

Course Title	Content Area	Grade Level	Credit (if applicable)
Grade 8 Math: Pathways to Algebra	Mathematics	Grade 8	N/A

Course Description

IM Grade 8 begins with transformational geometry. Students study rigid transformations and congruence, and then dilations and similarity. This provides background for understanding the slope of a line in the coordinate plane.

Next, students build on their understanding of proportional relationships, from IM Grade 7, to study linear relationships. They use equations, tables, and graphs to represent linear relationships, and make connections across these representations. Students expand their ability to work with linear equations in one and two variables, extending their understanding of a solution to an equation in one or two variables to comprehend a solution to a system of equations in two variables. They learn that linear relationships are an example of a special kind of relationship called a function. Students apply their understanding of linear relationships and functions to contexts involving data with variability.

The course ends the year with students extending their understanding of exponents to include all integers, and in the process codifying the properties of exponents. They learn about orders of magnitude and scientific notation in order to represent and compute with very large and very small quantities. They encounter irrational numbers for the first time and informally extend the rational-number system to the real-number system, motivated by their work with the Pythagorean Theorem.

Aligned Core Resources **Connection to the *BPS Vision of the Graduate***

- CT Core Standards**
(aligned to [National Common Core Standards](#))
<https://accessim.org>
- [Imagine Learning iM Resources \(Imagine 6-8\)](#)
BPS teacher login through ClassLink required
- <https://accessim.org/6-8/grade-8/course-guide/further-reading?a=teacher>
- [Empowering All Storytellers: Tips for Engaging Special Populations Using IM® v.360 for Grade 6-12](#)
 - [Tackling Wordy Problems: How the Three Reads Math Language Routine Supports Access for All Learners](#)
 - [Think Pair Share](#)
 - [Making Sense of Story Problems](#)
 - [Math Language Routines: Discourse with a Purpose](#)
 - [Unlocking Learners' Thinking Using the Mathematical Language Routines](#)

Common Core State Standards: Math Practice (MP) Standards

MP 1: Make sense of problems and persevere in solving them.
 MP 2: Reason abstractly and quantitatively.
 MP 3: Construct viable arguments and critique the reasoning of others.
 MP 4: Model with mathematics.
 MP 5: Use appropriate tools strategically.
 MP 6: Attend to precision.
 MP 7: Look for and make use of structure.
 MP 8: Look for and express regularity in repeated reasoning.

	Lessons that Showcase Math Practice Standards							
	Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6	Unit 7	Unit 8
MP 1	1, 6, 13	5,10,13	3, 5, 9, 10	8,	1,4,10,15,19-21	1,9	1,14	1,8,12,18
MP 2	6	11,	1, 4, 5, 12, 14	1,4,7,9	3-8,10,11,17,18	1,3,4,6-9,11	2,10-11-12,14,15,16	3,4,11,14
MP 3	2, 6-9, 11-14	2,3,6,8,12	4, 14	13,	6,14-17	5,10	4,13,15	5,6
MP 4	2		2, 3, 11	15	5,6,9-13	2,4-6,8,10,11	12-16	18
MP 5	1, 3, 7, 11	4,	5	10,13,	7	2,11		2
MP 6	1, 3, 4, 8, 10-14	1,4,5,6,	1, 3, 6	6,11,12,	2,5,6,11,12,21	3,5-7,9	1,7-9,11,14-16	1,7,8,9,12,13,15,16
MP 7	1,, 3, 4, 5, 9, 10,15-17	1,2,3,4,8,9,12,	2, 5	3,4,5,10,	2,13,14,16,19,20	1,2,4,7,9	4,5,7,8,10,11,13	1,2,5,6,8,10,13,14,17
MP 8	8,15	6,9,10,	6, 7, 8, 10, 13, 15	5,6,7,8,13,14,15,16	3,22		1-6,8	7,16,17

Bristol Public Schools Vision of the Graduate

Problem Solving

- iM's focus on real-world modeling and problem-solving strategies
- Multiple solution pathways are encouraged and explored
- Students develop perseverance through challenging tasks

Critical Thinking

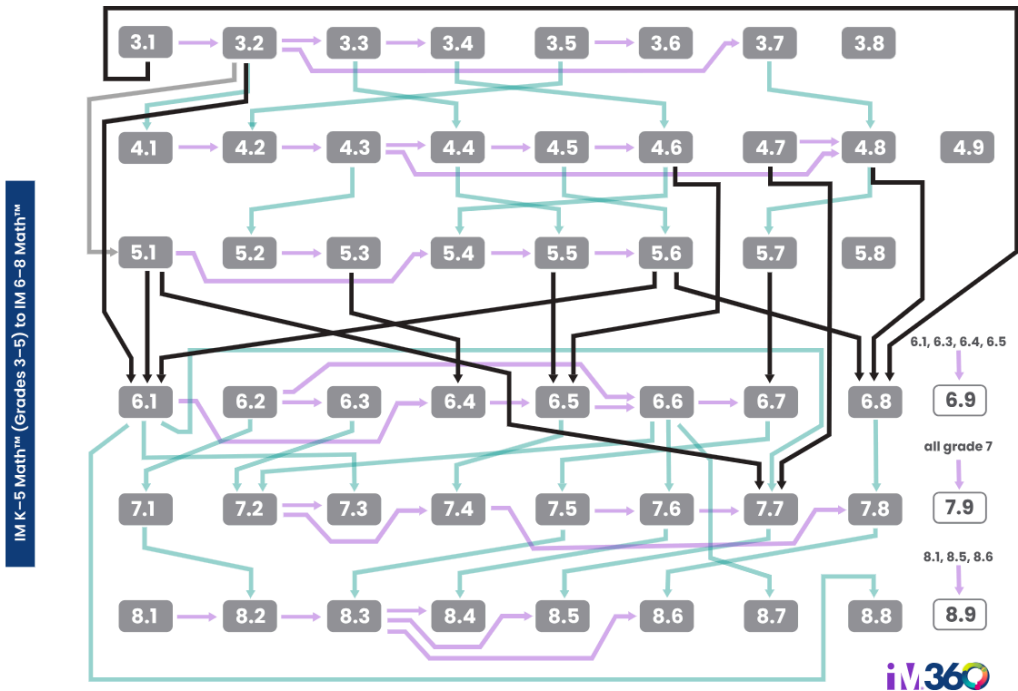
- Students analyze mathematical relationships and justify their reasoning
- Regular opportunities to critique others' reasoning
- Emphasis on understanding "why" not just "how"

Communication and Collaboration

- Structured mathematical discourse is built into lessons
- Students explain their thinking both verbally and in writing
- Many activities involve partner and group work

Link to *Equity Audit* **[Equity Curriculum Review Audit \(Gr. 8 Path\)](#)**

Additional Course Information:
Knowledge/Skill Dependent courses/prerequisites



Standard Matrix

Standard	Lessons
8.EE.A	Unit 8, Lesson 16
8.EE.A.1	Unit 7, Lesson 2 Unit 7, Lesson 3 Unit 7, Lesson 4
8.EE.A.2	Unit 8, Lesson 3 Unit 8, Lesson 4 Unit 8, Lesson 5
8.EE.A.3	Unit 7, Lesson 9 Unit 7, Lesson 10 Unit 7, Lesson 11
8.EE.A.4	Unit 7, Lesson 10 Unit 7, Lesson 11 Unit 7, Lesson 12
8.EE.B	Unit 3, Lesson 1 Unit 3, Lesson 2 Unit 3, Lesson 3 Unit 3, Lesson 4
8.EE.B.5	Unit 3, Lesson 2
8.EE.B.6	Unit 2, Lesson 10 Unit 2, Lesson 11 Unit 2, Lesson 12
8.EE.C	Unit 3, Lesson 13 Unit 3, Lesson 14
8.EE.C.7	Unit 4, Lesson 3 Unit 4, Lesson 4
8.EE.C.7.a	Unit 4, Lesson 7
8.EE.C.7.b	Unit 4, Lesson 6
8.EE.C.8	Unit 4, Lesson 9 Unit 4, Lesson 10
8.EE.C.8.a	Unit 3, Lesson 14
8.EE.C.8.b	Unit 4, Lesson 12
8.EE.C.8.c	Unit 4, Lesson 15

8.F.A	Unit 5, Lesson 3	Unit 5, Lesson 22		
8.F.A.1	Unit 5, Lesson 1 Unit 5, Lesson 2	Unit 5, Lesson 3 Unit 5, Lesson 4	Unit 5, Lesson 5 Unit 5, Lesson 17	Unit 9, Lesson 4
8.F.A.2	Unit 5, Lesson 7	Unit 5, Lesson 8		
8.F.A.3	Unit 5, Lesson 4 Unit 5, Lesson 7	Unit 5, Lesson 8	Unit 5, Lesson 17	Unit 5, Lesson 18
8.F.B	Unit 5, Lesson 10 Unit 5, Lesson 11	Unit 5, Lesson 17 Unit 5, Lesson 18	Unit 8, Lesson 3	Unit 9, Lesson 4 Unit 9, Lesson 6
8.F.B.4	Unit 5, Lesson 8	Unit 5, Lesson 9	Unit 5, Lesson 10	Unit 5, Lesson 11
8.F.B.5	Unit 5, Lesson 5	Unit 5, Lesson 6	Unit 5, Lesson 10	
8.G.A	Unit 1, Lesson 17 Unit 2, Lesson 2	Unit 2, Lesson 3 Unit 2, Lesson 4 Unit 2, Lesson 5	Unit 2, Lesson 8 Unit 2, Lesson 9 Unit 2, Lesson 12	Unit 9, Lesson 1 Unit 9, Lesson 2 Unit 9, Lesson 3
8.G.A.1	Unit 1, Lesson 2 Unit 1, Lesson 3	Unit 1, Lesson 4 Unit 1, Lesson 6	Unit 1, Lesson 11 Unit 1, Lesson 14	
8.G.A.1.a	Unit 1, Lesson 7 Unit 1, Lesson 8	Unit 1, Lesson 9	Unit 1, Lesson 10	Unit 1, Lesson 13
8.G.A.1.b	Unit 1, Lesson 7	Unit 1, Lesson 8	Unit 1, Lesson 9	Unit 1, Lesson 10
8.G.A.1.c	Unit 1, Lesson 9			
8.G.A.2	Unit 1, Lesson 11 Unit 1, Lesson 12	Unit 1, Lesson 13	Unit 1, Lesson 15	Unit 2, Lesson 6 Unit 2, Lesson 7
8.G.A.3	Unit 1, Lesson 5 Unit 1, Lesson 6	Unit 2, Lesson 4	Unit 2, Lesson 5	Unit 2, Lesson 12
8.G.A.4	Unit 2, Lesson 6	Unit 2, Lesson 7	Unit 2, Lesson 9	
8.G.A.5	Unit 1, Lesson 14 Unit 1, Lesson 15	Unit 1, Lesson 16	Unit 2, Lesson 8 Unit 2, Lesson 13	Unit 9, Lesson 2
8.G.B	Unit 8, Lesson 7	Unit 8, Lesson 8	Unit 8, Lesson 10	Unit 8, Lesson 12
8.G.B.6	Unit 8, Lesson 8	Unit 8, Lesson 10		
8.G.B.7	Unit 8, Lesson 7 Unit 8, Lesson 8	Unit 8, Lesson 9 Unit 8, Lesson 11	Unit 8, Lesson 12	Unit 8, Lesson 18
8.G.B.8	Unit 8, Lesson 13			
8.G.C	Unit 5, Lesson 12	Unit 5, Lesson 17	Unit 5, Lesson 19	Unit 5, Lesson 20
8.G.C.9	Unit 5, Lesson 13 Unit 5, Lesson 14 Unit 5, Lesson 15	Unit 5, Lesson 16 Unit 5, Lesson 17 Unit 5, Lesson 18	Unit 5, Lesson 19 Unit 5, Lesson 20	Unit 5, Lesson 21 Unit 5, Lesson 22
8.NS.A	Unit 8, Lesson 2	Unit 8, Lesson 4	Unit 8, Lesson 11	Unit 8, Lesson 16
8.NS.A.1	Unit 8, Lesson 16	Unit 8, Lesson 17		
8.NS.A.2	Unit 8, Lesson 2 Unit 8, Lesson 5	Unit 8, Lesson 6	Unit 8, Lesson 14	Unit 8, Lesson 15
8.SPA	Unit 6, Lesson 11	Unit 9, Lesson 4	Unit 9, Lesson 5	Unit 9, Lesson 6
8.SPA.1	Unit 6, Lesson 1 Unit 6, Lesson 2	Unit 6, Lesson 3 Unit 6, Lesson 4	Unit 6, Lesson 5 Unit 6, Lesson 6	Unit 6, Lesson 7 Unit 6, Lesson 8
8.SPA.2	Unit 6, Lesson 4	Unit 6, Lesson 5	Unit 6, Lesson 6	Unit 6, Lesson 8
8.SPA.3	Unit 6, Lesson 3	Unit 6, Lesson 6	Unit 6, Lesson 8	
8.SPA.4	Unit 6, Lesson 9	Unit 6, Lesson 10		

