Mathematics Education for the Future: Critical Components of a High-Quality Program

June 5, 2018

The World Economic Forum Top 10 Skills

In 2020	In 2015
1. Complex Problem-solving	1. Complex Problem-Solving
2. Critical Thinking	2. Coordinating with Others
3. Creativity	3. People Management
4. People Management	4. Critical Thinking
5. Coordinating with Others	5. Negotiation
6. Emotional Intelligence	6. Quality Control
7. Judgment & Decision-Making	7. Service Orientation
8. Service Orientation	8. Judgment & Decision-Making
9. Negotiation	9. Active Listening
10. Cognitive Flexibility	10. Creativity

Source: *The World Economic Forum Top 10 Skills for Surviving the Fourth Industrial Revolution* (2013)

Automation of Knowledge Work

- In approximately 60% of occupations, up to 33% of constituent activities could be automated by 2030, implying substantial workplace transformation
- Computers are now capable of doing jobs that were assumed only humans could perform
- All future workers will need to be adaptable as occupations evolve due to rapidly evolving technologies
- The future will demand creativity, innovation, and problem-solving

Sources: *Jobs Lost, Jobs Gained: Workforce Transitions in a Time of Automation*, McKinsey Global Institute, December 2017; *Disruptive Technologies: Advances That Will Transform Life, Business, in the Global Economy*, McKinsey Global Institute, May 2013.

"The question is not whether all students can succeed in mathematics but whether the adults organizing the mathematics learning opportunities can alter traditional beliefs and practices to promote success for all."

Source: *Principles to Actions: Ensuring Mathematical Success for All, Access and Equity.* National Council of Teachers of Mathematics, (2014)

Beliefs About Teaching and Learning Mathematics

Unproductive Beliefs	Productive Beliefs
Math learning should focus on practicing procedures and memorizing basic number combinations.	Math learning should focus on developing understanding of concepts and procedures through problem solving, reasoning, and discourse.
Students need only to learn and use the same standard computational algorithms and the same prescribed methods to solve algebraic problems.	All students need to have a range of strategies and approaches from which to choose in solving problems, including, but not limited to, general methods, standard algorithms, and procedures.
Students can learn to apply mathematics only after they have mastered the basic skills.	Students can learn mathematics through exploring and solving contextual and mathematical problems.
The role of the teacher is to tell students exactly what definitions, formulas, and rules they should know and demonstrate how to use this information to solve math problems	The role of the teacher is to engage students in tasks that promote reasoning and problem solving and facilitate discourse that moves students toward shared understanding of mathematics
The role of the student is to memorize information that is presented and then use it to solve routine problems on homework, quizzes, and tests.	The role of the student is to be actively involved in making sense of math tasks by using varied strategies and representations, justifying solutions, making connections to prior knowledge, and consider the reasoning of others.
An effective teacher makes the math easy for students by guiding them step by step through problem solving to ensure they are not frustrated or confused	An effective teacher provides students with appropriate challenge, encourages perseverance in solving problems, and supports productive struggle in learning math.

Source: *Principals to Actions: Ensuring Mathematical Success for All*, National Council of Teachers of Mathematics (2014)

High Leverage Practices and Essential Teaching Skills

Eight Mathematical Teaching Practices	Effective teaching…
1. Establish math goals to focus learning	establishes clear goals for learning, situates goals with progressions, and uses the goals to guide instruction.
2. Implement tasks that promote reasoning and problem solving	engages students in solving and discussing tasks that promote reasoning and problems solving and allow multiple entry points and varied solution strategies.
3. Use and connect mathematical representations	engages students in making connections among math representations to deepen understanding of concepts and procedures and as tools of problem solving.
4. Facilitate meaningful math discourse	facilitates discourse among students to build shared understanding of math ideas by analyzing and comparing student approaches and arguments

Source: *Principles to Actions: Ensuring Mathematical Success For All,* National Council of Teachers of Mathematics (2014)

High Leverage Practices and Essential Teaching Skills

Eight Mathematical Teaching Practices	Effective teaching…
5. Pose purposeful questions	uses purposeful questions to assess and advance students' reasoning and sense making about important math ideas and relationships.
6. Build procedural fluency from conceptual understanding	builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problem.
7. Support productive struggle in learning mathematics	consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with math ideas and relationships.
8. Elicit and use evidence of student thinking	uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction in ways that support and extend learning.

