.2	I can find the volume of a figure composed of rectangular prisms.	X Selected Response X Constructed Response Performance X Observation	**Lesson Progression:** Students learn about the additive nature of volume. They use previous understandings of the additive nature of area to decompose three-dimensional figures composed of two right rectangular prisms and find the total volume. They solve word problems involving the volume of three dimensional figures composed of two right rectangular prisms with whole number edge lengths.	**Mandatory Lessons/Activities:** iM Lessons 8, 9, 10, 11

			This section also allows students to engage in multiplication strategies learned in 3rd and 4th grades, by operating with whole number edge lengths of greater magnitude than in previous sections. Students incorporate place value understanding to decompose tens and use strategies that build toward understanding the standard algorithm for multiplying multi-digit numbers.	
Pacing:	5 days		**Math Practices:** SMP 1, 2, 3, 4, 5, 6, 7, 8	**Assessments:** Cool-down 8, 9 Checkpoint C

ADDITIONAL CONSIDERATIONS			
COMMON MISCONCEPTIONS	**PRIOR KNOWLEDGE NEEDED TO MASTER STANDARDS FOR THIS UNIT**	**ADVANCED STANDARDS FOR STUDENTS WHO HAVE DEMONSTRATED PRIOR MASTERY**	**OPPORTUNITIES FOR STUDENT-DIRECTED LEARNING WITHIN THE UNIT**
Students may believe the problems should be solved left to right regardless of symbols such as parentheses or Order of Operations. When students hear the word volume, they often think of sound. Students will need real world examples of mathematical volume as well as hands-on experiences in order to fully grasp this concept. By stacking geometric solids with cubic units in layers, students can begin understanding the concept of how addition plays a part in finding volume. This will lead to an understanding of the formula for the volume of a right rectangular prism, b × h, where b is the area of	5.OA.A.2: 5.OA.A.1 5.MD.C.3: 3.MD.C.5 5.MD.C.4: 5.MD.C.3 5.MD.C.5: 4.MD.A.3, 5.MD.C.3. 5.MD.C.4	Choose from iM leveled centers and exploration problems to differentiate for students who are ready.	iM Centers District-approved online resources

the base. Students may think the term "base" only applies to the bottom layer of a prism rather than any two parallel faces. When solving the volume of composite shapes, students often struggle to properly visualize the shape as two separate rectangular prisms. They may also believe a prism can only be decomposed in one way, or have difficulty determining the lengths of the various sides of the composite shape.			

RESOURCES
Kendall Hunt Flourish Blackline masters and materials from Teacher Resource Pack Connecting/snap cubes, colored pencils, crayons, markers, paper

UNIT 2: FRACTIONS AS DIVISION AND FRACTION MULTIPLICATION

Illustrative Mathematics Unit Focus: Students solve problems involving division of whole numbers with answers that are fractions (which could be in the form of mixed numbers). They develop an understanding of fractions as the division of the numerator by the denominator, that is a ÷ b = a/b. They then solve problems that involve the multiplication of a whole number by a fraction or mixed number.

Essential Questions:
How is a fraction related to division?
How does whole number computation relate to fraction computation?
How can models help us understand the multiplication and division of fractions?

Unit Pacing: 25 days (16 required lessons, 7 flex, 2 assessment and reaction)

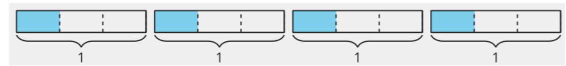
UNWRAPPED STANDARDS

Grade Level Standard	Standard Progression	Concepts (Big Ideas/ Understandings)	Academic Vocabulary (Standard Based)
5.OA.A.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them.	Students evaluate and interpret expressions, e.g., using their conceptual understanding of multiplication to interpret 3 x (18932 + 921) as being three times as large as 18932 + 921, without having to calculate the indicated sum or product.	Mathematical situations can be translated and represented abstractly using variables, expressions, and equations. You can make observations about the relative magnitude of the value of an expression without having to perform any calculations.	Expression Parentheses Variable Evaluate Interpret
5.NF.B.3 Interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound	In Grade 5, students connect fractions with division, understanding that 5 ÷ 3 = 5/3, or, more generally, a/b = a ÷ b for whole numbers a and b, with b not equal to zero.	We can interpret a fraction as a division problem, where the numerator is divided by the denominator. The denominator describes what number of equal parts a whole has been divided into. The numerator describes how many of the parts are considered. The numerator is a multiplier, e.g., 4/5 = 4 × 1/5 . A fraction represents division, so $a \div b$ = a/b , e.g., 3 ÷ 4 = 3/4 . The denominator	Fraction Mixed Number Numerator Denominator Equal parts Interpret Solve Represent Division Dividend Divisor Quotient

sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?		is the divisor. The numerator is the dividend. Equal shares means each sharer gets the same sized part and no parts are discarded. The solution to an equal sharing problem can be shown with a fraction representing the relationship of the sharers and the amount.	
5.NF.B.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. 5.NF.B.4.A Interpret the product (a/b) × q as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a × q ÷ b. For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = (ac)/(bd). 5.NF.B.4.B Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.	Students have, since Grade 1, been using language such as "third of" to describe one part when a whole is partitioned into three parts. With their new understanding of the connection between fractions and division, students now see that 5 ÷ 3 is one third of 5, which leads to the meaning of multiplication by a unit fraction: $\frac{1}{3} \times 5 = \frac{5}{3}$ This in turn extends to multiplication of any quantity by a fraction. Using this understanding of multiplication by a fraction, students develop the general formula for the product of two fractions, $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$ for whole numbers a, b, c, d, with b, d not zero. Students also calculate the area of a rectangular region with fractional side lengths, dividing it up into rectangles whose sides are the corresponding unit fractions.	The idea of the numerator as a multiplier can be used when a fraction is being multiplied by a whole number, e.g., Just as 5/8 = 5× 1/8 , 5 groups of 3/8 equals 5 × 3/8 = (5×3)× 1/8 which equals 15/8 . A variety of models, including arrays, number lines, fraction strips, etc. can be used to represent the multiplication of a whole number by a fraction. The properties of whole number computation can be applied to computation with fractions.	Fraction Numerator Denominator Product Partition Equal parts Equivalent Factor Unit fraction Area Side lengths

UNIT 2: FRACTIONS AS DIVISION AND FRACTION MULTIPLICATION
How is a fraction related to division? How does whole number computation relate to fraction computation?

How can models help us understand the multiplication and division of fractions?				
CCSS Standards #	**Learning Targets**	**Summative Assessment Strategy**	**Lesson Progression and Connection to Math Practices**	**Common Learning Experiences and Assessments**
Section A: Fractions as Quotients				
5.NF.B.3	I can interpret and represent a fraction as a division problem.	X Selected Response X Constructed Response Performance X Observation	**Lesson Progression:** Students learn that fractions are quotients and can be interpreted as division of the numerator by the denominator. Building on concepts of division and multiplication from grades 3 and 4, students draw and analyze tape diagrams that represent sharing situations. Through a sequence of first sharing 1, then sharing more than 1, then sharing a number of things with increasingly more people, students notice patterns and begin to understand that in general a/b = a ÷ b. For example, students use the diagram below to show 4 objects being shared by 3 people, or 4 ÷ 3 , which can also be written as a fraction, 4/3 . In this case, the fraction is greater than 1, and students may write this as a mixed number 1 ⅓ , though it is not required.  	**Mandatory Lessons/Activities:** iM Lessons 1, 2, 3, 4, 5
Pacing:	5 days		**Math Practices:** SMP 1, 2, 3, 4, 5, 6, 7, 8	**Assessments:** Cool-downs: 4, 5 Checkpoint A
Section B: Fractions of Whole Numbers				
5.NF.B.3 5.NF.B.4 5.NF.B.4.a 5.OA.A.2	I can represent a fraction x whole number with a model and solve to find the product.	X Selected Response X Constructed Response Performance X Observation	**Lesson Progression:** In grade 4, students interpret a fraction as a multiple of a unit fraction a/b = a x 1/b. In this section, as students develop their understanding that fractions can be interpreted in terms of division, they relate multiplication and division, and learn that a/b = a ÷ b = a x 1/b. This relationship allows students to use the same visual representation to express both division and	**Mandatory Lessons/Activities:** iM Lessons 6, 7, 8