Unit Links

[Grade 8 Mathematics: Pathways to Algebra](#)
[Unit 1: Rigid Transformations and Congruence](#)
[Unit 2: Dilation, Similarity, and Introducing Slope](#)
[Unit 3: Linear Relationships](#)
[Unit 4: Linear Equations and Linear Systems](#)
[Unit 5: Functions and Volume](#)
[Unit 6: Associations in Data](#)
[Unit 7: Exponents and Scientific Notation](#)
[Unit 8: Pythagorean Theorem and Irrational Numbers](#)
[Course Assessment Map](#)

Use of Instructional Time (181 School Days)

- 162 iM Content and Assessment Days
- 6 Climate and Culture Days: 2 days at start of year, 2 shortened days before breaks, and 2 days at end of year
- 9 IAB Days: 1 day Strategic Review and 2 day IAB in fall, winter, and spring
- 4 SBA Days: 1 day Strategic Review and 3 day SBA

Unit Title:**Unit 1: Rigid Transformations and Congruence****Relevant Standards: Bold indicates priority**

Lesson	Standards	Lesson	Standards
Lesson 1		Lesson 10	8.G.A.1.a 8.G.A.1.b
Lesson 2	8.G.A.1	Lesson 11	8.G.A.1 8.G.A.2
Lesson 3	8.G.A.1	Lesson 12	8.G.A.2
Lesson 4	8.G.A.1	Lesson 13	8.G.A.1.a 8.G.A.2
Lesson 5	8.G.A.3	Lesson 14	8.G.A.1 8.G.A.5
Lesson 6	8.G.A.1, 8.G.A.3	Lesson 15	8.G.A.2 8.G.A.5
Lesson 7	8.G.A.1.a 8.G.A.1.b	Lesson 16	8.G.A.5
Lesson 8	8.G.A.1.a 8.G.A.1.b	Lesson 17	8.G.A
Lesson 9	8.G.A.1.a 8.G.A.1.b 8.G.A.1.c		

Essential Question(s):

- How do different transformations (translation, rotation, reflection) affect the position and properties of a figure?
- What does it mean for two geometric figures to be "congruent"?
- What universal relationship exists among the interior angles of any triangle?

Enduring Understanding(s):

- Rigid transformations change a figure's position in the plane but preserve all side lengths and angle measures
- Two figures are congruent if there is a sequence of rigid transformations that takes one figure exactly onto the other
- The sum of the interior angle measures of any triangle is always 180 degrees, a fact that can be justified using rigid motions and parallel line relationships

Demonstration of Learning:

CFA 1: Checkpoint A (after lesson 6)
 CFA 2: Checkpoint B (after lesson 10)
 CFA 3: Checkpoint C (after lesson 13)
 CFA 4: Lesson 14 and Lesson 15 Cool Downs

Checkpoint D (after lesson 16) is an opportunity for feedback and/or review before the EOU assessment

NOTE: Could combine MOU and EOU Assessments - potentially???

MoU: Assessment A (after lesson 10)
 EoU: Assessment A (after lesson 16)

Pacing for Unit

14 Days (12 instructional days, 1 review, 1 assess)

- Keep as is: Lessons 1, 2, 7, 8, 11, 14, 15, 16
- Combine Lessons 3+4 (L4 Warmup, 3.1, 4,2)
- Combine Lessons 5+6 (L5 Warmup, 5.1, L6 Warmup)
- Combine Lessons 9+10 (L9 Warmup, 9.2, 10.2)2
- Suggestion: Combine mid and end of unit assessments into one.
- Combine Lessons 12+13 (12 Warmup, 12.1, 13.1, 13.2)

Lesson to Remove or Modify:

- Remove 8.1.14. In this lesson, students are introduced to transversal and types of angles. This is an additional standard for Geometry and can be revisited in high school Geometry
- Remove 8.1.15. In this lesson, students focus on interior angles of a triangle, this is also an additional standard for grade 8 that can be revisited in Geometry
- Remove 8.1.16. This lesson focused on triangles on and off a grid can be removed and revisited in Geometry
- Move 8.1.17 to outside of class as a culminating activity for the unit

Family Overview

<https://accessim.org/6-8/grade-8/unit-1?a=family>

Integration of Technology:

- Desmos Online Graphing Calculator
- Pear Assessment (Edulastic)
- iM v.360 Digital Applets (see below)

Unit-specific Vocabulary:

Lesson	New Terminology	
	receptive	productive
8.1.1	vertex plane measure	slide turn

Aligned Unit Materials, Resources, and Technology

Digital Applets

- 8.1.1 Lesson Synthesis Applet: Triangle Square Dance
- 8.1.2 Digital Applet: Move Card Sort
- 8.1.3 Digital Applet: Transformation Information
- 8.1.4 Digital Applets: Make That Move, A to B to C
- 8.1.5 Digital Applets: Reflecting Points on the

	direction	
8.1.2	clockwise counterclockwise reflection rotation translation	opposite
8.1.3	image angle of rotation center (of rotation) line of reflection	vertex
8.1.4	transformation sequence of transformations distance	clockwise counterclockwise reflect rotate translate
8.1.5	coordinate plane point segment coordinates x-axis y-axis	
8.1.6	polygon	angle of rotation center (of rotation) line of reflection
8.1.7	rigid transformation corresponding measurements preserve	reflection rotation translation measure point
8.1.8	midpoint	segment
8.1.9	vertical angles parallel intersect	distance
8.1.10		Image rigid transformation midpoint parallel
8.1.11	congruent perimeter area	
8.1.12		right angle x-axis y-axis area
8.1.13		corresponding
8.1.14	alternate interior angles transversal	vertical angles congruent supplementary angles
8.1.15	straight angle	
8.1.16		alternate interior angles transversal straight angle
8.1.17	tessellation symmetry	

- Coordinate Plane, Transformations of a Segment
- 8.1.7 Activity 2 Applet: Which One?
- 8.1.8 Digital Applet: A Pattern of Four Triangles
- 8.1.16 Digital Applet: Angle Plus Two

For whole course:

<https://accessim.org/6-8/grade-7/course-guide/required-materials?a=teacher>

Provide access as needed throughout the unit:

- Chart paper
- Geometry toolkits (Tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles. ruler and a protractor. Clear protractors with no holes and with radial lines printed on them are recommended.)
Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6–8, they are listed as separate Required Material.
- Math Community Chart
- Rulers
- Scissors
- Sticky notes
- Toothpicks, pencils, straws, or other objects
- Tracing paper or "patty paper"

Lesson	Materials to Gather	Materials to Copy
Lesson 1	<ul style="list-style-type: none"> Geometry toolkits: Lesson, Activity 1 Chart paper: Activity 1 Sticky notes: Activity 1 	<ul style="list-style-type: none"> Math Community Chart (1 copy for every 30 students): Activity 1 Triangle Square Dance Handout (1 copy for every 2 students): Activity 2
Lesson 2	Geometry toolkits: Activity 1, Activity 2	Move Cards (1 copy for every 3 students): Activity 3
Lesson 3	<ul style="list-style-type: none"> Math Community Chart: Activity 1 Sticky notes: Activity 1 Geometry toolkits: Activity 2 	
Lesson 4	Geometry toolkits: Activity 3	Make that Move Cards (1 copy for every 4 students): Activity 2
Lesson 5	Geometry toolkits: Activity 3	
Lesson 6	<ul style="list-style-type: none"> Geometry toolkits: Activity 1 Math Community Chart: Activity 1 	Transformation Information Cards (1 copy for every 2 students): Activity 2
Lesson 7	Geometry toolkits: Activity 3	
Lesson 8	<ul style="list-style-type: none"> Math Community Chart: Lesson, Activity 1 	

	<ul style="list-style-type: none"> Sticky notes: Activity 1 Geometry toolkits: Activity 2, Activity 3 	
Lesson 9	<ul style="list-style-type: none"> Tracing paper: Activity 1, Activity 2 Geometry toolkits: Activity 3 	
Lesson 10	Geometry toolkits: Activity 1, Activity 2, Activity 3, Activity 4	
Lesson 11	<ul style="list-style-type: none"> Geometry toolkits: Activity 1, Activity 2, Activity 3 Math Community Chart: Activity 1, Activity 2 	
Lesson 12	<ul style="list-style-type: none"> Geometry toolkits: Activity 1, Activity 2, Activity 3 Toothpicks, pencils, straws, or other objects: Activity 4 	
Lesson 13	<ul style="list-style-type: none"> Chart paper: Activity 1 Math Community Chart: Activity 1 Geometry toolkits: Activity 2, Activity 3, Activity 4 Rulers: Activity 3 	
Lesson 14	Geometry toolkits: Activity 1, Activity 2, Activity 3	
Lesson 15	<ul style="list-style-type: none"> Sticky notes: Activity 1 Geometry toolkits: Activity 2, Activity 3 Scissors: Activity 3 	Find All Three Cards (1 copy for every 15 students): Activity 2
Lesson 16	Geometry toolkits: Activity 2	
Lesson 17	Geometry toolkits: Activity 1, Activity 2, Activity 3	Deducing Angle Measures Handout (1 copy for every 2 students): Activity 1
Opportunities for Interdisciplinary Connections:		Anticipated misconceptions:
<ul style="list-style-type: none"> Entertainment/Media: Animation frames are used to describe how shapes move through space Art: Students create complex patterns and tessellations using sequences of rigid transformation 		<p>Congruence vs. Position: Students may think two figures are not congruent simply because they are in different positions or orientations</p> <p>See teacher's guide for specific misconceptions aligned to each lesson.</p>
Connections to Prior Units:		Connections to Future Units:
<p>Essential prior concepts to engage with this unit:</p> <ul style="list-style-type: none"> Plotting points on a coordinate plane Naming angles with the same vertex <p>Relevant Unit(s)/Lesson(s) to Review:</p> <ul style="list-style-type: none"> Grade 7 Unit 7: Angles, Triangles, and Prisms While there are connections to Grade 7 Unit 7, students should be able to engage with this unit without any additional review. If the initial Check Your Readiness assessment indicates students 		<p>Unit 2: Continues working with polygons on a plane, rigid transformations are built upon using dilations</p>

are not familiar with important concepts, consider proceeding through the unit without any modification continuing to monitor their readiness.

Differentiation through *Universal Design for Learning*

Engagement:

- Develop effort and persistence by connecting new concepts for curved shapes to success with polygons (Lesson 13, Activity 1)

Representation:

- Internalize comprehension through a physical demonstration using tracing paper to perform each type of transformation (Lesson 3, Activity 1 Launch)

Action & Expression:

- Internalize executive functions by inviting students to rephrase Info Gap directions in their own words (Lesson 6, Activity 1 Launch)

Related *CELP standards* aligned to Learning Targets:

Math Language Routines

The Illustrative Mathematics curriculum incorporates eight Mathematical Language Routines (MLRs) that support English Language Learners:

- MLR1: *Stronger and Clearer Each Time* - Students revise and refine their mathematical language through multiple drafts
- MLR2: *Collect and Display* - Students capture and organize language in visual displays
- MLR3: *Clarify, Critique, Correct* - Students analyze mathematical writing/talk
- MLR4: *Information Gap* - Students share information to solve problems
- MLR5: *Co-Craft Questions* - Students create and improve questions
- MLR6: *Three Reads* - Students analyze complex mathematical text
- MLR7: *Compare and Connect* - Students connect different mathematical representations
- MLR8: *Discussion Supports* - Students participate in mathematical discussions

In this unit:

- MLR2: Collect and Display (Lessons 1, 2, 3, 7, 8, 12, 14, 16)
- MLR8: Discussion Supports (Lessons 3, 7, 8, 10, 12, 13, 14, 15, 16, 17)
- MLR1: Stronger and Clearer Each Time (Lessons 8, 9, 10, 12, 13)

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes, such as describing, generalizing, and justifying. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Justify

- Whether or not rigid transformations could produce an image (Lesson 7).
- Whether or not shapes are congruent (Lesson 11).
- Whether or not polygons are congruent (Lesson 12).
- Whether or not ovals are congruent (Lesson 13).
- Whether or not triangles can be created from given angle measurements (Lesson 15).