Source: *Principles to Actions: Ensuring Mathematical Success For All*, National Council of Teachers of Mathematics (2014)

Essential Elements of Effective School Math Programs

- Commitment to equity and access
- Powerful curriculum
- Best practices instruction
- Appropriate tools and technology
- Meaningful and aligned assessment
- Culture of professionalism

Source: *It's Time: Themes and Imperatives for Mathematics Education.* National Council for Supervisors of Mathematics (2014); *Principles to Actions: Ensuring Mathematical Success for All, Access and Equity.* National Council of Teachers of Mathematics, (2014)

Mathematical Mindsets

- All students are capable reaching and engaging in high-level mathematics
- No such thing as a "math brain" or "math gene"
- Productive struggle and embracing mistakes is key to growth in mathematics
- Depth must be valued over speed
- Student capacity to persevere must be developed and supported

Source: *Mathematical Mindsets: Unleashing Potential Through Creative Math, Inspiring Messages, and Innovative Teaching*, Jo Boaler (2016)

Five Strands of Mathematical Proficiency

Five Strands	Definition
1. Understanding	Comprehending Mathematical concepts, operations, and relations - knowing what mathematical symbols, diagrams, and procedures mean.
2. Computing	Carrying our mathematical procedures, such as adding, subtracting, multiplying, and dividing numbers flexibly, accurately, efficiently, and appropriately.
3. Applying	Being able to formulate problems mathematically and to devise strategies for solving them using concepts and procedures appropriately.
4. Reasoning	Using logic to explain and justify a solution to a problem or to extend from something known to something not yet known.
5. Engaging	Seeing mathematics as sensible, useful, and doable - *if* you work at it - and being willing to do the work.

Source: *Helping Children Learn Mathematics*, National Research Council (2002)

Standards for Mathematical Practice

Overarching Habits of Mind

1. Make sense of problems and persevere in solving them
6. Attend to precision

Reasoning and Explaining

2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others

Modeling and Using Tools

4. Model with mathematics
5. Use appropriate tools strategically

Seeing Structure and Generalizing

7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

Source: corestandards.org

Conceptually Based Fluency

- Involves the ability to use numbers flexibly
- Reflects deep understanding of numbers and the ways they relate to each other
- Requires the ability to compose and decompose numbers to be flexible and efficient with solving problems
- Develops through strategy work that builds over time
- De-emphasizes memorization and speed
- Supports highest levels of math achievement
- Focuses on big mathematical ideas and connections between those ideas
- Introduces the standard algorithm after conceptual foundation is created

Source: *"Fluency Without Fear: Research Evidence on the Best Ways to Learn Math Facts."* Jo Boaler, YouCubed at Stanford University

Reasoning and Problem-Solving Facilitates….

- multiple entry points and varied solution strategies
- active inquiry and exploration
- used of procedures that are meaningfully connected with concepts or understanding
- complex and non-algorithmic thinking
- exploration of concepts, processes, and relationships
- self-monitoring of one's own cognitive processes
- task analysis and examination of constraints that limit solutions

Source: Adapted from *Characteristics of Mathematical Tasks at Four Levels of Cognitive Demand*. From Smith and Stein (1998)

Example:

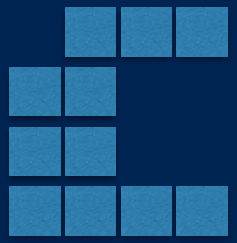

Fig. 2

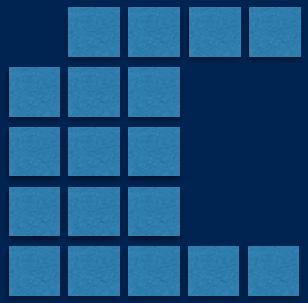

Fig. 3

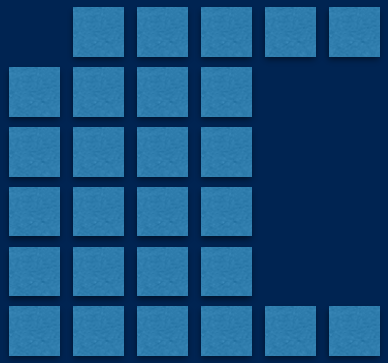

Fig. 4

What would figure 100 look like?

Imagine you could continue your pattern backward. How many tiles would there be in figure - 1 (negative one)?

What would figure -1 (negative one) look like?

Source: Adapted from Carlos Cabana. Mathematical Mindsets (2016)

High-Quality Assessment Practices

- Requires tight alignment to learning goals
- Must be consistent across grade levels
- Include common assessment tasks collaboratively reviewed by teaching teams
- Provides formative feedback to students
- Should focus on reasoning and problem-solving
- Must shift from performance to understanding

Source: *It's Time: Themes and Imperatives for Mathematics Education.* National Council for Supervisors of Mathematics (2014); *Principles to Actions: Ensuring Mathematical Success for All, Access and Equity.* National Council of Teachers of Mathematics, (2014); *Mathematical Mindsets: Unleashing Potential Through Creative Math, Inspiring Messages, and Innovative Teaching*, Jo Boaler (2016)

Challenge of Using CogAT to Determine Math Placement

- Emphasizes more “traditional” math learning outcomes
- Not aligned to math program goals or learning standards
- Does not assess conceptual based fluency or strategy work
- Does not provide a window into student reasoning or explanation
- Does not assess conceptual understanding or real-world problem-solving
- Does not allow for “out-of-the-box” thinking