			multiplication. This means that the same diagram we used to show 4 ÷ 3 can be used to represent 4 groups of 1/3, or 4 x 1/3.  Students conceptualize multiplication of a whole number by a fraction. In grade 4 students understand multiplication of a fraction by a whole number as equal groups of fractional pieces, and generalize that q x a/b = (q x a)/b. In this section, students learn that the product a/b x q means a fraction of a whole number, and they interpret this meaning through situations and diagrams. Students discover that the product a/b x q can be found by dividing q by b and then multiplying by a, or a/b x q = a x (q ÷ b) . Although the commutative property is not taught explicitly in this unit, equivalent expressions such as q x a/b = a/b x q become apparent to students as they analyze diagrams and interpret situations.	
Pacing:	3 days		**Math Practices:** SMP 1, 2, 3, 4, 5, 6, 7, 8	**Assessments:** Cool-down 6, 7 Checkpoint B
		Section C: Area and Fractional Side Lengths		
5.NF.B.3 5.NF.B.4 5.NF.B.4.a 5.NF.B.4.b	I can represent a fraction x whole number with a model and solve to find the product. I can find the area of a rectangle with fractional side lengths.	X Selected Response X Constructed Response Performance X Observation	**Lesson Progression:** In earlier grades, students calculate the area of a rectangle by counting unit squares or multiplying the side lengths. In this section, students learn that these concepts still apply when the rectangle has one whole number side length and one fractional side length. Students first analyze the shaded region of area diagrams made up of unit squares. They notice that the shaded region represents a fractional area, or a fraction of the whole. Students generalize that to find the area, they can multiply the side lengths of the shaded rectangular region. Students develop their understanding of area through a sequence of finding the area of a rectangle with one whole number side length, and the other a unit fraction side length, a non-unit	**Mandatory Lessons/Activities:** iM Lessons 9, 10, 11, 12, 13, 14, 15, 16

			fraction side length, a fractional side length greater than 1, and a mixed number side length. As they write numerical expressions that represent the strategies they use, and share these strategies with their classmates, they recognize that the same diagram can be interpreted in different ways. In some instances one area diagram can be interpreted as a fraction of a whole number, equal groups of a fraction, or division of two whole numbers. When students analyze diagrams where one side length is a mixed number, they decompose the shaded region to group the whole units and the fractional units. The area diagram helps students understand why the distributive property is a useful strategy to multiply a whole number and a mixed number.	
Pacing:	9 days		**Math Practices:** SMP 1, 2, 3, 4, 5, 6, 7, 8	**Assessments:** Cool-down 10, 13, 15 Checkpoint C

COMMON MISCONCEPTIONS	PRIOR KNOWLEDGE NEEDED TO MASTER STANDARDS FOR THIS UNIT	ADVANCED STANDARDS FOR STUDENTS WHO HAVE DEMONSTRATED PRIOR MASTERY	OPPORTUNITIES FOR STUDENT-DIRECTED LEARNING WITHIN THE UNIT
Students may believe that you can not divide a smaller number by a larger number. (i.e. 3 ÷ 4 = ¾). When creating a model to represent fraction multiplication and division situations, students do not correctly attend to the whole. When working with contextual problems, students may not attend to the meaning of the numerals in the problem in relation to the operation involved. Students may believe that multiplication always results in a	5.OA.A.2: 5.OA.A.1 5.NF.B.3: 4.MD.A.2, 4.OA.A.1, 4.OA.A.2 5.NF.B.4: 4.NF.B.4	Choose from iM leveled centers and exploration problems to differentiate for students who are ready.	iM Centers District-approved online resources

larger number (i.e. 2 x 8 = 16 vs. ½ x 8 = 4).			

RESOURCES
Kendall Hunt Flourish Blackline masters and materials from Teacher Resource Pack Grid paper, glue, paper, rulers, scissors

UNIT 3: FRACTION MULTIPLICATION AND DIVISION

Illustrative Mathematics Unit Focus: Students use area concepts to represent and solve problems involving the multiplication of two fractions, and generalize that $\frac{a}{b} \times \frac{c}{d} = \frac{(a \times c)}{(b \times d)}$. They also reason about the relationship between multiplication and division to divide a whole number by a unit fraction and a unit fraction by a whole number.

Essential Questions:
How is a fraction related to division?
How can models help us understand the multiplication and division of fractions?
How does whole number computation relate to fraction computation?

Unit Pacing: 28 days (19 required lessons, 7 flex, 2 assessment and reaction)

<table>
<tr><th colspan="4">UNWRAPPED STANDARDS</th></tr>
<tr><th>Grade Level Standard</th><th>Standard Progression</th><th>Concepts
(Big Ideas/ Understandings)</th><th>Academic Vocabulary
(Standard Based)</th></tr>
<tr><td>5.NF.B.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.</td><td rowspan="3">Students have, since Grade 1, been using language such as “third of” to describe one part when a whole is partitioned into three parts. With their new understanding of the connection between fractions and division, students now see that 5 ÷ 3 is “one third of 5”, which leads to the meaning of multiplication by a unit fraction:
$$\frac{1}{3} \times 5 = \frac{5}{3}$$
This in turn extends to multiplication of any quantity by a fraction.

Using this understanding of multiplication by a fraction, students develop the general formula for the product of two fractions,
$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$$
for whole numbers a, b, c, d, with b, d not zero.

Students also calculate the area of a rectangular region with fractional side lengths, dividing it up into rectangles whose sides are the corresponding unit fractions.</td><td rowspan="3">The idea of the numerator as a multiplier can be used when a fraction is being multiplied by a whole number, e.g., Just as 5/8 = 5× 1/8 , 5 groups of 3/8 equals 5 × 3/8 = (5×3)× 1/8 which equals 15/8.

A variety of models, including arrays, number lines, fraction strips, etc. can be used to represent the multiplication of a whole number by a fraction.

The properties of whole number computation can be applied to computation with fractions.</td><td rowspan="3">Fraction
Numerator
Denominator
Product
Partition
Equal parts
Equivalent
Factor
Unit fraction
Area
Side lengths</td></tr>
<tr><td>5.NF.B.4.A Interpret the product (a/b) × q as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a × q ÷ b. For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = (ac)/(bd).</td></tr>
<tr><td>5.NF.B.4.B Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.</td></tr>
</table>

5.NF.B.6 Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.	Students attend carefully to the underlying unit quantities when solving problems. For example, if ½ of a fund-raiser's funds were raised by the 6th grade, and if ⅓ of the 6th grade's funds were raised by Ms. Wilkin's class, then ⅓ x ½ gives the fraction of the fund-raiser's funds that Ms. Wilkin's class raised, but it does not tell us how much money Ms. Wilkin's class raised.	Solving word problems with addition, subtraction, multiplication and division of fractions follow the same problem solving structures as for whole number situations.	Fraction Numerator Denominator Product Quotient Partition Equal parts Equivalent Factor Unit fraction Fraction model
5.NF.B.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.	Using the relationship between division and multiplication, students start working with simple fraction division problems. Having seen that division of a whole number by a whole number, e.g. 5 ÷ 3, is the same as multiplying the number by a unit fraction, ⅓ x 5, they now extend the same reasoning to division of a unit fraction by a whole number, seeing for example that $$\frac{1}{6} \div 3 = \frac{1}{6 \times 3} = \frac{1}{18}$$ Also, they reason that since there are 6 portions of ⅙ in 1, there must be 3 x 6 in 3, and so $$3 \div \frac{1}{6} = 3 \times 6 = 18$$ Students use story problems to make sense of division problems: How much chocolate will each person get if 3 people share ½ lb of chocolate equally? How many ⅓ cup servings are in 2 cups of raisins?	A variety of models, including arrays, number lines, fraction strips, etc. can be used to represent the division of a whole number by a fraction. The relationship between multiplication and division can help us reason about fraction division. Contextual situations are also imperative to help students reason about the computation. The properties of whole number computation can be applied to computation with fractions.	Fraction Numerator Denominator Quotient Product Partition Equal parts Equivalent Factor Unit fraction
5.NF.B.7.a Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.			
5.NF.B.7.b Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.			
5.NF.B.7.c Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3			

people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?			

UNIT 3: FRACTION MULTIPLICATION AND DIVISION

How is a fraction related to division?
How can models help us understand the multiplication and division of fractions?
How does whole number computation relate to fraction computation?