Generalize

- About categories for movement (Lesson 2).
- About rotating line segments 180° (Lesson 8).
- About the relationship between vertical angles (Lesson 9).
- About transformations and congruence (Lesson 12).
- About corresponding segments and length (Lesson 13).
- About alternate interior angles (Lesson 14).
- About the sum of angles in a triangle (Lesson 16).

Describe

- Movements of figures (Lessons 1 and 2).
- Observations about transforming parallel lines (Lesson 9).
- Transformations using corresponding points, line segments, and angles (Lesson 10).
- Observations about angle measurements (Lesson 16).
- Transformations found in tessellations and in designs with rotational symmetry (Lesson 17).

Sentence Frames and Stems

Section A

- The coordinates of the image are ... after a _____ (transformation).

- When a figure is transformed using a _____ (transformation), the coordinates change by ...
- A _____ (transformation) of a figure on a grid looks like ...
- To draw an image of figure _____ using a _____ (transformation), first I _____, then I ...

Section B

- Between the original figure _____ and its image, side _____ corresponds to side _____ ...
- The image of line _____ is _____ to the original because ...
- I know the image was created using a _____ (transformation) because ...

Section C

- Figure _____ is/is not congruent to figure _____ because ...
- I can prove figure _____ is congruent to figure _____ by using the following transformations ...

Section D

- Given the parallel lines _____ and transversal _____, angles _____ and _____ are congruent because ...
- If I know the measure of angle _____ is _____ degrees, then angle _____ must be _____ degrees because ...
- The sum of the measures of angles _____ and _____ is _____ degrees because ...
- To find the angle measures in the triangle, first I _____, then I ...

Section E

- To create a tessellation, first I _____, then I ...
- _____ are shapes that work well in tessellations because ...
- To create a figure with rotational symmetry, I have to think about ...

Unit Outline

In this unit, students explore translations, rotations, and reflections of plane figures in order to understand the structure of rigid transformations. They use the properties of rigid transformations to formally define what it means for shapes to be congruent.

In earlier grades, students studied geometric measurement to find angle measures and side lengths of two-dimensional figures as well as applied area and perimeter formulas for polygons including rectangles, parallelograms, and triangles. In this unit, students build on this work as they identify corresponding congruent angles and side lengths of figures and their images under rigid transformations. In an upcoming unit, students will explore dilations and similar figures in the plane.

In the first section, students begin with an informal exploration of transformations in the plane, then increase their precision of language to describe translations, rotations, and reflections with formal descriptions, including coordinates (MP6).

Then students identify corresponding parts of figures and conclude that angles and distances are preserved under rigid transformations. Students use this property to reason about plane figures, including parallel lines cut by a transversal.

Students then learn the formal definition of "congruent" and use this definition to show that corresponding parts of congruent figures are also congruent. Finally, students apply their understanding of congruence and rigid motions to justify that the sum of the interior angles in a triangle must be .

The lessons in this unit ask students to work on geometric figures that are not set in a real-world context. Students have opportunities to engage in real-world applications in the culminating lesson of the unit where they examine tessellations and other symmetric designs.

In this unit, students reason about congruence and justify properties of figures using rigid transformations, but they are not required to create a formal proof. They will prove these and other geometric properties more formally in later courses.

Lesson Sequence	Learning Target(s)	Success Criteria/Assessment
Section A Rigid Transformations (Lessons 1-6)	Learning Target #1 Determine coordinates that represent the image of a polygon or line segment in the coordinate plane after a transformation.	Lesson 1 Moving in the Plane <ul style="list-style-type: none"> • I can describe how a figure moves and turns to get from one position to another.
	Learning Target #2 Draw and label the image of figures that result from translations, rotations, and reflections on a square or isometric grid.	Lesson 2 Naming the Moves <ul style="list-style-type: none"> • I can identify corresponding points before and after a transformation. • I know the difference between translations, rotations, and reflections.
	Learning Target #3 Explain the sequence of transformations that takes one figure to its image.	Lesson 3 Grid Moves <ul style="list-style-type: none"> • I can decide which type of transformations will work to move one figure to another. • I can use grids to carry out transformations of figures.
		Lesson 4 Making the Moves <ul style="list-style-type: none"> • I can use the terms "translation," "rotation," and "reflection" to precisely describe transformations. Lesson 5 Coordinate Moves

		<ul style="list-style-type: none"> I can apply transformations to points on a grid if I know their coordinates. <p>Lesson 6 Describing Transformations</p> <ul style="list-style-type: none"> I can apply transformations to a polygon on a grid if I know the coordinates of its vertices.
Checkpoint A	<p>Responding to Student Thinking</p> <ul style="list-style-type: none"> Problem 1: Points to Emphasize: If students struggle with identifying coordinates of a point or line segment after a translation or reflection, spend time addressing this in a future lesson. For example, in the Activity Synthesis of the activity referred to here, draw a set of axes on a grid and ask students to identify the coordinates of a few points on the figure and its image. <ul style="list-style-type: none"> Grade 8, Unit 1, Lesson 7, Activity 2 Sides and Angles Problem 2: Points to Emphasize: If students struggle with describing a translation, rotation, or reflection, spend time in a future lesson addressing the description of a sequence of transformations. For example, in the activity referred to here, ask students to describe the sequence of transformations needed to get from the first figure to the second. <ul style="list-style-type: none"> Grade 8, Unit 1, Lesson 8, Activity 1 Notice and Wonder: Building a Quadrilateral NOTE: Add a sentence to clarify for students that they do not need to be specific about the number of units the object needs to move. 	
<p>Section B Properties of Rigid Transformations (Lessons 7-10)</p>	<p>Learning Target #4 Draw and label rigid transformations of lines and parallel lines and explain the relationship between the original and its image under the transformation.</p> <p>Learning Target #5 Identify a rigid transformation using a drawing of a figure and its image.</p> <p>Learning Target #6 Identify side lengths and angles that have equivalent measurements in composite shapes and explain why they are equivalent.</p>	<p>Lesson 7 No Bending or Stretching</p> <ul style="list-style-type: none"> I can describe the effects of a rigid transformation on the lengths and angles in a polygon. <p>Lesson 8 Rotation Patterns</p> <ul style="list-style-type: none"> I can describe how to move one part of a figure to another using a rigid transformation. <p>Lesson 9 Moves in Parallel</p> <ul style="list-style-type: none"> I can describe the effects of a rigid transformation on a pair of parallel lines. If I have a pair of vertical angles and know the angle measure of one of them, I can find the angle measure of the other. <p>Lesson 10 Composing Figures</p> <ul style="list-style-type: none"> I can find missing side lengths or angle measures using properties of rigid transformations.
Checkpoint B	<p>Responding to Student Thinking</p> <ul style="list-style-type: none"> Problem 1: Points to Emphasize: If students struggle with constructing the image of a segment after a rotation, spend time in a future lesson addressing strategies for constructing transformations. For example, in the activity referred to here, select students to demonstrate or describe their strategy for rotating the congruent figures. <ul style="list-style-type: none"> Grade 8, Unit 1, Lesson 12, Activity 2 Congruent Pairs (Part 1) Problem 2: More Chances: Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons. 	
<p>Section C Congruence (Lessons 11-13)</p>	<p>Learning Target #7 Compare and contrast side lengths, angle measures, and other features of shapes using rigid transformations to explain why a shape is or is not congruent to another.</p> <p>Learning Target #8 Justify that two polygons on a grid are congruent using the definition of congruence in terms of rigid transformations.</p>	<p>Lesson 11 What Is the Same?</p> <ul style="list-style-type: none"> I can decide whether or not two figures are congruent using rigid transformations. <p>Lesson 12 Congruent Polygons</p> <ul style="list-style-type: none"> I can decide using rigid transformations whether or not two figures are congruent. <p>Lesson 13 Congruence</p> <ul style="list-style-type: none"> I can use distances between points to decide if two figures are congruent.
Checkpoint C	<p>Responding to Student Thinking</p>	

	<ul style="list-style-type: none"> ● Problem 1: Press Pause: By this point in the unit, there should be some student mastery of describing rigid transformations between congruent figures and justifying why two figures are not congruent. If most students struggle with these concepts, make time to examine related work in the section referred to here. The Course Guide provides additional ideas for revisiting earlier work. <ul style="list-style-type: none"> ○ Grade 8, Unit 1, Section C Congruence 	
Mid-Unit Assessment		
<p>Section D Angles in a Triangle (Lessons 14-16)</p>	<p>Learning Target #9 Calculate angle measures using alternate interior, vertical, and supplementary angles to solve problems.</p> <p>Learning Target #10 Generalize that the sum of angles in a triangle is 180 degrees using rigid transformations or the congruence of alternate interior angles of parallel lines cut by a transversal.</p>	<p>Lesson 14 Alternate Interior Angles</p> <ul style="list-style-type: none"> ● If I have two parallel lines cut by a transversal, I can identify alternate interior angles and use that to find missing angle measurements. <p>Lesson 15 Adding the Angles in a Triangle</p> <ul style="list-style-type: none"> ● I can determine whether three angles could make a triangle using their sum. <p>Lesson 16 Parallel Lines and the Angles in a Triangle</p> <ul style="list-style-type: none"> ● I can explain using pictures why the sum of the angles in any triangle is 180 degrees.
<p>Checkpoint D</p>	<p>Responding to Student Thinking</p> <ul style="list-style-type: none"> ● Problem 1: Press Pause: By this point in the unit, there should be some student mastery of solving problems using corresponding angles and properties of parallel lines. If students struggle with these concepts, make time to examine related work in the section referred to here. The Course Guide provides additional ideas for revisiting earlier work. <ul style="list-style-type: none"> ○ Grade 8, Unit 1, Section D Angles in a Triangle ● Problem 2: More Chances: Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons. 	
<p>Section E Let's Put it to Work (Lessons 17)</p>	<p>Learning Target #13 Create tessellations and designs with rotational symmetry using rigid transformations.</p> <p>Learning Target #14 Explain (orally and in writing) the rigid transformations needed to move a tessellation or design with rotational symmetry onto itself.</p>	<p>Lesson 17 Rotate and Tessellate</p> <ul style="list-style-type: none"> ● I can repeatedly use rigid transformations to make interesting repeating patterns of figures. ● I can use properties of angle sums to reason about how figures will fit together.
End of Unit Assessment		

Unit Title:

Unit 2: Dilation, Similarity, and Introducing Slope

Relevant Standards: Bold indicates priority

Lesson	Standards	Lesson	Standards
Lesson 1		Lesson 8	8.G.A 8.G.A.5
Lesson 2	8.G.A	Lesson 9	8.G.A 8.G.A.4
Lesson 3	8.G.A	Lesson 10	8.EE.B.6
Lesson 4	8.G.A 8.G.A.3	Lesson 11	8.EE.B.6
Lesson 5	8.G.A 8.G.A.3	Lesson 12	8.EE.B.6 8.G.A 8.G.A.3
Lesson 6	8.G.A.2 8.G.A.4	Lesson 13	8.G.A.5
Lesson 7	8.G.A.2 8.G.A.4		

Essential Question(s):

- How is a dilation different from a rigid transformation?
- What defines similarity between two geometric figures?
- How does triangle similarity explain the concept of slope?

Enduring Understanding(s):

- Unlike rigid transformations, a dilation changes the size of a figure based on a scale factor and a center of dilation while keeping the shape (angles) the same
- What defines similarity between two geometric figures
- The slope of a line is constant because any two "slope triangles" drawn on the same line are similar, meaning the ratio of vertical change to horizontal change is always the same

Demonstration of Learning:

CFA 1: Checkpoint A (after lesson 4)
 CFA 2: Checkpoint B (after lesson 9)
 CFA 3 Checkpoint C (after lesson 12)
 EoU: Assessment A (after lesson 12)

Pacing for Unit

11 Days (9 instructional Days 2 assessment Days)
 Lessons to Add/Review

- Combine 7.1.4 and 7.1.5; Activity 1 Three Quadrilaterals; Activity 3 Missing Figure, Factor or Copy

Lessons to Remove/Modify

- Combine 8.2.3 and 8.2.4. Focus on dilations with no grid and square grids.
- Remove 8.2.8 Activity 2. Can be done outside of class as an extension.
- Move to outside of class 8.2.13—culminating lesson incorporating work from the unit.