Research on Acceleration and Tracking

- All students should be engaged in productive struggle when solving problems and persevere when doing so
- Research shows that all students benefit from heterogenous math learning environments
- Early advancement can be detrimental for students and can undermine their future math potential in upper grade levels
- All students should have a pathway toward high levels of mathematics
- Metrics utilized to determine placement must be aligned with expectations for student understanding and mastery of established critical areas

Source: *"Equity and Deeper Learning in Mathematics."* Pedro Noguera, Keynote Speaker, National Council for the Supervisors of Mathematics National Conference (2018); *Principles to Actions: Ensuring Mathematical Success For All*, National Council of Teachers of Mathematics (2014); *Raising Expectations and Achievement: The Impact of Wide-Scale Mathematics Reform Giving All Students Access to High-Quality Mathematics.* Jo Boaler and David Foster (2016).

What Makes a Mathematics Curriculum Rigorous?

- Balance of procedures, concepts, and applications
- Increasing the variety and quality of student work
- Infusion of performance tasks / projects
- Enhanced written explanations:
  - ✳Procedures: show steps, explain how it was calculated
  - ✳Conceptual understanding: explain why something is true
  - ✳Problem-Solving: explain which method you used to solve the problem
  - ✳Modeling: explain the assumptions made and implications

Source: *"Systemic Consequences of Misunderstanding Mathematical Rigor."* Patrick Callahan, National Council for the Supervisors of Mathematics National Conference (2018)

What Makes a Mathematics Curriculum Rigorous?

Misconception #1: Input vs Output

- It is not what the teacher or materials do (inputs) but what students do (outputs) that matters.
- Inputs tend to focus on coverage and introducing topics at earlier ages vs. looking at topics in greater depth facilitating substantive student outputs.

Misconception #2: Difficulty and Workload

- Using unfriendly numbers for calculations vs. looking deeply into patterns and structure of mathematics
- Results in many hours of rote homework vs. one conceptually challenging problem

Source: *"Systemic Consequences of Misunderstanding Mathematical Rigor."* Patrick Callahan, National Council for the Supervisors of Mathematics National Conference (2018)

Misconception #3: Selectivity and Exclusivity

- Selective or exclusive courses do not necessarily incorporate greater rigor
- Significant systemwide implications for equity and access

Source: *"Systemic Consequences of Misunderstanding Mathematical Rigor."* Patrick Callahan, National Council for the Supervisors of Mathematics National Conference (2018)

Access and Equity Requires….

- critical examination of who the math program truly serves and who it structurally disenfranchises
- all students be held to high expectations and have access to high-quality curriculum and instruction
- recognition that inequitable learning opportunities can exist in any setting
- consistency of instructional delivery across all classrooms

Source: *"Equity and Deeper Learning in Mathematics."* Pedro Noguera, Keynote Speaker, National Council for the Supervisors of Mathematics National Conference (2018); *Principles to Actions: Ensuring Mathematical Success For All*, National Council of Teachers of Mathematics (2014); *Raising Expectations and Achievement: The Impact of Wide-Scale Mathematics Reform Giving All Students Access to High-Quality Mathematics.* Jo Boaler and David Foster (2016).

Access and Equity Requires….

- acknowledgment that tracking/low-level math classes limits participation and achievement of students
- understanding that heterogenous classrooms do not reduce the achievement levels of strong math students
- math programs that offer a pathway to the highest-level courses for all students

Source: *"Equity and Deeper Learning in Mathematics."* Pedro Noguera, Keynote Speaker, National Council for the Supervisors of Mathematics National Conference (2018); *Principles to Actions: Ensuring Mathematical Success For All*, National Council of Teachers of Mathematics (2014); *Raising Expectations and Achievement: The Impact of Wide-Scale Mathematics Reform Giving All Students Access to High-Quality Mathematics.* Jo Boaler and David Foster (2016).

Essential Elements of Effective Math Programs	Progress to Date	Next Steps
Equity and Access	• All student have access to high-quality materials	• Identify research-based intervention materials aligned to core curriculum
Powerful Curriculum	• Adopted Investigations (K-5) and Connected Math Project 3 (6-8) • Aligned curriculum with clearly defined student progressions	• Identify supplementary materials and projects to enhance mathematical depth and rigor
Appropriate tools and technology	• Conducted trial of conceptually based fluency program • Scout assessment tool	• Adopt conceptually-based fluency program • Align tech platforms to units
Meaningful aligned assessment	• Grade level teams completed unit assessments and applied student benchmarks • Developed shared assessment expectations	• Develop common assessments K-8 • Determine appropriate metrics and grade levels for math acceleration
Culture of professionalism	• Development of common expectations to support alignment across grade level teams	• Utilize weekly meetings to analyze student work and select high-quality tasks

Q & A