CCSS Standards #	Learning Targets	Summative Assessment Strategy	Lesson Progression and Connection to Math Practices	Common Learning Experiences and Assessments
Section A: Fraction Multiplication				
5.NF.B.4 5.NF.B.4.a 5.NF.B.4.b 5.NF.B.6	I can represent a fraction multiplication problem using models and equations and solve to find the product. I can find the area of a rectangle with fractional side lengths.	X Selected Response X Constructed Response Performance X Observation	**Lesson Progression:** Students work with fraction multiplication using area concepts. They connect taking a fraction of a fraction to multiplication of a fraction times a fraction. Students begin by interpreting situations and drawing diagrams to represent the fractional area. They recall that the number of equal parts of the whole defines the denominator of the fraction and use this information when representing a situation such as, “Kiran eats macaroni and cheese from a pan that is ⅓ full. He eats ¼ of the remaining macaroni and cheese in the pan. How much of the whole pan did Kiran eat?” Students then connect the area of a rectangle with fractional side lengths to the multiplication of two fractions. In unit fraction by unit fraction multiplication, students see the denominator as the number of small rectangles in the unit square, structured as an array. They begin to generalize that when multiplying a unit fraction by a unit fraction, they can multiply the denominators together to find the product. Students extend this conceptual understanding to multiply a unit fraction times a non-unit fraction (including	**Mandatory Lessons/Activities:** iM Lessons 1, 2, 3, 4, 5, 6, 7, 8

			fractions greater than 1), and a non-unit fraction times a non-unit fraction. In each case, they relate this multiplication to area and see the numerators as an array of small rectangles that make up the shaded region, and see the denominators as an array of small rectangles within a unit square. After noticing this pattern, they generalize a "rule" that works for all fraction multiplication including fractional side lengths greater than 1. $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$	
Pacing:	8 days		**Math Practices:** SMP 1, 2, 3, 4, 5, 6, 7, 8	**Assessments:** Cool-downs: 3, 4, 6 Checkpoint A
Section B: Fraction Division				
5.NBT.B.7 5.NF.B.7 5.NF.B.7.a 5.NF.B.7.b 5.NF.B.7.c	I can represent a fraction division problem using models and equations and solve to find the quotient.	X Selected Response X Constructed Response Performance X Observation	**Lesson Progression:** Students use their understanding of multiplication of a unit fraction by a whole number to divide by unit fractions. Students learn about fraction division through situations. They revisit whole number division to conceptualize and describe what happens to the value of the quotient when the dividend or divisor changes. As students draw tape diagrams and write expressions to represent situations involving the division of unit fractions, students recognize the inverse relationship between multiplication and division. They use this relationship to reason that ⅕ ÷ 2 = 1/10 because 2 groups of tenths can fit inside of each fifth, or 2 x 1/10 = ⅕ . Similarly, they reason that 2 ÷ ⅓ = 6 because 6 groups of thirds can fit inside of 2, or 6 x ⅓ = 2 . As students make sense of division situations involving unit fractions, they represent them with diagrams and equations. They then write and solve their own problems involving fraction division.	**Mandatory Lessons/Activities:** iM lessons 9, 10, 11, 12, 13, 14, 15, 16
Pacing:	8 days		**Math Practices:** SMP 1, 2, 3, 4, 5, 6, 7, 8	**Assessments:** Cool-down 10, 12, 14

				Checkpoint B
Section C: Problem Solving with Fractions				
5.NF.B.4 5.NF.B.6 5.NF.B.7	I can solve problems involving fraction multiplication and division.	X Selected Response X Constructed Response Performance X Observation	**Lesson Progression:** Students apply what they have learned in the previous sections through problem solving. The situations students engage with allow them to see how fraction multiplication and division are relevant in everyday life. Students collaborate with one another to create and solve problems involving fraction multiplication and division. They use their understanding of the meaning of multiplication and division to strategize about when to use a particular operation. The inverse relationship becomes apparent when students realize that they can think about multiplication to solve a division problem, and vice versa.	**Mandatory Lessons/Activities:** iM lessons 17, 18, 19
Pacing:	3 days		**Math Practices:** SMP 1, 2, 3, 4, 5, 6, 7, 8	**Assessments:** Cool-down 19 Checkpoint C

COMMON MISCONCEPTIONS	PRIOR KNOWLEDGE NEEDED TO MASTER STANDARDS FOR THIS UNIT	ADVANCED STANDARDS FOR STUDENTS WHO HAVE DEMONSTRATED PRIOR MASTERY	OPPORTUNITIES FOR STUDENT-DIRECTED LEARNING WITHIN THE UNIT
Students may believe that multiplication always results in a larger number (i.e. 2 x 8 = 16 vs. ½ x 8 = 4). Students may believe that you can not divide a smaller number by a larger number. (i.e. 3 ÷ 4 = ¾). When creating a model to represent fraction multiplication and division situations, students do not correctly attend to the whole. When working with contextual problems,	5.NBT.B.7: 4.NBT.B.4, 5.NBT.A.1, 5.NF.A.1, 5.NF.B.4, 5.NF.B.7 5.NF.B.4: 4.NF.B.4 5.NF.B.6 : 4.MD.A.2, 4.OA.A.1, 4.OA.A.2 5.NF.B.7: 4.NF.B.4	Choose from iM leveled centers and exploration problems to differentiate for students who are ready.	iM Centers District-approved online resources

students may not attend to the meaning of the numerals in the problem in relation to the operation involved.			
RESOURCES			
Kendall Hunt Flourish Blackline masters and materials from Teacher Resource Pack			

UNIT 4: WHOLE NUMBER MULTIPLICATION AND DIVISION

Illustrative Mathematics Unit Focus: Students multiply and divide multi-digit whole numbers using place value understanding, properties of operations, and the relationship between multiplication and division. They use the standard algorithm to multiply multi-digit whole numbers and partial quotients algorithms to divide whole numbers up to four digits by two digits. They then apply these skills as they solve problems involving volume.

Essential Questions:
How can understanding place value help us?
How do the properties of operations make computation simpler?
How are multiplication and division related?
How can we represent mathematical situations?

Unit Pacing: 28 days (20 required lessons, 6 flex, 2 assessment and reaction)

UNWRAPPED STANDARDS

Grade Level Standard	Standard Progression	Concepts (Big Ideas/ Understandings)	Academic Vocabulary (Standard Based)
5.OA.A.1 Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.	Students write expressions to express a calculation, e.g. writing 2 x (8 + 7) to express the calculation "add 8 and 7, then multiply by 2."	Parentheses and brackets can be used in writing expressions to group numbers and operations together. There are multiple ways to write equivalent expressions to represent a given situation. There is a specific convention used to interpret and evaluate expressions. The information within parentheses and brackets are evaluated first.	Expression Parentheses Brackets Braces Evaluate
5.OA.A.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them.	Students evaluate and interpret expressions, e.g., using their conceptual understanding of multiplication to interpret 3 x (18932 + 921) as being three times as large as 18932 + 921, without having to calculate the indicated sum or product.	Mathematical situations can be translated and represented abstractly using variables, expressions, and equations. You can make observations about the relative magnitude of the value of an expression without having to perform any calculations.	Expression Parentheses Variable Evaluate Interpret

5.NBT.B.5 Fluently multiply multi-digit whole numbers using the standard algorithm.	Students extend their whole number work with multiplication to multi-digit computation using the standard algorithm. Underlying this algorithm are the properties of operations and the base-ten system.	Understanding place value enables us to represent, compare order and round numbers and perform computations. Properties of operations allow us to reorder, decompose and/or compose numbers in order to make computation simpler. There are different algorithms that can be used to multiply. Real-world mathematical situations can be represented using concrete models or drawings. Patterns and structures can be generalized when multiplying and dividing whole numbers.	Distributive property Factors Product Rectangular arrays Partial Products
5.NBT.B.6 Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	Division in Grade 5 extends Grade 4 methods to two-digit divisors using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Students continue to decompose the dividend into base-ten units and find the quotient place by place, starting from the highest place. They illustrate and explain their calculations using equations, rectangular arrays, and/or area models. Estimating the quotients is a new aspect of dividing by a two-digit number. Even if students round the dividend appropriately, the resulting estimate may need to be adjusted up or down. Sometimes multiplying the ones of a two-digit divisor composes a new thousand, hundred, or ten. These newly composed units can be written as part of the division computation, added mentally, or as part of a separate multiplication computation	Understanding place value enables us to represent, compare order and round numbers and perform computations. Properties of operations allow us to reorder, decompose and/or compose numbers in order to make computation simpler. There is a relationship between multiplication and division. Equations, rectangular arrays, and/or area models can be used to illustrate and explain division. Real-world mathematical situations can be represented using concrete models or drawings. Patterns and structures can be generalized when multiplying and dividing whole numbers.	Quotient Dividend Divisor Calculate Partial Quotients

5.MD.C.5.B Apply the formulas V = l × w × h and V = b × h for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.	Students understand that multiplying the length times the width of a right rectangular prism can be viewed as determining how many cubes would be in each layer if the prism were packed with or built up from unit cubes. They also learn that the height of the prism tells how many layers would fit in the prism.	The area of a base of a rectangular prism is found by multiplying the length by width (b = ℓ × w). In a right rectangular prism, any two parallel faces can be the bases. The volume of a rectangular prism can be found by multiplying the length by width by height (ℓ × w × h) or by multiplying the area of the base by height (b x h).	Volume Cubic units Multiplication Addition Rectangular prism Length Width Height Base area Formula Associative Property Distributive Property Commutative Property Decompose

UNIT 4: WHOLE NUMBER MULTIPLICATION AND DIVISION

How can understanding place value help us?
How do the properties of operations make computation simpler?
How are multiplication and division related?
How can we represent mathematical situations?