BPS Lesson Modifications:

- Lesson 1 SKIP
- Keep Lessons: 2,3,4,7,9,10,11,12
- Combine Lessons 5+6 (L5 Warmup, 6.1, 6.3)
- Lesson 8 SKIP

Family Overview

<https://accessim.org/6-8/grade-8/unit-2?a=family>

Integration of Technology:

- Desmos Online Graphing Calculator
- Pear Assessment (Edulastic)
- iM v.360 Digital Applets (see below)

Unit-specific Vocabulary:

Lesson	New Terminology	
	receptive	productive
8.2.1	scale factor scaled copy scaling	
8.2.2	dilation center of dilation dilate	
8.2.4		center of dilation

Aligned Unit Materials, Resources, and Technology

Digital Applets

- 8.2.2 Digital Applets: A Droplet on the Surface, Quadrilateral on a Circular Grid, A Quadrilateral and Concentric Circles
- 8.2.3 Digital Applets: Dilation Obstacle Course, Getting Perspective, Perspective Drawing
- 8.2.5 Digital Applet: Many Dilations of a Triangle
- 8.2.6 Digital Applet: Similarity Transformations

Provide access as needed throughout the unit:

		scale factor
8.2.6	similar	dilate
8.2.7		dilation
8.2.9	quotient	
8.2.10		slope slope triangle
8.2.11	similarity x-coordinate y-coordinate equation of a line	quotient
8.2.13	estimate approximate/approximately	

- Blank paper
- Dried linguine pasta (We specified linguine since it is flatter and less likely to roll around than spaghetti.)
- Geometry toolkits (ongoing)
- Long straightedge
- Math Community Chart
- Measuring tapes
- Protractors
- Clear protractors with no holes and with radial lines printed on them are recommended.
- Rulers
- Scissors
- Straightedges
- Tape
- Tracing paper
- Yardsticks

Lesson	Materials to Gather	Materials to Copy
Lesson 1	<ul style="list-style-type: none"> • Math Community Chart: Warm-up • Blank paper: Activity 1 • Long straightedge: Activity 1 • Scissors: Activity 1 • Rulers: Activity 2 	
Lesson 2	Straightedges: Activity 1, Activity 2, Activity 3	
Lesson 3	Geometry toolkits: Warm-up, Activity 1, Activity 2	
Lesson 4	Geometry toolkits: Warm-up, Activity 1	Matching Dilations on a Coordinate Plane Cards (1 copy for every 2 students): Activity 2
Lesson 5	<ul style="list-style-type: none"> • Math Community Chart: Warm-up • Geometry toolkits: Activity 1 	Dilations Cards (1 copy for every 2 students): Activity 1
Lesson 6	Geometry toolkits: Activity 1, Activity 2	Methods for Translations and Dilations Cards (1 copy for every 2 students): Activity 3
Lesson 7		Find Someone Similar Cards (1 copy for every 10 students): Activity 2
Lesson 8	<ul style="list-style-type: none"> • Blank paper: Activity 1 • Dried linguine pasta: Activity 1 • Geometry toolkits: Activity 1, Activity 2 • Protractors: Activity 1 • Rulers: Activity 1 • Tape: Activity 1 	Making Pasta Angles and Triangles Cards (1 copy for every 4 students): Activity 1
Lesson 9	Geometry toolkits: Warm-up	
Lesson 10	Geometry toolkits: Warm-up	

	<ul style="list-style-type: none"> Tracing paper: Warm-up Straightedges: Activity 2 	
Lesson 11	<ul style="list-style-type: none"> Rulers: Warm-up Straightedges: Warm-up 	
Lesson 12	<ul style="list-style-type: none"> Geometry toolkits: Warm-up, Activity 1, Activity 2 Straightedges: Activity 2 	
Lesson 13	<ul style="list-style-type: none"> Measuring tapes: Activity 2, Activity 3 Rulers: Activity 2, Activity 3 Yardsticks: Activity 2, Activity 3 	

Opportunities for Interdisciplinary Connections:	Anticipated misconceptions:
<ul style="list-style-type: none"> Optics: Dilation is introduced through the lens of projecting images. Physics: Students use triangle similarity and shadows to calculate the height of tall objects that are difficult to measure directly 	<p>Similarity Criteria: Students might believe triangles are similar if they share only one congruent angle</p> <p>See teacher's guide for specific misconceptions aligned to each lesson.</p>

Connections to Prior Units:	Connections to Future Units:
<p>Essential prior concepts to engage with this unit:</p> <ul style="list-style-type: none"> Identify scale factors. Identify scaled copies. Identify angle lengths and side measurements in a triangle. <p>Relevant Unit(s)/Lesson(s) to Review:</p> <ul style="list-style-type: none"> Grade 7 Unit 1 Scale Drawings Grade 7 Unit 7 Angles, Triangles and Prisms 	Triangle similarity explains the concept of slope, which is a primary focus of Unit 3: Linear Relationships

Differentiation through [Universal Design for Learning](#)

<p>Engagement:</p> <ul style="list-style-type: none"> Recruit interest by providing choices in which pair of triangles students will show are similar (Lesson 10, Activity 1) <p>Representation:</p> <ul style="list-style-type: none"> Use color coding and annotations to highlight connections between representations of similar triangles and ratios (Lesson 9, Activity 2) <p>Action & Expression:</p> <ul style="list-style-type: none"> Support expression and communication by providing blank or partially completed graphs with slope triangles (Lesson 12, Activity 2)

Related [CELP standards](#) aligned to Learning Targets:

<p>Math Language Routines</p> <p>The Illustrative Mathematics curriculum incorporates eight Mathematical Language Routines (MLRs) that support English Language Learners:</p> <p>MLR1: <i>Stronger and Clearer Each Time</i> - Students revise and refine their mathematical language through multiple drafts</p> <p>MLR2: <i>Collect and Display</i> - Students capture and organize language in visual displays</p> <p>MLR3: <i>Clarify, Critique, Correct</i> - Students analyze mathematical writing/talk</p> <p>MLR4: <i>Information Gap</i> - Students share information to solve problems</p> <p>MLR5: <i>Co-Craft Questions</i> - Students create and improve questions</p> <p>MLR6: <i>Three Reads</i> - Students analyze complex mathematical text</p> <p>MLR7: <i>Compare and Connect</i> - Students connect different mathematical representations</p> <p>MLR8: <i>Discussion Supports</i> - Students participate in mathematical discussions</p> <p>In this unit:</p> <ul style="list-style-type: none"> MLR8: Discussion Supports (Lessons 2, 4, 5, 9, 10) MLR7: Compare and Connect (Lessons 1, 3, 5, 8, 9, 10) MLR1: Stronger and Clearer Each Time (Lessons 6, 7, 10)
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Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes, such as describing, explaining, representing, and justifying. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Describe

- Observations about scaled rectangles (Lesson 1).
- Observations about dilated points, circles, and polygons (Lesson 2).
- Sequences of transformations (Lesson 6).
- Observations about side lengths in similar triangles (Lesson 9).

Explain

- How to apply dilations to find specific images (Lesson 5).
- How to determine whether triangles are congruent, similar, or neither (Lesson 8).
- Strategies for finding missing side lengths (Lesson 9).
- How to apply dilations to find specific images of points (Lesson 12).
- Reasoning for a conjecture (Lesson 13).

Represent

- Dilations using given scale factors and coordinates (Lesson 4).
- Figures using specific transformations (Lesson 6).
- Graphs of lines using equations (Lesson 12).

Sentence Frames and Stems

Section A

- To create a dilation of figure _____ with center _____, first I _____, then I ...
- I know the scale factor is _____ because ...
- I know the center of dilation is _____ because ...
- The image of figure _____ was dilated with a scale factor of _____ and a center _____. The coordinates of the image are ...
- I used _____ to dilate figure _____ because ...

Section B

- The ratio of side lengths _____ and _____ is equivalent to the ratio of the corresponding side lengths _____ and _____. This means ...
- The transformations ... move figure _____ to figure _____. The figures are similar because ...
- Figure _____ is similar to figure _____ because ...
- Triangle _____ is similar to triangle _____ because ...

Section C

- The slope of a line is a value that describes ...
- I know that line _____ has a slope of _____ because ...
- To draw a line with a slope of _____, first I _____, then I ...
- The point _____ is on the line because the equation for the line is _____ and ...

Section D

- If I know the shadow length of the _____ is _____, then the height of the lamppost is _____ because ...
- The triangles created by the objects and their shadows are similar because ...

Unit Outline

In this unit students learn what makes figures similar and justify claims of similarity. They are introduced to the slope of a line and use properties of similar triangles to write equations that can describe all points (x,y) on a given line.

In prior grades, students learned about the relationship between scale factors and scaled copies. Students expand on this in the first section where they learn about dilations as a new transformation that creates scaled copies.

In the next section, students connect dilations to earlier work with rigid transformations as they explain why two figures are similar by describing a sequence of translations, reflections, rotations, and dilations that take one figure to the other. They discover that angle measures in similar figures are preserved, which can be used to justify that two triangles are similar if they share two (or three) angle measures. Students also find that the quotients of corresponding side lengths in similar figures are equal. This along with the fact that side lengths in similar figures are all multiplied by the same scale factor allows students to calculate unknown lengths in similar figures.

In the following section, students use the similarity of slope triangles to understand why any two distinct points on a line determine the same slope. Using these same properties of similar triangles, students practice writing equations for a given line, though students are not expected at this time to write equations in the form $y=mx+b$.

The lessons in this unit ask students to work on geometric figures that are not set in a real-world context, as those tasks are sometimes contrived and hinder rather than help understanding. Students do have opportunities to tackle real-world applications in the culminating activity of the unit where students examine shadows cast by objects.

In this unit, several lesson plans suggest that each student have access to a geometry toolkit. Each toolkit contains tracing paper, graph paper, colored pencils, scissors, ruler, protractor, and an index card to use as a straightedge or to mark right angles, giving students opportunities to develop their abilities to select appropriate tools and use them strategically to solve problems. Note that even students in a digitally enhanced classroom should have access to such tools; apps and simulations should be considered additions to their toolkits, not replacements for physical tools.

Lesson Sequence	Learning Target(s)	Success Criteria/Assessment
<p>Section A Dilations (Lessons 1-5)</p>	<p>Learning Target #1 Create a dilation of a figure given a scale factor and center of dilation.</p> <p>Learning Target #2 Describe a figure on a coordinate grid and its image under a dilation, using coordinates to refer to points.</p> <p>Learning Target #3 Identify the center, scale factor, and image of a dilation.</p>	<p>Lesson 1 Projecting and Scaling</p> <ul style="list-style-type: none"> I can decide if one rectangle is a scaled copy of another rectangle. <p>Lesson 2 Circular Grid</p> <ul style="list-style-type: none"> I can apply dilations to figures on a circular grid when the center of dilation is the center of the grid. <p>Lesson 3 Dilations with No Grid</p> <ul style="list-style-type: none"> I can apply a dilation to a polygon using a ruler. <p>Lesson 4 Dilations on a Square Grid</p> <ul style="list-style-type: none"> I can apply dilations to figures on a square grid. <p>Lesson 5 More Dilations</p> <ul style="list-style-type: none"> I can apply dilations to polygons on a rectangular grid if I know the coordinates of the vertices and of the center of dilation.
<p>Checkpoint A</p>	<p>Responding to Student Thinking</p> <ul style="list-style-type: none"> Problem 1: Points to Emphasize: If most students struggle with finding the center of dilation, revisit how to describe dilations when showing that two figures are similar. For example, in the activity refer to here, emphasize the location of the center of the dilation. <ul style="list-style-type: none"> Grade 8, Unit 2, Lesson 6, Activity 2 Similarity Transformations (Part 1) Problem 2: Points to Emphasize: If most students struggle with drawing a dilation on the coordinate plane, revisit how to do this before starting the activity referred to here. <ul style="list-style-type: none"> Grade 8, Unit 2, Lesson 12, Activity 2 Dilations and Slope Triangles 	
<p>Section B Similarity (Lessons 6-9)</p>	<p>Learning Target #4 Calculate unknown side lengths in similar triangles using the ratios of side lengths within the triangles and the scale factor between similar triangles.</p> <p>Learning Target #5 Justify that two triangles are similar by finding a sequence of transformations that takes one triangle to the other or by checking that two pairs of corresponding angles are congruent.</p>	<p>Lesson 6 Similarity</p> <ul style="list-style-type: none"> I can apply a sequence of transformations to one figure to a similar figure. I can use a sequence of transformations to explain why two figures are similar. <p>Lesson 7 Similar Polygons</p> <ul style="list-style-type: none"> I can use angle measures and side lengths to conclude that two polygons are not similar. I know the relationship between angle measures and side lengths in similar polygons. <p>Lesson 8 Similar Triangles</p> <ul style="list-style-type: none"> I know how to decide if two triangles are similar just by looking at their angle measures. <p>Lesson 9 Side Length Quotients in Similar Triangles</p> <ul style="list-style-type: none"> I can decide if two triangles are similar by looking at quotients of lengths of corresponding sides. I can find missing side lengths in a pair of similar triangles using quotients of side lengths.
<p>Checkpoint B</p>	<p>Responding to Student Thinking</p> <ul style="list-style-type: none"> Problem 1: Points to Emphasize: If most students struggle with explaining why two figures are similar, before starting the activity referred to here, emphasize the different ways to show similarity. <ul style="list-style-type: none"> Grade 8, Unit 2, Lesson 10, Activity 2 Similar Triangles on the Same Line Problem 2: More chances: Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons. 	
<p>Section C Slope (Lessons 10-12)</p>	<p>Learning Target #6 Comprehend the term “slope” to mean a number that tells how steep a line is.</p>	<p>Lesson 10 Meet Slope</p> <ul style="list-style-type: none"> I can draw a line on a grid with a given slope. I can find the slope of a line on a grid. <p>Lesson 11 Writing Equations for Lines</p>