CCSS Standards #	Learning Targets	Summative Assessment Strategy	Lesson Progression and Connection to Math Practices	Common Learning Experiences and Assessments
Section A: Multi-Digit Multiplication and the Standard Algorithm				
5.MD.C.5 5.NBT.B.5 5.NF.B.5 5.OA.A.2	I can solve multi-digit multiplication problems using the standard algorithm.	X Selected Response X Constructed Response Performance X Observation	**Lesson Progression:** Using the familiar context of volume, students begin by estimating products and quotients. Estimation allows students to naturally work with powers of 10, employ the relationship between multiplication and division, and think about the reasonableness of answers. It also helps students to recall the strategies they have learned in previous grades to multiply and divide whole numbers, which supports the goals of this unit. Students develop fluency with the standard algorithm for multiplication by connecting what	**Mandatory Lessons/Activities:** iM Lessons 1, 2, 3, 4, 5, 6, 7, 8, 9

			they learned about multiplying multi-digit whole numbers in previous grades. Prior to this point, students multiplied single-digit by up to four-digit numbers, and 2 two-digit numbers using partial products strategies that are based on place value understanding. Students have used a variety of representations, including area diagrams and equations to support their thinking. Students multiply three-digit by two-digit numbers in a progression that allows them to use their understanding of partial products diagrams and equations to develop an understanding of the standard multiplication algorithm as a strategy for multiplying large numbers. Initially, students analyze partial products strategies and begin to think about how partial products can be organized in a systematic way. When students first learn to record partial products using the standard algorithm, the factors involved do not require composition of new units, or carrying. As the lessons progress, students learn to record the composition of one new unit, and then multiple new units using the standard algorithm.	
Pacing:	9 days		**Math Practices:** SMP 3, 5, 6, 7, 8	**Assessments:** Cool-downs: 4, 7 Checkpoint A
Section B: Multi-digit Division using Partial Quotients				
5.NBT.B.5 5.NBT.B.6 5.NF.B.3 5.OA.A.1 5.OA.A.2	I can represent and solve multi-digit division problems.	X Selected Response X Constructed Response Performance X Observation	**Lesson Progression:** Students begin with an exploration that allows them to connect a real-world context to division with large numbers. Students use strategies based on place value and the relationship between multiplication and division to estimate how the world's longest noodle could be shared. In previous grades, students found whole-number quotients and remainders with up to four-digit dividends and one-digit divisors using partial quotients strategies. In grade 5, they find whole-number quotients with up to four-digit dividends and two-digit divisors	**Mandatory Lessons/Activities:** iM Lessons 10, 11, 12, 13, 14, 15, 16, 17

			using partial quotients strategies. In a previous unit students learn that fractions are quotients, in that they are equivalent to dividing a numerator by the denominator. In grade 4, they learned to decompose fractions into sums of smaller fractions with like denominators. At the start of this section students find whole-number quotients using fractions, before they engage in a partial quotients algorithm. For example, $364 \div 13 = \frac{130}{13} + \frac{130}{13} + \frac{65}{13} + \frac{39}{13}$ As students decompose the dividend, they notice that although there are many ways to decompose a number, there are some ways that are more helpful for finding whole-number quotients. This reasoning naturally involves students' understanding of the relationship between multiplication and division, which is key to learning more complex partial quotients algorithms.	
Pacing:	8 days		**Math Practices:** SMP 3, 5, 6, 7, 8	**Assessments:** Cool-down 11, 13, 15 Checkpoint B
Section C: Let's Put it to Work: Multiplication, Division, and Volume				
5.MD.C.5 5.NBT.B.5 5.NBT.B.6 5.NF.B.7	I can apply a formula to find the volume of a rectangular prism.	X Selected Response X Constructed Response Performance X Observation	**Lesson Progression:** Students practice their multiplication and division skills as they solve problems involving volume. Students have learned that volume can be measured as the number of cubic units of a solid figure. They found the volume of rectangular prisms with relatively small dimensions by applying the formulas V = l x w x h and V = b x h. The work in this section offers a review of topics from previous units, and it supports the multiplication and division work of this unit. For example, they use estimation and their understanding of the volume of a rectangular prism to determine how many children can fit inside of a giant wagon. Students engage with relatively large numbers to multiply and divide using these volume formulas,	**Mandatory Lessons/Activities:** iM Lessons 18, 19, 20

			developing fluency with the standard algorithm for multiplication and the partial quotients algorithms for division.	
Pacing:	4 days		**Math Practices:** SMP 1, 2, 3, 4, 5, 6, 7, 8	**Assessments:** Cool-down 18 Checkpoint C

COMMON MISCONCEPTIONS	PRIOR KNOWLEDGE NEEDED TO MASTER STANDARDS FOR THIS UNIT	ADVANCED STANDARDS FOR STUDENTS WHO HAVE DEMONSTRATED PRIOR MASTERY	OPPORTUNITIES FOR STUDENT-DIRECTED LEARNING WITHIN THE UNIT
When students don't attend to place value, they may not understand the magnitude of the numbers they are multiplying or dividing and therefore make computational errors. When working with contextual problems, students may not attend to the meaning of the numerals in the problem in relation to the operation involved.	5.NBT.B.5: 4.NBT.B.4, 4.NBT.B.5, 5.NBT.A.1 5.NBT.B.6: 4.NBT.B.4, 4.NBT.B.6, 5.NBT.A.1, 5.NBT.B.5 5.NF.B.3 : 4.MD.A.2, 4.OA.A.1, 4.OA.A.2 5.NF.B.5: 4.MD.A.2, 4.OA.A.1, 4.OA.A.2, 4.NF.A.1, 5.NF.B.4 5.NF.B.7 : 4.NF.B.4 5.OA.A.2: 5.OA.A.1 5.MD.C.5: 4.MD.A.3, 5.MD.C.3, 5.MD.C.4	Choose from iM leveled centers and exploration problems to differentiate for students who are ready.	iM Centers District-approved online resources

RESOURCES
Kendall Hunt Flourish Blackline masters and materials from Teacher Resource Pack Rulers, yardsticks, index cards

UNIT 5: PLACE VALUE PATTERNS AND DECIMAL OPERATIONS

Illustrative Mathematics Unit Focus: Students build from place value understanding in grade 4 to recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and one tenth of what it represents in the place to its left. They use this place value understanding to round, compare, order, add, subtract, multiply, and divide decimals.

Essential Questions:
How is our number system organized?
How can understanding place value help us?
How do the properties of operations make computation simpler?

Unit Pacing: 35 days (23 required lessons, 10 flex, 2 assessment and reaction)

UNWRAPPED STANDARDS

Grade Level Standard	Standard Progression	Concepts (Big Ideas/ Understandings)	Academic Vocabulary (Standard Based)
5.OA.A.1 Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.	Students write expressions to express a calculation, e.g. writing 2 x (8 + 7) to express the calculation “add 8 and 7, then multiply by 2.”	Parentheses and brackets can be used in writing expressions to group numbers and operations together. There are multiple ways to write equivalent expressions to represent a given situation. There is a specific convention used to interpret and evaluate expressions. The information within parentheses and brackets are evaluated first.	Expression Parentheses Brackets Braces Evaluate
5.NBT.A.1 Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.	Students understand why multiplying by a power of 10 shifts the digits of a whole number or decimal that many places to the left. Multiplying by 10 once shifts every digit of the multiplicand one place to the left in the product (the product is ten times as large) because in the base-ten system the value of each place is 10 times the value of the place to its right.	Our number system is a base-ten system. A given place value is ten times greater than the value of the place to its right (500 is ten times greater than 50) and 1/10 the value of the place to its left (0.3 is 1/10 the value of 3).	Place value Period Decimal Decimal point Tenths Hundredths Thousandths Place value chart

5.NBT.A.3 Read, write, and compare decimals to thousandths. 5.NBT.A.3.a Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000). 5.NBT.A.3.b Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.	Students extend their understanding of the base-ten system to decimals to the thousandths place, building on their Grade 4 work with tenths and hundredths. They examine the relationship between adjacent places and how numbers compare through thousandths.	Understanding place value enables us to represent, compare order and round numbers and perform computations. When reading a decimal number, the decimal point is read as and. In a decimal number, digits to the right of the decimal point are named by the appropriate unit: tenths, hundredths, thousandths. and are read followed by the name of the appropriate unit, i.e. 1.438 is read as one and four hundred thirty eight thousandths. Decimals to thousandths can be expressed in standard form, word form, and expanded form. Two decimals to thousandths can be compared using the symbols >, =, and <.	Greater than > Less than < Equal to = Comparison Expanded form Tenths Hundredths Thousandths Decimal
5.NBT.A.4 Use place value understanding to round decimals to any place.	Students extend their understanding of the base-ten system to decimals to the thousandths place, building on their Grade 4 work with tenths and hundredths. They examine how numbers round for decimals to thousandths.	Understanding place value enables us to represent, compare, order and round numbers and perform computations.	Tenths Hundredths Thousandths Decimal Round Place value
5.NBT.B.7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.	Because of the uniformity of the structure of the base-ten system, students use the same place value understanding for adding and subtracting decimals that they used for adding and subtracting whole numbers. Like base-ten units must be added and subtracted, so students need to attend to aligning the corresponding places correctly (this also aligns the decimal points). General methods used for computing products of whole numbers extend to products of decimals. Because the expectations for decimals are limited to thousandths and expectations for factors are limited to hundredths at this grade level, students	Understanding place value enables us to represent, compare and round numbers and perform computations. These patterns exist for both whole numbers and decimals. Properties of operations allow us to reorder, decompose and/or compose numbers in order to make computation simpler. The same properties of operations for whole numbers work with decimal numbers.	Associative Property Commutative Property Identity Property Sum Difference Product Quotient Decimal Tenths Hundredths Thousandths Patterns

	will multiply tenths with tenths and tenths with hundredths, but they need not multiply hundredths with hundredths. General methods used for computing quotients of whole numbers extend to decimals with the additional issue of placing the decimal point in the quotient.		

UNIT 5: PLACE VALUE PATTERNS AND DECIMAL OPERATIONS				
How is our number system organized? How can understanding place value help us? How do the properties of operations make computation simpler?				
CCSS Standards #	**Learning Targets**	**Summative Assessment Strategy**	**Lesson Progression and Connection to Math Practices**	**Common Learning Experiences and Assessments**
Section A: Numbers to Thousandths				
5.NBT.A.1 5.NBT.A.3 5.NBT.A.3.a 5.NBT.A.3.b 5.NBT.A.4 5.OA.A	I can read, write, and represent decimals to the thousandths place. I can compare two decimals using the symbols <, > or =. I can round decimals to any place.	X Selected Response X Constructed Response Performance X Observation	**Lesson Progression:** Students clarify and extend their understanding of the value of the digits of multi-digit numbers as they are introduced to the thousandths place. They begin by representing decimals on gridded area diagrams, building on their work with tenths and hundredths in grade 4, where the large square has a value of 1, and each small square within represents 1/100. Students learn that if they partition each small square into tenths, each of those parts represents a thousandth of the large square. Beyond understanding the size of thousandths, students learn about the value of the digits of multidigit numbers to the thousandths. Through a context of measurement, students reason about the “10 times” and “ 1/10 of” relationship between	**Mandatory Lessons/Activities:** iM Lessons 1, 2, 3, 4, 5, 6, 7, 8

			tenths, hundredths and thousandths. For example, students analyze equivalent weights and recognize that 1 hundredth-ounce weight is equivalent to 10 thousandth-ounce weights. They then use weights to represent the weight of gold in ounces, recognizing that gold that weighs 0.124 ounces can balance a scale using 1 tenth-ounce weight, 2 hundredth-ounce weights, and 4 thousandth-ounce weights. Students use this reasoning to write decimals in expanded form using sums of multiplication expressions. For example, 0.124 in expanded form can be written as $\left(1 \times \frac{1}{10}\right) + \left(2 \times \frac{1}{100}\right) + \left(4 \times \frac{1}{1000}\right)$ because students can count the number of each type of weight. As students become flexible with representing fractions using decimal notation, they can use decimals to represent numbers in expanded form, writing 0.124 as (1 x 0.1) + 2 x 0.01) + (4 x 0.001). Although students will not formally learn decimal multiplication until the third section of this unit, they can reason about the decimal multiplication expressions by thinking of the number of parts in each place value within a given decimal number. Students use their developing understanding of place value to the thousandths to locate decimals on a number line. They then use the number line to round, compare, and order decimals. They recognize that they can use the same strategies used in previous years to round, compare, and order whole numbers.	
Pacing:	8 days		**Math Practices:** SMP 1, 2, 3, 4, 5, 6, 7, 8	**Assessments:** Cool-downs: 2, 4, 5, 8 Checkpoint A
Section B: Add and Subtract Decimals				
5.NBT.B.7	I can add and subtract decimals to the hundredths using a variety of strategies.	X Selected Response	**Lesson Progression:** Students use place value understanding and their knowledge of whole-number addition and	**Mandatory Lessons/Activities:** iM Lessons 9, 10, 11, 12, 13, 14

		X Constructed Response Performance X Observation	subtraction to add and subtract decimals to the hundredths. Initially, students add and subtract in ways that make sense to them. This allows students to relate adding and subtracting decimals to operating with whole numbers. It also allows teachers to take note of the strategies they are using, including expanded form, and the standard algorithm. Students then use place value reasoning to estimate the value of sums and differences before conducting an error analysis to address the challenge of lining up the place values when using the standard algorithm to add or subtract decimals.	
Pacing:	6 days		**Math Practices:** SMP 1, 2, 3, 4, 5, 6, 7, 8	**Assessments:** Cool-down 10, 11, 13 Checkpoint B
Section C: Multiply Decimals				
5.NBT.A.1 5.NBT.B.7 5.OA.A 5.OA.A.1 5.OA.A.2	I can multiply decimals using a variety of strategies.	X Selected Response X Constructed Response Performance X Observation	**Lesson Progression:** Students multiply decimals with products up to the hundredths. Students initially multiply decimals in ways that make sense to them. The area diagram is used to help students understand decimal multiplication, as it is a familiar representation from earlier units. With these diagrams, students can relate whole-number multiplication to decimal multiplication. For example, to solve 2 x 0.06, students can think of 2 groups of 6 hundredths, or 12 hundredths, which can be written as the equation 2 x 0.06 = 0.12.Students may also see this as 2 times 6 groups of 1 hundredth or 2 x 6 x 0.01 = 12 x 0.01 = 0.12. As students represent these diagrams with multiplication equations, they begin to see how they can use the Associative Property of Multiplication to multiply decimals. They use this strategy early in this section when they multiply a whole number times some tenths or some hundredths. For example, students recognize that	**Mandatory Lessons/Activities:** iM Lessons 15, 16, 17, 18, 19

			4 x 0.7 is equivalent to 4 x 7 tenths or (4 x 7) x 0.1 . They then generalize that they can use the unit form of the decimal to operate in terms of whole numbers, multiply the whole numbers together, and then multiply the product of the whole numbers by 1 tenth or 1 hundredth. To multiply tenths by tenths, students revisit area concepts from previous units. Using area diagrams, they find the area of the shaded region by multiplying side lengths. Instead of fractions, they use decimal notation to mark the side lengths. The diagrams provide students access to multiple solution pathways. As the section progresses, students find the products of larger numbers, such as 285 x 0.17. Although students can use any strategy that makes sense to them to find the product of numbers like these, they realize that drawing a diagram would not be helpful, and instead rely on the Associative Property of Multiplication. In this case students can use what they learned about the standard algorithm for multiplication to first multiply 285 x 17 and then multiply that result by 0.01. Students may also notice patterns in the placement of the decimal point while engaging in the work of this section. That is, students may notice that when a whole number is multiplied by a tenth or hundredth, the product has one or two digits after the decimal point, respectively. You may encourage this discourse, but these patterns will be analyzed and discussed in detail in the next unit.	
Pacing:	5 days		**Math Practices:** SMP 1, 2, 3, 4, 5, 6, 7, 8	**Assessments:** Cool-down 15, 18 Checkpoint C
Section D: Divide Decimals				
5.NBT.A.3 5.NBT.B.7	I can divide decimals using a variety of strategies.	X Selected Response	**Lesson Progression:** Just as with whole numbers and fractions, students use the relationship between multiplication and	**Mandatory Lessons/Activities:** iM Lessons 20, 21, 22, 23