	<p>Learning Target #7 Create an equation relating the quotient of the vertical and horizontal side lengths of a slope triangle to the slope of a line and use it to justify whether a point (x,y) is on the line by verifying that the values of x and y satisfy the equation.</p>	<ul style="list-style-type: none"> I can decide whether a point is on a line by finding quotients of horizontal and vertical distances. <p>Lesson 12 Using Equations for Lines</p> <ul style="list-style-type: none"> I can find an equation for a line and use it to decide which points are on that line.
Checkpoint C	<p>Responding to Student Thinking</p> <ul style="list-style-type: none"> Problem 1, 2, & 3: More Chances: Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons. 	
<p>Section D Let's Put it to Work (Lessons 13)</p>	<p>Learning Target #8 Calculate the unknown heights of objects by using proportional reasoning and explain (orally) the solution method.</p> <p>Learning Target #9 Justify (orally) why the relationship between the height of objects and the length of their shadows cast by the sun is approximately proportional.</p>	<p>Lesson 13 The Shadow Knows</p> <ul style="list-style-type: none"> I can model a real-world context with similar triangles to find the height of an unknown object.
End of Unit Assessment		

Unit Title:

Unit 3: Linear Relationships

Relevant Standards: Bold indicates priority

Lesson	Standards	Lesson	Standards
Lesson 1	8.EE.B	Lesson 9	8.EE.B
Lesson 2	8.EE.B 8.EE.B.5	Lesson 10	8.EE.B 8.EE.B.6
Lesson 3	8.EE.B 8.EE.B.5	Lesson 11	8.EE.B 8.EE.B.6
Lesson 4	8.EE.B 8.EE.B.5	Lesson 12	8.EE.B 8.EE.B.6
Lesson 5	8.44.B	Lesson 13	8.EE.B 8.EE.C
Lesson 6	8.EE.B 8.EE.D.5	Lesson 14	8.EE.C 8.EE.C.8.a
Lesson 7	8.EE.B 8.EE.B.6	Lesson 15	8.EE.B.6 8.EE.C.8.a
Lesson 8	8.EE.B		

Essential Question(s):

- What defines a linear relationship and how does it differ from a proportional one?
- How do the slope and y-intercept of a line communicate the "story" of a real-world situation?
- How are decreasing relationships represented mathematically?

Enduring Understanding(s):

- A linear relationship is characterized by a constant rate of change between two quantities; it is only proportional if its graph passes through the origin (0, 0)
- In a linear context, the slope represents the rate of change (how much the dependent variable changes for every unit increase in the independent variable), and the y-intercept represents the initial or starting value
- A negative slope indicates a linear relationship where one quantity decreases at a constant rate in relation to a second quantity

Demonstration of Learning:

CFA 1: Checkpoint A (after lesson 4)
 CFA 2: Checkpoint B (after lesson 8)
 CFA 3: Checkpoint C (after lesson 12)
 CFA 4: Checkpoint D After lesson 14)
 EoU: Assessment A (after lesson 15)

Pacing for Unit

18 Days (16 instructional days, 1 review, 1 assess)
 Lessons to Add/Review

- Grade 7 Unit 2 Lesson 10: Activities 1 and 2 can be completed digitally. This lesson launches the introduction to graphing proportional relationships.
- Grade 7 Unit 2 Lesson 11 Activity 2: This activity can be completed digitally. This is essential prior knowledge for this unit.

Lessons to Remove/Modify

- Combine Lessons 2 and 3.
- Combine Lessons 6 and 7.
- Optional: Remove Lesson 14 if needed.

BPS Lesson Modifications:

- Keep As Is: 1,4-10,12-14
- Combine Lesson 2+3
- Add in 4 Practice Problems Days (After 4,8,12 and 14)

Family Overview

<https://accessim.org/6-8/grade-8/unit-3?a=family>

Integration of Technology:

- Desmos Online Graphing Calculator
- Pear Assessment (Edulastic)

Unit-specific Vocabulary:

Lesson	New Terminology	
	receptive	productive
8.3.1	represent scale label	constant of proportionality
8.3.2	equation	

Aligned Unit Materials, Resources, and Technology

Provide access as needed throughout the unit:

- Colored pencils
- Geometry toolkits (ongoing)
- Graduated cylinders
- Graph paper
- Math Community Chart
- Straightedges

8.3.3	rate of change	equation	<ul style="list-style-type: none"> Teacher's collection of objects Tools for creating a visual display Water 																																								
8.3.5	linear relationship constant rate rate of change	slope																																									
8.3.6	vertical intercept y-intercept																																										
8.3.7	initial (value or amount)	Constant rate																																									
8.3.8	relate																																										
8.3.9	horizontal intercept x-intercept																																										
8.3.10		rate of change vertical intercept y-intercept																																									
8.3.12	constraint	horizontal line vertical line																																									
8.3.13	solution to an equation with two variables variable combination set of solutions																																										
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Opportunities for Interdisciplinary Connections:			Anticipated misconceptions:																																								
Physical Science: The unit uses helium balloons and their buoyancy to explore rates of change and vertical intercepts			<p>Scale Misinterpretation: One graph may look steeper than another, but if the axes are scaled differently, they may represent the same relationship</p> <p>See teacher's guide for specific misconceptions aligned to each lesson.</p>																																								
Connections to Prior Units:			Connections to Future Units:																																								

Essential prior concepts to engage with this unit:

- This unit builds heavily on the work of Grade 7 Unit 2 and Grade 8 Unit 2. In Grade 7 Unit 2, students dive deep into different representations of proportional relationships and understand the constant of proportionality.
- At the end of Grade 8 Unit 2, students learned the terms “slope” and “slope triangle,” used the similarity of slope triangles on the same line to understand that any two distinct points on a line determine the same slope, and found an equation for a line with a positive slope and vertical intercept.

Direct preparation for linear systems in Unit 4. Establishes linear relationships as a special type of function explored in Unit 5

Relevant Unit(s)/Lesson(s) to Review:

- Grade 7 Unit 2: Introducing Proportional Relationships

Differentiation through *Universal Design for Learning***Engagement:**

- Promote self-regulation by having students self-assess their confidence that their description matches a chosen line (Lesson 15, Activity 1)

Representation:

- Develop language and symbols by displaying vocabulary like “vertical intercept” with diagrams (Lesson 6, Activity 1)

Action & Expression:

- Support organizational skills by chunking the task of marking bug positions into discrete steps (Lesson 1, Activity 2)

Related *CELP standards* aligned to Learning Targets:**Math Language Routines**

The Illustrative Mathematics curriculum incorporates eight Mathematical Language Routines (MLRs) that support English Language Learners:

- MLR1: *Stronger and Clearer Each Time* - Students revise and refine their mathematical language through multiple drafts
- MLR2: *Collect and Display* - Students capture and organize language in visual displays
- MLR3: *Clarify, Critique, Correct* - Students analyze mathematical writing/talk
- MLR4: *Information Gap* - Students share information to solve problems
- MLR5: *Co-Craft Questions* - Students create and improve questions
- MLR6: *Three Reads* - Students analyze complex mathematical text
- MLR7: *Compare and Connect* - Students connect different mathematical representations
- MLR8: *Discussion Supports* - Students participate in mathematical discussions

In this unit:

- MLR5: Co-Craft Questions (Lessons 1, 5, 8, 11, 15)
- MLR7: Compare and Connect (Lessons 1, 5, 8, 10, 12)
- MLR6: Three Reads (Lessons 2, 4, 6, 13, 14)

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes, such as representing, generalizing, and explaining. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Represent

- Situations involving proportional relationships (Lesson 1).
- Constants of proportionality in different ways (Lesson 3).
- Slope using expressions (Lesson 10).
- Linear relationships using graphs, tables, equations, and verbal descriptions (Lesson 5).
- Situations using negative slopes and slopes of zero (Lesson 9).
- Situations by graphing lines and writing equations (Lesson 13).
- Situations involving linear relationships (Lesson 15).

Generalize

- Categories for graphs (Lesson 2).
- About equations and linear relationships (Lesson 7).
- In order to make predictions about the slope of lines (Lesson 10).

Explain

- How to graph proportional relationships (Lesson 3).
- How to use a graph to determine information about a linear situation (Lessons 5 and 6).
- How to graph linear relationships (Lesson 10 and 11).
- How slope relates to changes in a situation (Lesson 11).

Sentence Frames and Stems

Section A

- The relationship between _____ and _____ is proportional because ...
- The equation _____ represents this proportional relationship because ...
- To create a graph of a proportional relationship, first I _____, then I ...
- I can substitute the value _____ into the equation _____ to find the value of _____.

Section B

- The graphs of the lines with equations _____ and _____ are/aren't parallel. I know because in the equations ...
- The slope of the line is _____ because ...
- The slope in this situation is _____ and represents _____.
- The y-intercept of the line is _____ and represents _____.
- The equation _____ represents this situation because ...

Section C

- I used _____ to represent the linear relationship between _____ and _____ because ...
- The slope in this situation is _____ and represents _____.
- To draw a line with a slope _____ and the point _____ on the line, first I _____, then I ...
- The equation _____ describes the line because ...

Section D

- The points ... are solutions to the equation _____ because ...
- The pair of values _____ do/do not satisfy the equation _____ because ...

Section E

- The equation _____ represents this situation because ...
- The account starts at _____ and increases/decreases by _____ per _____.

Unit Outline

Lesson Sequence	Learning Target(s)	Success Criteria/Assessment
Section A Proportional Relationships (Lessons 1-4)	Learning Target #1 Create an equation and a graph to represent proportional relationships, including an appropriate scale and axes. Learning Target #2 Interpret multiple representations of a proportional relationship in context.	Lesson 1 Understanding Proportional Relationships <ul style="list-style-type: none"> • I can graph a proportional relationship from a story. • I can use the constant of proportionality to compare the pace of different animals. Lesson 2 Graphs of Proportional Relationships <ul style="list-style-type: none"> • I can graph a proportional relationship from an equation. • I can tell when two graphs are of the same proportional relationship even if the scales are different. Lesson 3 Representing Proportional Relationships <ul style="list-style-type: none"> • I can scale and label coordinate axes in order to graph a proportional relationship. Lesson 4 Comparing Proportional Relationships <ul style="list-style-type: none"> • I can compare proportional relationships represented in different ways.
Checkpoint A	Responding to Student Thinking <ul style="list-style-type: none"> • Problem 1 & 2: More Chances: Students will have more opportunities to develop this understanding later lessons. There is no need to slow down or add additional work to review this concept at this time. 	
Section B Representing Linear Relationships (Lessons 5-8)	Learning Target #3 Create and compare graphs that represent linear relationships with the same rate of change but different initial values. Learning Target #4 Create an equation that represents a linear relationship. Learning Target #5 Interpret the slope and y-intercept of the graph of a line in context.	Lesson 5 Introduction to Linear Relationships <ul style="list-style-type: none"> • I can find the rate of change of a linear relationship by figuring out the slope of the line representing the relationship. Lesson 6 More Linear Relationships <ul style="list-style-type: none"> • I can interpret the vertical intercept of a graph of a real-world situation. • I can match graphs to the real-world situations they represent by identifying the slope and the vertical intercept. Lesson 7 Representations of Linear Relationships <ul style="list-style-type: none"> • I can use patterns to write a linear equation to represent a situation. • I can write an equation for the relationship between the total volume in a graduated cylinder and the number of objects added to the graduated cylinder. Lesson 8 Translating to $y=mx+b$