		<table><tr><td>X</td><td>Constructed Response</td></tr><tr><td></td><td>Performance</td></tr><tr><td>X</td><td>Observation</td></tr></table>	division to make sense of division with decimals in this section. Students begin this section by considering how many tenths or hundredths are in whole numbers. This understanding provides a foundation for students to divide a whole number by any amount of tenths or hundredths. For example, when students divide 4 ÷ 0.2, it helps to understand that there are 10 tenths in 1 whole, which is 5 groups of 2 tenths. If students think about 4 wholes, then there must be 4 times as many groups of 2 tenths, or 20 groups of 2 tenths in 4 wholes. Students can represent this work using an area diagram if needed. Students also use expression equivalence to divide decimals using whole-number division. For example, students reason that 6 ÷ 0.4 is equivalent to 60 ÷ 4 because both the dividend and divisor are multiplied by 10. They then divide the whole numbers to find the quotient, 60 ÷ 4 = 15.	
Pacing:	5 days		**Math Practices:** SMP 1, 2, 3, 4, 5, 6, 7, 8	**Assessments:** Cool-down 21, 22 Checkpoint D

COMMON MISCONCEPTIONS	PRIOR KNOWLEDGE NEEDED TO MASTER STANDARDS FOR THIS UNIT	ADVANCED STANDARDS FOR STUDENTS WHO HAVE DEMONSTRATED PRIOR MASTERY	OPPORTUNITIES FOR STUDENT-DIRECTED LEARNING WITHIN THE UNIT
When trying to extend their understanding of whole number place value to decimal place value, students may believe that as you move to the right of the decimal point, the number increases in value, i.e. 6 hundredths is larger than 6 tenths. Students may also try to apply whole number concepts when comparing decimals by looking only at the number of digits. However, with decimals a	5.OA.A.2: 5.OA.A.1 5.NBT.A.1: 4.NBT.A.1, 4.NF.C.5, 4.NF.C.6, 4.NF.C.7 5.NBT.A.3: 4.NBT.A.2, 4.NF.C.7, 5.NBT.A.1 5.NBT.A.4 : 4.NBT.A.3, 5.NBT.A.1, 5.NBT.A.3	Choose from iM leveled centers and exploration problems to differentiate for students who are ready.	iM Centers District-approved online resources

number with one decimal place may be greater than a number with two or three decimal places. For example, 0.5 is greater than 0.12, 0.009 or 0.499. Students may not attend to place value when adding or subtracting decimals by ignoring the idea that they need to add like place values as with whole number addition and subtraction. For example, students might line up decimals to add or subtract from left to right or right to left not attending to the place value of the digits. Students may believe that multiplication always results in a larger product. Students may just perform decimal computation without assessing the reasonableness of their answer. It is essential that students think about the relative magnitude of the numbers in both the problem as well as their answer.			

RESOURCES
Kendall Hunt Flourish Blackline masters and materials from Teacher Resource Pack Chart paper, markers, number cubes,

UNIT 6: MEASUREMENT CONVERSIONS AND FRACTION OPERATIONS

Illustrative Mathematics Unit Focus: Students use their understanding of place value to the thousandths learned in the previous unit and measurement conversions learned in grade 4 to convert standard measurement units within a given system of measurement. They also build on grade 4 ideas of fraction equivalence and operations to add and subtract fractions with unlike denominators, and interpret multiplication as scaling

Essential Questions:
How is the size of a factor related to the size of the product when multiplying a given fraction?
How can equivalent fractions be used to add and subtract fractions with unlike denominators?
What is the relationship between units of measure in each system?
Why do we collect, organize, represent and analyze data?

Unit Pacing: 30 days (21 required lessons, 7 flex, 2 assessment and reaction)

UNWRAPPED STANDARDS

Grade Level Standard	Standard Progression	Concepts (Big Ideas/ Understandings)	Academic Vocabulary (Standard Based)
5.NBT.A.1 Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.	Students understand why multiplying by a power of 10 shifts the digits of a whole number or decimal that many places to the left. Multiplying by 10 once shifts every digit of the multiplicand one place to the left in the product (the product is ten times as large) because in the base-ten system the value of each place is 10 times the value of the place to its right.	Our number system is a base-ten system. A given place value is ten times greater than the value of the place to its right (500 is ten times greater than 50) and 1/10 the value of the place to its left (0.3 is 1/10 the value of 3).	Place value Period Decimal Decimal point Tenths Hundredths Thousandths Place value chart
5.NBT.A.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.	New at Grade 5 is the use of whole number exponents to denote powers of 10. Students understand why multiplying by a power of 10 shifts the digits of a whole number or decimal that many places to the left. For example, multiplying by 104 is multiplying by 10 four times. Multiplying by 10 once shifts every digit of the multiplicand one place to the left in the product (the product is ten times as large) because in the base-ten system the value of each place is 10 times the value of the place to its right. So multiplying by 10 four	In the base-ten system, the value of each place is 10 times the value of the place to the immediate right and 1/10 of the value to its immediate left	Exponent Base Power Squared Cubed

	times shifts every digit 4 places to the left. Patterns in the number of 0s in products of a whole number and a power of 10 can be explained in terms of place value.		
5.NF.A.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)	In Grade 4, students have some experience calculating sums of fractions with different denominators (tenths/hundredths) where one denominator is a divisor of the other, so that only one fraction has to be changed. Grade 5 students extend this reasoning to situations where it is necessary to re-express both fractions in terms of a new denominator.	Equivalent fractions can be used to replace given fractions to make calculations simpler. We decompose fractions into sums or products of fractions to make computation easier or to simplify expressions. Fractions can be added and subtracted when the wholes are the same size and the fractional parts (denominators) are the same. Fractions with different denominators can be added and subtracted by replacing each fraction with an equivalent fraction expressed with a like denominator. A fraction with a numerator larger than the denominator can be expressed as a mixed number or a fraction greater than one; both are correct representations. Expressing a mixed number as a fraction, e.g., 2 3/5 = 13/5 , may be useful when solving a fraction problem.	Equivalent fraction Sum Difference Numerator Denominator Unlike denominator Like denominator Mixed numbers
5.NF.A.2 Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness	Students make sense of fractional quantities when solving word problems, estimating answers mentally to see if they make sense. For example in the problem: Ludmilla and Lazarus each have a lemon. They need a cup of lemon juice to make hummus for a party. Ludmilla squeezes 1/2 a cup from hers and Lazarus squeezes 2/5 of a cup from his. How much lemon juice do they have? Is it	An equation can be used to describe a mathematical situation involving fractions. There is usually more than one way to describe and solve a mathematical situation involving fractions.	Equivalent fraction Sum Difference Numerator Denominator Unlike denominator Like denominator Mixed numbers Benchmark fractions

of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.*	enough? Students estimate that there is almost but not quite one cup of lemon juice, because 2/5 < 1/2. They calculate 1/2 + 2/5 = 9/10, and see this as 1/10 less than 1, which is probably a small enough shortfall that it will not ruin a recipe.	Benchmark fractions may be used to estimate and to check whether answers are reasonable.	Estimate
5.NF.B.5.a Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.	In preparation for Grade 6 work in ratios and proportional relationships, students learn to see products such as 5 x 3 or 1/2 x 3 as expressions that can be interpreted in terms of a quantity, 3, and a scaling factor, 5 or 1/2. Thus, in addition to knowing that 5 x 3 = 15, they can also say that 5 x 3 is 5 times as big as 3, without evaluating the product. Likewise, they see 1/2 x 3 as half the size of 3.	We can make observations and assumptions about how factors will impact the product based upon the size of one factor compared to the size of the second factor. For example, we can say that 7 x 56 is 7 times as big as 56 without performing the calculation to find the product. The resulting product will be larger than either of the two factors.	Product Factor Scaling Comparing Larger Smaller Equivalent Fraction Numerator Denominator
5.NF.B.5.b Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence a/b = (n×a)/(n×b) to the effect of multiplying a/b by 1.	Grade 5 work with multiplying by unit fractions, and interpreting fractions in terms of division, enables students to see that multiplying a quantity by a number smaller than 1 produces a smaller quantity, as when the budget of a large state university is multiplied by 1/2, for example.	When multiplying a given fraction by a factor, the product will either be greater than, equal to, or less than the fraction depending on how the factor compares to 1.	
5.NF.B.6 Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.	Students attend carefully to the underlying unit quantities when solving problems. For example, if ½ of a fund-raiser's funds were raised by the 6th grade, and if ⅓ of the 6th grade's funds were raised by Ms. Wilkin's class, then ⅓ x ½ gives the fraction of the fund-raiser's funds that Ms. Wilkin's class raised, but it does not tell us how much money Ms. Wilkin's class raised.	Solving word problems with addition, subtraction, multiplication and division of fractions follow the same problem solving structures as for whole number situations.	Fraction Numerator Denominator Product Partition Equal parts Equivalent Factor Unit fraction