		<ul style="list-style-type: none"> I can explain where to find the slope and vertical intercept in both equation and its graph. I can write equations of lines using $y=mx+b$.
Checkpoint B	Responding to Student Thinking <ul style="list-style-type: none"> Problem 1: Points to Emphasize: If most students struggle with interpreting the slope of a line in context revisit the concept of rate of change. For example, in the section referred to here, discuss the similarity between graphing and interpreting negative rates of change and positive rates of change. <ul style="list-style-type: none"> Grade 8, Unit 3, Section C Finding Slopes 	
Section C Finding Slope (Lessons 9-12)	Learning Target #6 Create multiple representations of a linear relationship, including a graph, equation, and table. Learning Target #7 Interpret the slope of a non-increasing line in context.	Lesson 9 Slopes Don't Have to be Positive <ul style="list-style-type: none"> I can create a graph of a situation that has a negative slope. I can determine if a situation or a graph has a slope that is positive, negative, or zero and explain how I know. Lesson 10 Calculating Slope <ul style="list-style-type: none"> I can calculate positive and negative slopes given two points on the line. Lesson 11 Line Designs <ul style="list-style-type: none"> I can describe a line precisely enough that another student can draw it. Lesson 12: Equations of All Kinds of Line <ul style="list-style-type: none"> I can write equations of lines that have a positive or a negative slope. I can write equations of vertical and horizontal lines.
Checkpoint C	Responding to Student Thinking <ul style="list-style-type: none"> Problem 1: Press Pause: By this point in the unit, there should be some student mastery of interpreting slope in context. If most students struggle, make time to revisit related work in the section referred to here. See the Course Guide for ideas to help students re-engage with earlier work. 	
Section D Linear Equations (Lessons 13-14)	Learning Target #8 Determine pairs of values that satisfy or do not satisfy a linear relationship using an equation or graph.	Lesson 13 Solutions to Linear Equations <ul style="list-style-type: none"> I know that the graph of an equation is a visual representation of the solutions to the equation. I understand what the solution to an equation in two variables is. Lesson 14 More Solutions to Linear Equations <ul style="list-style-type: none"> I can find solutions (x,y) to linear equations given either the x - or y -value to start from.
Checkpoint D	Responding to Student Thinking <ul style="list-style-type: none"> Problem 1: Points To Emphasize: If most students struggle with determining whether or not a pair of values satisfies a linear relationship, revisit the concept of what it means to be a solution to an equation. For example, in the activity referred to here, emphasize how to check that the coordinates of the point of intersection of two lines satisfies the equations of both lines. <ul style="list-style-type: none"> Grade 8, Unit 4, Lesson 12, Activity 2 Passing on the Trail 	
Section E Let's Put it to Work (Lessons 15)	Learning Target #9 Describe (orally) limitations of a graphical representation of a situation based on real-world constraints on the quantities. Learning Target #10 Interpret the graph of a linear equation in context, including slope, intercept, and solution, in contexts using multiple representations of non-proportional linear relationships.	Lesson 15 Using Linear Relations to Solve Problems <ul style="list-style-type: none"> I can write linear equations to reason about real-world situations.
End of Unit Assessment		

Unit Title:**Unit 4: Linear Equations and Linear Systems****Relevant Standards: Bold indicates priority**

Lesson	Standards	Lesson	Standards
Lesson 1	7.EE.A	Lesson 9	8.EE.C 8.EE.C.7 8.EE.C.8
Lesson 2	8.EE.C	Lesson 10	8.EE.C 8.EE.C.8
Lesson 3	8.EE.C 8.EE.C.7	Lesson 11	8.EE.C.8
Lesson 4	8.EE.C 8.EE.C.7	Lesson 12	8.EE.C.8 8.EE.C.8.a 8.EE.C.8.b
Lesson 5	8.EE.C 8.EE.C.7	Lesson 13	8.EE.C.8 8.EE.C.8.a
Lesson 6	8.EE.C.7 8.EE.C.7.b	Lesson 14	8.EE.C.8
Lesson 7	8.EE.C.7.a	Lesson 15	8.EE.C.8 8.EE.C.8.b 8.EE.C.8.c
Lesson 8	8.EE.C.7.a	Lesson 16	8.EE.C.8.c

Essential Question(s):

- How can algebraic moves be used to solve complex linear equations?
- How can the structure of an equation indicate whether it has one, zero, or infinitely many solutions?
- What does it mean to find a solution to a system of equations?

Enduring Understanding(s):

- Equivalent equations are created by performing the same operations on both sides to maintain balance (adding, subtracting, multiplying, or dividing by the same nonzero value) or by using properties like the distributive property to combine like terms
- An equation has one solution if variable coefficients are different, no solution if variable coefficients are the same but constants differ, and infinitely many solutions if both sides are identical
- A solution to a system of equations is a pair of values (x,y) that makes both equations true at the same time; graphically, this corresponds to the point where the two lines intersect

Demonstration of Learning:

CFA 1: Checkpoint A (after lesson 6)
 CFA 2: Checkpoint B (after lesson 9)
 CFA 3 Checkpoint C (after lesson 15)
 EoU: Assessment A

Pacing for Unit

20 Days (15 Instructional Days, 3 Practice Days, 1 Review and 1 Assessment Day)
 Lessons to Add/Review

- Combine and add 7.6.7 and 7.6.8
- 7.6.9
- 7.6.10
- 7.6.11

Lessons to Remove/Modify

- Remove 8.4.1: This introductory lesson can be skipped because the inclusion of the lessons from grade 7 addresses the material in more depth.
- Remove 8.4.8: This lesson contains an optional activity.
- Remove 8.4.15: This lesson focuses on an Info Gap that is not necessary to understand the concepts of the unit.
- Remove 8.4.16: This lesson is an application of the concepts from the unit. It can be moved to outside of class.

BPS Lesson Modifications:

- Teach Lesson 1-15
- Add in 3 Practice Problems days (After 6,9 and 14)
- 1 Day of Review
- 1 Day of Assessment

Family Overview

<https://accessim.org/6-8/grade-8/unit-4?a=family>

Integration of Technology:

- Desmos Online Graphing Calculator

			<ul style="list-style-type: none"> • Pear Assessment (Edulastic) • iM v.360 Digital Applets (see below)
Unit-specific Vocabulary:			Aligned Unit Materials, Resources, and Technology
New Terminology			Digital Applets
Lesson	receptive	productive	<ul style="list-style-type: none"> • 8.4.2 Digital Applet: Keeping the Equation Balanced • 8.4.12 Digital applets: Systems of Equations • 8.4.13 Digital Applet: Different Types of Systems
8.4.1	representation equivalent expression		Provide access as needed throughout the unit: <ul style="list-style-type: none"> • Math Community Chart • Scissors • Straightedges • Tools for creating a visual display
8.4.2	expression		
8.4.3	solution to an equation distribute		
8.4.4	substitute	equation	
8.4.5	term like terms distributive property factor		
8.4.6		term like terms distribute common denominator	
8.4.7	no solution (only) one solution		
8.4.8	constant term coefficient linear equation infinitely many solutions	expression variable	
8.4.11	ordered pair		
8.4.12	system of equations solution to a system of equations		
8.4.13	substitution	substitute no solution (only) one solution infinitely many solutions	
8.4.14	algebraically		
8.4.15		system of equations substitution	
Opportunities for Interdisciplinary Connections:			Anticipated misconceptions:
Economics: The unit introduces "Supply and Demand" to provide a context for finding the intersection of two linear models			Unacceptable Division: Students may try to "divide each side by x," which can lead to losing a solution (like $x=0$) or creating an untrue statement ($2=6$) See teacher's guide for specific misconceptions aligned to each lesson.
Connections to Prior Units:			Connections to Future Units:
Essential prior concepts to engage with this unit: <ul style="list-style-type: none"> • In this unit, students build on their grade 6 and 7 work with equivalent expressions and equations with one occurrence of one variable, learning algebraic methods to solve linear equations with multiple occurrences of one variable. Students learn to use algebraic methods to solve systems of linear equations in two variables, building on their grade 7 and 8 work with graphs and equations of linear relationships. Understanding of linear relationships is, in turn, built on the understanding of proportional relationships developed in grade 7 that connected ratios and rates with lines and triangles. 			Solving systems of equations provides the algebraic foundation for mathematical modeling in Unit 6 and future courses.

Relevant Unit(s)/Lesson(s) to Review:

- Grade 7 Unit 6: Expressions, Equations, and Inequalities

Differentiation through *Universal Design for Learning***Engagement:**

- Develop effort and persistence by providing sentence frames to support peer collaboration during discussions on solution sets (Lesson 7, Activity 2 Launch)

Representation:

- Use color coding and annotations to highlight connections between connected variables through substitution (Lesson 14, Activity 1 Synthesis)

Action & Expression:

- Support organizational skills by chunking the card sort task into more manageable parts (Lesson 3, Activity 1 Launch)

Related *CELP standards* aligned to Learning Targets:**Math Language Routines**

The Illustrative Mathematics curriculum incorporates eight Mathematical Language Routines (MLRs) that support English Language Learners:

- MLR1: *Stronger and Clearer Each Time* - Students revise and refine their mathematical language through multiple drafts
- MLR2: *Collect and Display* - Students capture and organize language in visual displays
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- MLR4: *Information Gap* - Students share information to solve problems
- MLR5: *Co-Craft Questions* - Students create and improve questions
- MLR6: *Three Reads* - Students analyze complex mathematical text
- MLR7: *Compare and Connect* - Students connect different mathematical representations
- MLR8: *Discussion Supports* - Students participate in mathematical discussions

In this unit:

- MLR2: Collect and Display (Lessons 1, 3, 4, 7, 9)
- MLR7: Compare and Connect (Lessons 3, 4, 5, 8, 10)
- MLR8: Discussion Supports (Lessons 3, 5, 6, 7, 8, 10, 13, 14, 15, 16)

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes, such as critiquing, justifying, and generalizing. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Critique

- Strategies for writing equivalent equations (Lesson 1).
- Reasoning about maintaining balance in equations (Lesson 3).
- Solutions of linear equations (Lessons 4 and 5).
- Reasoning about structures of systems of equations (Lesson 14).
- Explanations of solutions (Lesson 16).

Justify

- Strategies for writing equivalent equations (Lessons 1 and 5).
- Predictions about maintaining balance (Lesson 2).
- Predictions about solutions of linear equations (Lesson 6).

Generalize

- About the structures of equations that have one, infinite, and no solutions (Lessons 7 and 8).
- About the structures of systems of equations (Lessons 14 and 15).

Sentence Frames and Stems**Section A**

- The moves ... show that the equation _____ is equivalent to the equation _____.
- To solve the equation _____, the first move was to _____ on both sides, then ...
- To find the unknown weight on the hanger diagram, first I _____, then I ...
- I know that equation _____ will have a positive/negative/zero solution because ...

Section B

- The equation _____ has _____ solution(s). I know this because ...
- The equation _____ represents ...
- The solution _____ to the equation _____ makes sense because ...

Section C

- The system of equations has _____ solution(s). I know this because ...
- The solution to the system of equations is _____ because ...
- This situation can be represented by the system of equations _____ because ...
- To solve the system of equations, first I _____, then I ...

Section D

- This situation can be represented by the system of equations _____ because ...
- To solve the system of equations, first I _____, then I ...
- The solution to the system of equations is _____ because ...

Unit Outline

In this unit, students work with writing equivalent equations and use reasoning to solve equations for a variable. Then students solve systems of linear equations using graphic and algebraic methods.

The unit begins with a focus on moves that can be done to write equivalent equations. At first, students use hanger diagrams as an intuitive representation of equality and represent their reasoning by labeling arrows that connect equivalent representations. With the reintroduction of negative values, students move away from hanger diagrams to algebraic equations and writing equivalent equations with the intention of solving for a variable.

Next, students examine the conditions under which equations could have 0, 1, or infinite solutions as a transition to thinking about similar situations involving systems of equations. Students finish the unit by examining systems of equations graphically and then finding solutions algebraically. They build on their understanding that the line representing an equation with 2 variables is made up of coordinate pairs that make the equation true. They find that the intersection of 2 lines is the point that makes both equations for the system true. Students also recognize when systems have no solution or infinite solutions based on the graphs and the slope and intercept.

Lesson Sequence	Learning Target(s)	Success Criteria/Assessment
<p>Section A Equivalent Equations (Lessons 1-6)</p>	<p>Learning Target #1 Write equivalent equations and describe the moves that are used.</p> <p>Learning Target #2 Write equivalent equations to solve linear equations in one variable.</p>	<p>Lesson 1 Tiling the Plane</p> <ul style="list-style-type: none"> • I can describe moves that change one equation into an equivalent equation. <p>Lesson 2 Keeping the Equation Balanced</p> <ul style="list-style-type: none"> • I can add or remove blocks from a hanger and keep the hanger balanced. • I can represent balanced hangers with equations. <p>Lesson 3 Balanced Moves</p> <ul style="list-style-type: none"> • I can add, subtract, multiply, or divide each side of an equation by same expression to get a new equation with the same solution. <p>Lesson 4 More Balanced Moves</p> <ul style="list-style-type: none"> • I can make sense of multiple ways to solve an equation. <p>Lesson 5 Solving Any Linear Equation</p> <ul style="list-style-type: none"> • I can solve an equation where the variable appears on both sides. <p>Lesson 6 Strategic Solving</p> <ul style="list-style-type: none"> • I can solve linear equations in one variable.
<p>Checkpoint A</p>	<p>Responding to Student Thinking</p> <ul style="list-style-type: none"> • Problem 1 & 2: More Chances: Students will have more opportunities to develop this understanding later lessons. There is no need to slow down or add additional work to review this concept at this time 	
<p>Section B Linear Equations in One Variable (Lessons 7-9)</p>	<p>Learning Target #3 Describe features of linear equations that have one solution, no solution, or many solutions.</p> <p>Learning Target #4 Interpret the solution of an equation in one variable in context.</p>	<p>Lesson 7 All, Same, or No Solutions</p> <ul style="list-style-type: none"> • I can determine whether an equation has no solutions, one solution, or infinitely many solutions. <p>Lesson 8 How Many Solutions?</p> <ul style="list-style-type: none"> • I can solve equations with different numbers of solutions. <p>Lesson 9 When Are They the Same?</p> <ul style="list-style-type: none"> • I can use an expression to find when two things, like height are the same in a real-world situation.
<p>Checkpoint B</p>	<p>Responding to Student Thinking</p> <ul style="list-style-type: none"> • Problem 1: Points to Emphasize: If students struggle to connect an equation to the number of solutions revisit the idea when a similar situation arises for systems of equations. For example, the inclusion of graphs to see why there are no solutions can help make the connection between coefficients and slope <ul style="list-style-type: none"> ○ Grade 8, Unit 4, Lesson 13 Solving Systems of Equations 	

	<ul style="list-style-type: none"> ● Problem 2: Points to Emphasize: If students struggle to solve the equation, make time for students to practice solving linear equations, especially those involving the distributive property. For example, encourage students to fully solve this practice problem: <ul style="list-style-type: none"> ○ Grade 8, Unit 4, Lesson 10, Practice Problem 4 	
Section C Systems of Linear Equations (Lessons 10-15)	Learning Target #5 Categorize systems of equations, including systems with infinitely many or no solutions, and calculate the solution for a system using a variety of strategies. Learning Target #6 Comprehend that solving a system of equations means finding values of the variables that make both equations true at the same time. Learning Target #7 Create a system of equations that represents a situation and interpret the solution in context.	Lesson 10 On or Off the Line? <ul style="list-style-type: none"> ● I can identify ordered pairs that are solutions to an equation. ● I can interpret ordered pairs that are solutions to an equation. Lesson 11 On Both of the Lines <ul style="list-style-type: none"> ● I can use graphs to find an ordered pair that two real-world situations have in common. Lesson 12 Systems of Equations <ul style="list-style-type: none"> ● I can explain the solution to a system of equations in a real-world context. ● I can explain what a system of equations is. ● I can make graphs to find an ordered pair that two real-world situations have in common. Lesson 13 <ul style="list-style-type: none"> ● I can graph a system of equations. ● I can solve systems of equations using algebra. Lesson 14 Solving More Systems <ul style="list-style-type: none"> ● I can use the structure of equations to help me figure out how many solutions a system of equations has. Lesson 15 Writing Systems of Equations <ul style="list-style-type: none"> ● I can write a system of equations from a real-world situation.
Checkpoint C	Responding to Student Thinking <ul style="list-style-type: none"> ● Problem 1: Press Pause: If students struggle to classify the number of solutions for a system, make time for students to revisit what a solution for a system means for each equation, in any situation given, and graphically. For example, revisit the situation with bug passing. Ask students to write a system for the situation and to interpret the intersection point in all 3 ways. Then, ask students what it might look like if the bugs never passed or were together the entire time. <ul style="list-style-type: none"> ○ Accelerated 7, Unit 5, Lesson 13, Activity 1 Bugs Passing in the Night ● Problem 2: Points to Emphasize: If students struggle to write a system of equations to represent a situation, move more slowly through the practice problems and activities related to the idea. For example, in this activity, ask students to identify the variables and constants involved and then to combine them into an equation for each person in the scenarios: <ul style="list-style-type: none"> ○ Grade 8, Unit 4, Lesson 16, Activity 2 Cycling, Fundraising, Working, and ___? 	
Section D Let's Put it to Work (Lessons 16)	Learning Target #8 Calculate the solution to a system of equations in context, and present (using words and other representations) the solution method. Learning Target #9 Create a system of equations to solve a problem in context. Learning Target #10 Critique (orally) peer solutions to a system of equations.	Lesson 16 <ul style="list-style-type: none"> ● I can use a system of equations to represent a real-world situation and answer questions about the situation.
End of Unit Assessment		

Unit Title:

Unit 5: Functions and Volume

Relevant Standards: Bold indicates priority

Lesson	Standards	Lesson	Standards
Lesson 1	8.F.A.1	Lesson 12	8.G.8.G,C,9C
Lesson 2	8.F.A.1	Lesson 13	8.G,C,9
Lesson 3	8.F.A 8.F.A.1	Lesson 14	8.G.C.9
Lesson 4	8.F.A.1 8.F.A.3	Lesson 15	8.G.C.9
Lesson 5	8.F.A.1 8.F.B.3	Lesson 16	8.G.C.9
Lesson 6	8.F.B.5	Lesson 17	8.F.A.1 8.F.A.3 8.F.B 8.G.C 8.G.C.9
Lesson 7	8.F.A.2 8.F.A.3	Lesson 18	8.F.A.3 8.F.B 8.G.C.9
Lesson 8	8.F.A.2 8.F.A.3 8.F.A.4	Lesson 19	8.G.C 8.G.C.9
Lesson 9	8.F.B.4	Lesson 20	8.G.C 8.G.C.9
Lesson 10	8.F.B 8.F.B.4 8.F.B.5	Lesson 21	8.G.C.9
Lesson 11	8.F.B 8.F.B.4		

Essential Question(s):	Enduring Understanding(s):
<ul style="list-style-type: none"> What is a mathematical function, and how is it represented? How are the volume formulas for cylinders, cones, and spheres related? How does changing a single dimension (like radius or height) affect the volume of a 3D object? 	<ul style="list-style-type: none"> A function is a rule that assigns to each allowable input exactly one output; it can be represented by verbal descriptions, tables, equations, or graphs The volume of a cylinder is $\pi r^2 h$, a cone with the same base and height has exactly $\frac{1}{3}$ that volume, and a sphere with the same radius and height ($h=2r$) has $\frac{2}{3}$ the volume of that cylinder Scaling the height of a cylinder or cone results in a proportional change in volume, but scaling the radius results in a non-proportional change because the radius is squared or cubed in volume formulas

Demonstration of Learning:	Pacing for Unit
CFA 1: Checkpoint A (after lesson 2) CFA 2: Checkpoint B (after lesson 7) CFA 3: Checkpoint C (after lesson 10) MoU: Assessment A CFA 4: Checkpoint D (after lesson 17) CFA 5: Checkpoint E (after lesson 21) EoU: Assessment A	19 Days Lessons to Add/Review <ul style="list-style-type: none"> This unit should provide sufficient background to engage with the material on its own. If additional resources are needed to increase familiarity with proportional relationships, Grade 7 Unit 2 may be useful to revisit. Lessons to Remove/Modify <ul style="list-style-type: none"> Remove 8.5.6 - Application lesson and activities that can be moved to outside of class for more practice Remove 8.5.17 - optional lesson for the unit Remove 8.5.18 - optional lesson for the unit Move 8.5.22 to outside of class

Family Overview	Integration of Technology:
https://accessim.org/6-8/grade-8/unit-5?a=family	<ul style="list-style-type: none"> Desmos Online Graphing Calculator Pear Assessment (Edulastic) iM v.360 Digital Applets (see below)

Unit-specific Vocabulary:	Aligned Unit Materials, Resources, and Technology											
<table border="1"> <thead> <tr> <th rowspan="2">Lesson</th> <th colspan="2">New Terminology</th> </tr> <tr> <th>receptive</th> <th>productive</th> </tr> </thead> <tbody> <tr> <td>8.5.1</td> <td>input output</td> <td></td> </tr> <tr> <td>8.5.2</td> <td>function</td> <td>input</td> </tr> </tbody> </table>	Lesson	New Terminology		receptive	productive	8.5.1	input output		8.5.2	function	input	Digital Applets <ul style="list-style-type: none"> 8.5.1 Digital Applet: Guess My Rule 8.5.6 Digital Applet: Sketching a Story about a Boy and a Bike 8.5.9 Digital Applet: Candlelight & Shadows 8.5.11 Digital Applet: Height and Volume
Lesson		New Terminology										
	receptive	productive										
8.5.1	input output											
8.5.2	function	input										

		output depends on	Provide access as needed throughout the unit: <ul style="list-style-type: none"> • Colored pencils • Graduated cylinders • Math Community Chart • Spherical objects • Straightedges • Tools for creating a visual display
8.5.3	independent variable dependent variable radius		
8.5.5	prediction		
8.5.7	volume cube		
8.5.8	functional relationship linear function	function	
8.5.9	mathematical model	prediction	
8.5.10	piecewise linear function	linear function constant rate	
8.5.11	cylinder three-dimensional		
8.5.12	cone sphere dimension	cylinder cube cubic centimeter rectangular prism	
8.5.13	base (of a cylinder or cone) approximation for π		
8.5.14		radius base (of a cylinder or cone)	
8.5.16		cone	
8.5.19	hemisphere		
8.5.20		sphere	
8.5.21	spherical	volume	
Opportunities for Interdisciplinary Connections:			Anticipated misconceptions:
Music/Acoustics: The relationship between drum size and volume provides a context for exploring round 3D object			Radius in Volume: Students often forget to square or cube the radius in volume formulas, or they use the diameter as the radius See teacher's guide for specific misconceptions aligned to each lesson.
Connections to Prior Units:			Connections to Future Units:
Essential prior concepts to engage with this unit: <ul style="list-style-type: none"> • Prior concepts that will allow students to engage with this unit stem for grades 6 and 7. These concepts are: <ul style="list-style-type: none"> ○ identifying independent and dependent variables ○ constant of proportionality ○ rate of change ○ slope Relevant Unit(s)/Lesson(s) to Review: <ul style="list-style-type: none"> • Grade 7 Unit 2: Introducing Proportional Relationships 			Content serves as a lead-in to nonlinear relationships and advanced function study in high school.
Differentiation through Universal Design for Learning			

Engagement:

- Recruit interest by inviting students to generate a list of personal examples with negative rates of change (Lesson 10, Activity 3 Synth)

Representation:

- Internalize comprehension by providing a range of different-sized physical containers for height/volume testing (Lesson 11, Activity 2 Launch)

Action & Expression:

- Provide access to digital applets to fill digital cylinders and collect data precisely (Lesson 11, Activity 1 Launch)

Related **CELP standards** aligned to Learning Targets:

Math Language Routines

The Illustrative Mathematics curriculum incorporates eight Mathematical Language Routines (MLRs) that support English Language Learners:

MLR1: *Stronger and Clearer Each Time* - Students revise and refine their mathematical language through multiple drafts

MLR2: *Collect and Display* - Students capture and organize language in visual displays

MLR3: *Clarify, Critique, Correct* - Students analyze mathematical writing/talk

MLR4: *Information Gap* - Students share information to solve problems

MLR5: *Co-Craft Questions* - Students create and improve questions

MLR6: *Three Reads* - Students analyze complex mathematical text

MLR7: *Compare and Connect* - Students connect different mathematical representations

MLR8: *Discussion Supports* - Students participate in mathematical discussions

In this unit:

- MLR1: Stronger and Clearer Each Time (Lessons 2, 8, 11, 14, 17, 18)
- MLR5: Co-Craft Questions (Lessons 4, 7, 10, 12, 13, 16)
- MLR7: Compare and Connect (Lessons 1, 6, 7, 8, 12)

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes such as generalizing, justifying, and comparing. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Generalize

- about what happens to inputs for each rule (Lesson 1)
- about dimensions of cylinders (Lesson 14)
- about the relationship between the volumes of cylinders and cones (Lesson 15)
- about dimensions of cones (Lesson 16)
- about volumes of spheres, cones, and cylinders as functions of their radii (Lesson 21)

Justify

- claims about what can be determined from given information (Lesson 2)
- claims about volumes of cubes and spheres based on graphs (Lesson 7)
- claims about approximately linear relationships (Lesson 10)
- reasoning about the volumes of spheres and cones (Lesson 21)

Compare

- different representations of functions (Lesson 3)
- features of graphs, equations, and situations (Lesson 4)
- features of a situation with features of a graph (Lesson 6)
- temperatures shown on a graph with different temperatures given in a table (Lesson 7)
- the volumes of cones with the volumes of cylinders (Lesson 16)
- methods for finding and approximating the volume of a sphere as function of its radius (Lesson 20)

Sentence Frames and Stems

Section A

- The rule between the input and the output of the function is ____ because ...
- If the input of the function is ____, then the output must be ____ because the rule is ...