5.MD.A.1 Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.	In Grade 5, students extend their abilities from Grade 4 (4.MD.A.1) to express measurements in larger or smaller units within a measurement system. This is an excellent opportunity to reinforce notions of place value for whole numbers and decimals, and make connections between fractions and decimals (e.g., 2 1/2 meters can be expressed as 2.5 meters or 250 centimeters).	Relationships between units vary depending on the measurement system. Conversions in the U.S. customary system vary depending upon what is being measured. Conversions in the metric system are based on powers of ten. When converting from a larger unit to a smaller unit, there will be more iterations of the smaller unit. For example, when converting from yards to feet, there will always be a greater number of feet than yards. When converting from a smaller unit to a larger unit, there will be less iterations of the larger unit. For example, when converting from cups to gallons, there will always be fewer gallons than cups. Measurements can be converted to solve multi-step real-world problems.	Convert Measurement units Metric system Customary system
5.MD.B.2 Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.	Grade 5 students grow in their skill and understanding of fraction arithmetic, including multiplying a fraction by a fraction, dividing a unit fraction by a whole number or a whole number by a unit fraction, and adding and subtracting fractions with unlike denominators. Students can use these skills to solve problems, including problems that arise from analyzing line plots. For example, given five graduated cylinders with different measures of liquid in each, students might find the amount of liquid each cylinder would contain if the total amount in all the cylinders were redistributed equally. (Students in Grade 6 will view the answer to this question as the mean value for the data set in questions.)	We collect, organize, represent, and analyze data in order to answer a question or solve a problem. Data can be organized and represented in a picture graph, a bar graph, or a line plot. Symbols used in line plots should be consistently spaced and sized for visual accuracy.	Line plot Interpret Data

UNIT 6: MEASUREMENT CONVERSIONS AND FRACTION OPERATIONS				
How is the size of a factor related to the size of the product when multiplying a given fraction? How can equivalent fractions be used to add and subtract fractions with unlike denominators? What is the relationship between units of measure in each system? Why do we collect, organize, represent and analyze data?				
CCSS Standards #	**Learning Targets**	**Summative Assessment Strategy**	**Lesson Progression and Connection to Math Practices**	**Common Learning Experiences and Assessments**
Section A: Measurement Conversions and Powers of 10				
5.MD.A.1 5.NBT.A.1 5.NBT.A.2 5.NF.B.6	I can explain patterns when multiplying and dividing by powers of 10. I can solve problems involving measurement conversions.	X Selected Response X Constructed Response Performance X Observation	**Lesson Progression:** Students have an opportunity to extend and apply what they learned about place value, measurement, and decimal and fraction arithmetic to convert units of measurement. Students begin by revisiting grade 4 measurement concepts as they convert measurements from larger units to smaller units within the metric system. This allows them to connect the decimal work learned in the previous unit and analyze patterns in the converted measurements when a number is multiplied by a power of 10. Subsequent lessons include an analysis of metric measurement conversions that express smaller units in terms of larger units, showing the effects of dividing by a power of 10. In addition to comparing the number of zeros in each number, students compare the placement of the decimal point and recognize the shifting of digits relative to the decimal point or vice versa. While this section begins with measurement units within the metric system, it shifts to problems involving customary units, allowing students to develop an understanding of the relative sizes of units of length, volume, and weight. As students solve multi-step word problems involving measurement conversions, they use the four operations with whole numbers, decimals, and	**Mandatory Lessons/Activities:** iM Lessons 1, 2, 3, 4, 5, 6, 7

			fractions. They have opportunities to think strategically about whether to convert from the larger unit to a smaller unit or from the smaller unit to a larger unit.	
Pacing:	7 days		**Math Practices:** SMP 1, 2, 3, 4, 5, 6, 7, 8	**Assessments:** Cool-downs: 1, 3 Checkpoint A
Section B: Add and Subtract Fractions with Unlike Denominators				
5.MD.B.2 5.NF.A.1 5.NF.A.2	I can add and subtract fractions with unlike denominators. I can solve word problems involving fraction addition and subtraction. I can create line plots to display fractional measurement data, and use the information to solve problems.	X — Selected Response X — Constructed Response — Performance X — Observation	**Lesson Progression:** Students learn to add and subtract fractions and mixed numbers with unlike denominators, and apply this learning to problem solving. In grade 4, students added and subtracted fractions with like denominators, and considered fraction equivalence in order to add tenths and hundredths. In order to develop an understanding of the need to generate equivalent fractions to add and subtract fractions with unlike denominators, number lines are employed. Number lines allow students to conceptualize addition and subtraction as distance, which is helpful when a student is adding 2/3 + 5/6, for example. Students can partition a number line that is already partitioned into thirds into sixths to count on from 2/3 a distance of 5/6. In the progression of lessons, students first encounter problems where one denominator is a factor of the other, so that they will only need to change one denominator, and then they encounter problems where neither denominator is a factor of the other. Students recognize that as the denominators get larger, the number line becomes less useful as a tool for finding sums and differences of fractions with unlike denominators. They analyze and then use numerical methods for finding common denominators, such as multiplying the denominators and finding a common multiple. In the final part of this section, students extend their understanding of line plots. They create line	**Mandatory Lessons/Activities:** iM Lessons 8, 9, 10, 11, 12, 13, 14, 15

			plots using measurement data in fractional units (halves, fourths, and eighths), and interpret the data on line plots to solve problems involving the four fraction operations.	
Pacing:	8 days		**Math Practices:** SMP 1, 2, 3, 4, 5, 6, 7, 8	**Assessments:** Cool-downs 8, 9, 11 Checkpoint B
Section C: The Size of Products				
5.MD.B.2 5.NF.A.2 5.NF.B.5 5.NF.B.5.a 5.NF.B.5.b	I can explain the magnitude (size) of a product based on the factors.	X Selected Response X Constructed Response Performance X Observation	**Lesson Progression:** Students build on their understanding of multiplication to include the concept of scaling. Students interpret multiplication expressions as a quantity that is resized or scaled by a factor. This concept builds on the multiplicative comparison work students did with whole numbers in grade 4. To develop an understanding of this concept, students compare the value of multiplication expressions without performing the multiplication. Early in the section, the expressions are such that one factor is the same and the other one is different. In the example shown, students reason that 7/6 x 4 is greater than the other two expressions because in each expression, 4 is being multiplied by a fraction, and 7/6 is the largest fraction of the three.	**Mandatory Lessons/Activities:** iM Lessons 16, 17, 18, 19, 20, 21
Pacing:	6 days		**Math Practices:** SMP 1, 2, 3, 4, 5, 6, 7, 8	**Assessments:** Cool-downs 16, 18 Checkpoint C

COMMON MISCONCEPTIONS	PRIOR KNOWLEDGE NEEDED TO MASTER STANDARDS FOR THIS UNIT	ADVANCED STANDARDS FOR STUDENTS WHO HAVE DEMONSTRATED PRIOR MASTERY	OPPORTUNITIES FOR STUDENT-DIRECTED LEARNING WITHIN THE UNIT
Students may incorrectly evaluate powers of ten, for example thinking 10 to the 2nd power is 20.	5.NBT.A.1: 4.NBT.A.1, 4.NF.C.5, 4.NFC.6, 4.NF.C.7 5.NBT.A.2: 5.NBT.A.1	Choose from iM leveled centers and exploration problems to differentiate for students who are ready.	iM Centers District-approved online resources

Students may not pay attention to the unit of measurement when solving problems that require renaming units. For example, when subtracting 2 inches from 5 feet, students may simply subtract 2 from 5 and say the answer is 3. Students may add or subtract fractions without first finding equivalent fractions with the same denominator. Students may misapply whole number concepts to adding and subtracting fractions, for example treating the numerator and denominator as separate numbers rather than understanding fractions as numbers. Students may not understand that when multiplying a given fraction by a factor, the product can either be greater than, equal to, or less than the fraction depending on how the factor compares to 1.	5.MD.A.1: 4.MD.A.1, 4.MD.A.2 5.MD.B.2: 4.MD.B.4, 5.NF.A.2, 5.NF.B.6, 5.NF.B.7 5.NF.A.1: 4.NF.A.1, 4.NF.B.3 5.NF.A.2: 4.NF.A.2, 5.NF.A.1 5.NF.B.5: 4.MD.A.2, 4.OA.A.3, 4.NF.A.1, 4.OA.A.1, 4.OA.A.2, 5.NF.B.4 5.NF.B.6: 4.MD.A.2, 4.OA.A.1, 4.OA.A.2		

RESOURCES
Kendall Hunt Flourish Blackline masters and materials from Teacher Resource Pack Meter sticks, rulers, yardsticks, paper clips, pencils

UNIT 7: GEOMETRY AND THE COORDINATE PLANE

Illustrative Mathematics Unit Focus: Students plot coordinate pairs on a coordinate grid and classify triangles and quadrilaterals in a hierarchy based on properties of side length and angle measure. They generate, identify, and graph relationships between corresponding terms in two numeric patterns, given two rules, and represent and interpret real world and mathematical problems on a coordinate grid.

Essential Questions:
How can we describe an object's location in space?
How can two-dimensional figures be described, classified and analyzed?

Unit Pacing: 15 days (10 required lessons, 3 flex, 2 assessment and reaction)

UNWRAPPED STANDARDS

Grade Level Standard	Standard Progression	Concepts (Big Ideas/ Understandings)	Academic Vocabulary (Standard Based)
5.OA.B.3 Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. For example, given the rule "Add 3" and the starting number 0, and given the rule Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.	Students extend their Grade 4 pattern work by working briefly with two numerical patterns that can be related and examining these relationships within sequences of ordered pairs and in the graphs in the first quadrant of the coordinate plane. This work prepares students for studying proportional relationships and functions in middle school.	We analyze patterns to determine how they change and identify relationships. Ordered pairs generated from given rules can be graphed on a coordinate plane.	Numerical pattern Rule Ordered pair Coordinate plane
5.G.A.1 Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the	Although students can often "locate a point," these understandings are beyond simple skills. For example, initially, students often fail to distinguish between two different ways of viewing the point (2, 3), say, as instructions: "right 2, up 3"; and as the point defined by being a distance 2 from the y -axis and a distance 3 from the x -axis. In these two descriptions the 2 is first associated with the x -axis, then with the y -axis.	Coordinate graphs show relationships between numbers on a coordinate grid. The coordinate system is created from a horizontal number line (x-axis) and a vertical number line (y-axis) with the intersection of the lines at zero (the origin).	Coordinate grid Grid lines Vertical Horizontal Intersect Point Axis X-axis Y-axis

second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).		A given point can be located in the plane by using an ordered pair of numbers (x, y). The origin of the coordinate plane is represented by the ordered pair (0, 0). The first number in an ordered pair, the x-coordinate or x, indicates how far to travel from the origin in the horizontal direction. The second number in an ordered pair, the y-coordinate or y, indicates how far to travel in the vertical direction. Distance is found by counting intervals rather than counting the grid marks.	Origin Ordered pair X-coordinate Y-coordinate
5.G.A.2 Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.	Students connect ordered pairs of (whole number) coordinates to points on the grid, so that these coordinate pairs constitute numerical objects and ultimately can be operated upon as single mathematical entities. Students solve mathematical and real-world problems using coordinates. For example, they plan to draw a symmetric figure using computer software in which students input coordinates that are then connected by line segments.	Real-world situations can be represented by graphing points in the coordinate plane. Coordinate values can be interpreted in the context of real-world situations.	
5.G.B.3 Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.	Based on analysis of properties, they classify two-dimensional figures in hierarchies. For example, they conclude that all rectangles are parallelograms, because they are all quadrilaterals with two pairs of opposite, parallel, equal-length sides. In this way, they relate certain categories of shapes as subclasses of other categories.	Shapes can be named and classified by angle measures, side lengths, or the presence or absence of parallel and perpendicular lines.	Attribute Category Sub category Properties Two-dimensional Hierarchy Quadrilateral

5.G.B.4 Classify two-dimensional figures in a hierarchy based on properties			Parallel lines Perpendicular lines Squares Rectangles Parallelograms Trapezoid Rhombus Classify

UNIT 7: GEOMETRY AND THE COORDINATE PLANE

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CCSS Standards #	Learning Targets	Summative Assessment Strategy	Lesson Progression and Connection to Math Practices	Common Learning Experiences and Assessments
Section A: The Coordinate Plane				
5.G.A.1	I can identify and plot points on a coordinate grid. .	X Selected Response X Constructed Response Performance X Observation	**Lesson Progression:** Students have an opportunity to extend and apply what they learned about grids in previous grades to the concept of the coordinate grid. In grade 3, students measure areas of rectangles by counting unit squares on a grid and use the structure of the grid to confirm that the area can also be found by multiplying the side lengths. In grade 4, they plot polygons on grids and use the structure of the grids to identify perpendicular and parallel lines in two-dimensional figures. In this section, students learn about the coordinate grid. They learn that the horizontal and vertical axes have numbers that are useful for describing and locating vertices. They locate and name given points on the coordinate grid by using an ordered pair. They also plot points on a coordinate grid while recognizing the importance of attending to precision when naming and plotting coordinates. For example, students notice that the first number in the ordered pairs (3,	**Mandatory Lessons/Activities:** iM Lessons 1, 2, 3

			7) and (7, 3) corresponds to the horizontal axis while the second number corresponds to the vertical axis.	
Pacing:	3 days		**Math Practices:** SMP 3, 4, 5, 6, 7	**Assessments:** Cool-downs: 2, 3 Checkpoint A
Section B: The Hierarchy of Shapes				
5.G.B 5.G.B.3 5.G.B.4	iM I can classify shapes in a hierarchy based on properties.	X Selected Response X Constructed Response Performance X Observation	**Lesson Progression:** Students learn about the hierarchy of two-dimensional figures. They begin to classify different types of triangles and quadrilaterals into categories and subcategories. Students first explore the exclusive and inclusive definitions of a trapezoid, and see how each definition impacts the hierarchy of quadrilaterals. In this curriculum, we will be using the inclusive definition, which states that a trapezoid has at least one pair of opposite sides parallel. Students continue to build their understanding of the hierarchy of quadrilaterals by analyzing the properties of parallelograms, rhombuses, squares, and rectangles. Using toothpicks to keep the side lengths constant, students describe the angles, parallelism, and perpendicularity of each shape. They determine that parallelograms are trapezoids, and squares are both rhombuses and rectangles. They also learn that rhombuses and rectangles are 2 different categories of parallelograms. The section concludes with a lesson that helps to classify subcategories of triangles. Students sort triangles by their side-length and angle measurements and describe their properties.	**Mandatory Lessons/Activities:** iM Lessons 4, 5, 6, 7, 8
Pacing:	5 days		**Math Practices:** SMP 1, 2, 3, 4, 5, 6, 7, 8	**Assessments:** Cool-down 4, 6, 7 Checkpoint B
Section C: Numerical Patterns				
5.G.A.2	I can generate a pattern from a given		**Lesson Progression:**	**Mandatory Lessons/Activities:**

5.OA.B.3	rule. I can identify and describe the relationships between corresponding terms in two patterns.	X Selected Response X Constructed Response Performance X Observation	This section allows students to bring together all of the concepts learned in this unit as they generate numerical patterns and interpret the relationship between them. Students learn that they can form ordered pairs using corresponding terms from each pattern, and graph the coordinates. As students apply these concepts to situations, they use their understanding of the properties of shapes to graph the length and width of rectangles with given areas, and find the perimeter.	iM Lessons 9, 10, 11, 12, 13
Pacing:	5 days		**Math Practices:** SMP 1, 2, 3, 4, 5, 6, 7, 8	**Assessments:** Cool-down 9, 10 Checkpoint C

COMMON MISCONCEPTIONS	PRIOR KNOWLEDGE NEEDED TO MASTER STANDARDS FOR THIS UNIT	ADVANCED STANDARDS FOR STUDENTS WHO HAVE DEMONSTRATED PRIOR MASTERY	OPPORTUNITIES FOR STUDENT-DIRECTED LEARNING WITHIN THE UNIT
Students may reverse the order of the coordinates when plotting points on a coordinate plane. They count up first on the y-axis and then count over on the x-axis. Students think that when describing geometric shapes and placing them in subcategories, the last category is the only classification that can be used. When students are asked to describe the relationship between two patterns, they may focus on each pattern in isolation instead of looking for a multiplicative relationship between the two.	5.OA.B.3: 4.OA.C.5 5.G.B.3: 4.GA.2 5.G.B.4: 5.G.B.3	Choose from iM leveled centers and exploration problems to differentiate for students who are ready.	iM Centers District-approved online resources

RESOURCES
Kendall Hunt Flourish Blackline masters and materials from Teacher Resource Pack Markers (dry erase), sheet protectors, toothpicks, coins