Section B

- The equation _____ represents a function because ...
- The output of the function when the input is ____ is ____ because ...
- To draw the graph of a function that represents ____, first I ____, then I ...
- The graph represents a function because ...

Section C

- In the piecewise function, the rate of change from ____ to ____ is ____.

- The rate of change from ____ to ____ means _____.
- From ____ to ____ on the graph, ____ increases/decreases, which means ...
- The linear equation ____ can model the function because ...

Section D

- I know the volume of the cylinder/cone is ____, and the radius of the base is ____, so the height must be ____ because ...
- The volume of the cylinder is ____ because ...
- The volume of the cone is ____ because ...
- As the radius of the base increases by ____, the volume of the cylinder/cone increases by ____ because ...

Section E

- When the radius of the ____ is changed by ____, the volume ...
- The volume of the hemisphere is ____ because ...
- The volume of the sphere is ____ because ...

Section F

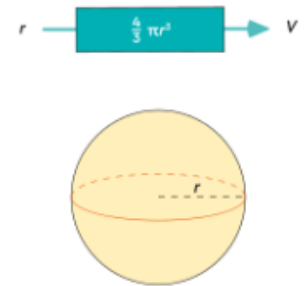
- The volume of a sphere with radius ____ is ____ the volume of a sphere with radius ____ because ...
- The ____ can hold the highest volume of water because ...

Unit Outline

In this unit, students are introduced to the concept of a function as a relationship between “inputs” and “outputs” in which each allowable input determines exactly one output.

In the first three sections of the unit, students work with relationships that are familiar from previous grades or units (perimeter formulas, proportional relationships, linear relationships), expressing them as functions. They study the different ways functions can be represented, making connections between the representations and interpreting what they mean in context. Linear functions are a focus of the third section, and students will continue to work with linear functions in a later unit to model data. The use of function notation is left for a future course.

r	V
0	0
2	$\frac{32}{3}\pi$
6	288π
r	$\frac{4}{3}\pi r^3$



In the remaining three sections of the unit, students build on their knowledge of the formula for the volume of a right rectangular prism from grade 7, learning formulas for volumes of cylinders, cones, and spheres. Students express functional relationships described by these formulas as equations, focusing on situations involving proportional relationships. They use these relationships to reason about how the volume of a figure changes as one of its dimensions changes, transforming algebraic expressions to get the information they need. In future courses, students will continue this thinking as they study nonlinear relationships and question how, for example, the volume of a sphere changes as the radius increases.

Lesson Sequence	Learning Target(s)	Success Criteria/Assessment
Section A Inputs and Outputs (Lessons 1-2)	Learning Target #1 Comprehend the structure of a function as having one and only one output for each allowable input.	Lesson 1 Inputs and Outputs <ul style="list-style-type: none"> • I can write rules when I know input-output pairs. • I know how an input-output diagram represents a rule. Lesson 2 Introduction to Functions <ul style="list-style-type: none"> • I know that a function is a rule with exactly one output for each allowable input. • I know that if a rule has exactly one output for each allowable input then the output depends on the input.
Checkpoint A	<i>Responding to Student Thinking</i> More Chances: Students will have more opportunities to develop this understanding in later lessons. There is need to slow down or add additional work to review this concept at this time.	
Section B Representing and Interpreting Functions (Lessons 3-7)	Learning Target #2 Draw the graph of a function that represents a context, and explain which quantity is a function of which. Learning Target #3 Interpret multiple representations of functions, including graphs, tables, and equations, and explain how to find information in each type of representation.	Lesson 3 Equations for Functions <ul style="list-style-type: none"> • I can find the output of a function when I know the input. • I can name the independent and dependent variables for a given function and represent the function with an equation. Lesson 4 Tables, Equations and Graphs of Functions <ul style="list-style-type: none"> • I can identify graphs that do, and do not, represent functions. • I can use a graph of a function to find the output for a given input and to find the input(s) for a given output. Lesson 5 More graphs of Functions <ul style="list-style-type: none"> • I can explain the story told by the graph of a function. Lesson 6 Even More Graphs of Functions <ul style="list-style-type: none"> • I can draw the graph of a function that represents a real-world situation. Lesson 7 Connecting Representations of Functions

		<ul style="list-style-type: none"> I can compare inputs and outputs of functions that are represented in different ways.
Checkpoint B	<p>Responding to Student Thinking</p> <ul style="list-style-type: none"> Problem 1: More Chances: Students will have more opportunities to develop this understanding in later lessons. There is no need to slow down or add additional work to review this concept at this time. Problem 2: Points To Emphasize: If most students struggle working with drawing a graph of a function that represents a context, focus on this connection as opportunities arise. For example, in the Lesson Synthesis of the lesson referred to here, invite the students that share to narrate how they interpreted the graph their partner made as they point to the shape of the container they drew from the graph. <ul style="list-style-type: none"> Grade 8, Unit 5, Lesson 11 Filling Containers 	
Section C Linear Functions and Rates of Change (Lessons 8-10)	<p>Learning Target #4 Calculate the different rates of change of a piecewise linear function using a graph, and interpret the rates of change in context.</p> <p>Learning Target #5 Comprehend that any linear function can be represented by an equation in the form $y=mx+b$, where m and b are the rate of change and initial value of the function, respectively.</p>	<p>Lesson 8 Linear Functions</p> <ul style="list-style-type: none"> I can determine whether a function is increasing or decreasing based on whether its rate of change is positive or negative. I can explain in my own words how the graph of a linear function relates to its rate of change and initial value. <p>Lesson 9 Linear Models</p> <ul style="list-style-type: none"> I can decide when a linear function is a good model for data and when it is not. I can use data points to model a linear function. <p>Lesson 10 Piecewise Linear Functions</p> <ul style="list-style-type: none"> I can create graphs of nonlinear functions with pieces of linear functions.
Checkpoint C	<p>Responding to Student Thinking</p> <ul style="list-style-type: none"> Problem 1: More Chances: Students will have more opportunities to develop this understanding in later units. There is no need to slow down or add additional work to review this concept at this time. Problem 2: Points to Emphasize: By this point in the unit, there should be some student mastery of writing linear equations and interpreting linear graphs. If most students struggle, make time to revisit this Cool-down. For example, after the Warm-up of the lesson referred to here, invite 2-3 students to share their equations for Program B, graphing each for all to see. <ul style="list-style-type: none"> Grade 8, Unit 5, Lesson 11 Filling Containers 	
Mid-Unit Assessment		
Section D Cylinders and Cones (Lessons 11-16)	<p>Learning Target #6 Calculate the value of one dimension of a cylinder or cone, and explain the reasoning.</p> <p>Learning Target #7 Calculate the volume of a cylinder or cone.</p>	<p>Lesson 11 Filling Containers</p> <ul style="list-style-type: none"> I can collect data about a function and represent it as a graph. I can describe the graph of a function in words. <p>Lesson 12 How Much Will Fit?</p> <ul style="list-style-type: none"> I know that volume is the amount of space contained inside a three-dimensional figure. I recognize the following three-dimensional shapes: cylinder, cone, rectangular prism, and sphere. <p>Lesson 13 The Volume of a Cylinder</p> <ul style="list-style-type: none"> I can find the volume of a cylinder in mathematical and real-world situations. I know the formula for the volume of a cylinder. <p>Lesson 14 Finding Cylinder Dimensions</p> <ul style="list-style-type: none"> I can find missing information about a cylinder if I know its volume and some other information. <p>Lesson 15 The Volume of a Cone</p> <ul style="list-style-type: none"> I can find the volume of a cone in mathematical and real-world situations. I know the formula for the volume of a cone. <p>Lesson 16 Finding Cone Dimensions</p> <ul style="list-style-type: none"> I can find missing information about a cone if I know its volume and some other information.
Checkpoint D	<p>Responding to Student Thinking</p> <ul style="list-style-type: none"> Problem 1: Points to Emphasize: If most students struggle with using the formula for volume of a cylinder, revisit this skill throughout the next section. For example, during the Activity Synthesis of the activity referred to here, invite 1-3 students to share how they calculated the volume of the cylinder. <ul style="list-style-type: none"> Grade 8, Unit 5, Lesson 19, Activity 3 Estimating Hemispheres Problem 2: Points to Emphasize: If most students struggle with calculating the radius of the cone, focus on this type of solving as opportunities arise. For example, in the Warm-up referred to here, invite students to calculate the radius of the cone if the volume was 9π or 72π ($r=3$ or $r=6$). 	

<ul style="list-style-type: none"> ○ Grade 8, Unit 5, Lesson 19, Activity 1 Notice and Wonder: Two Shapes 		
<p>Section E Dimensions and Spheres (Lessons 17-21)</p>	<p>Learning Target #8 Calculate the volume of a sphere.</p> <p>Learning Target #9 Solve problems involving cones, cylinders, and spheres.</p>	<p>Lesson 17 Scaling One Dimension</p> <ul style="list-style-type: none"> ● I can create a graph of the relationship between volume and height for all cylinders and cones with a fixed radius. ● I can explain in my own words why changing the height by a scale factor changes the volume by the same scale factor. <p>Lesson 18 Scaling Two Dimensions</p> <ul style="list-style-type: none"> ● I can create a graph representing the relationship between volume and radius for all cones (or cylinders) with a fixed height. ● I can explain in my own words why changing the radius by a scale factor changes the volume by the scale factor squared. <p>Lesson 19 Estimating a Hemisphere</p> <ul style="list-style-type: none"> ● I can estimate the volume of a hemisphere by calculating the volume of a shape I know is larger and the volume of a shape I know is smaller. <p>Lesson 20 Volume of a Sphere</p> <ul style="list-style-type: none"> ● I can find the volume of a sphere when I know the radius. <p>Lesson 21 Cylinders, Spheres and Cones</p> <ul style="list-style-type: none"> ● I can find the radius of a sphere if I know its volume. ● I can solve mathematical and real-world problems about the volume of cylinders, cones, and spheres.
<p>Checkpoint E</p>	<p>Responding to Student Thinking</p> <ul style="list-style-type: none"> ● Problem 1: Press Pause: If most students struggle with using the formula for volume, make time to do some or all of this optional lesson: <ul style="list-style-type: none"> ○ Grade 8, Unit 5, Lesson 22 Volume as a Function of ... ● Problem 2: Press Pause: If most students struggle with using formulas to order the four shapes by volume, make time to do some or all of the optional lesson referred to here: <ul style="list-style-type: none"> ○ Grade 8, Unit 5, Lesson 22 Volume as a Function of ... 	
<p>Section F Let's Put it To Work (Lesson 22)</p>	<p>Learning Target #10 Describe (orally) how a change in the radius of a sphere affects the volume.</p> <p>Learning Target #11 Interpret (orally and in writing) functions that represent the volumes of spheres, cones, and cylinders, using different representations.</p>	<p>Lesson 22 Volume of a Function of...</p> <ul style="list-style-type: none"> ● I can compare functions about volume represented in different ways.
<p>End of Unit Assessment</p>		

Unit Title:

Unit 6: Associations in Data

Relevant Standards: Bold indicates priority

Lesson	Standards	Lesson	Standards
Lesson 1	8.SP.A.1	Lesson 7	8.SP.A.1
Lesson 2	8.SP.A.1	Lesson 8	8.SP.A.1 8.SP.A.2 8.SP.A.3
Lesson 3	8.SP.A.1 8.SP.A.3	Lesson 9	8.SP.A.4
Lesson 4	8.SP.A.1 8.SP.A.2	Lesson 10	8.SP.A.4
Lesson 5	8.SP.A.1 8.SP.A.2	Lesson 11	8.SP.A
Lesson 6	8.SP.A.1 8.SP.A.2 8.SP.A.3		

Essential Question(s):

- How can scatter plots be used to identify associations between two numerical variables?
- What is the purpose of fitting a line to data in a scatter plot?
- How do we identify associations in categorical data?

Enduring Understanding(s):

- Scatter plots show the relationship between two variables; associations can be described as positive (both increase), negative (one increases as the other decreases), linear, or non-linear
- A fitted line, or linear model, is used to describe the general trend of the data and make predictions about the dependent variable based on given values of the independent variable
- Associations in categorical data are found by comparing relative frequencies in two-way tables or segmented bar graphs; an association exists if frequencies within categories are significantly different

Demonstration of Learning:

CFA 1: Checkpoint A (after lesson 3)
 CFA 2: Checkpoint B (after lesson 8)
 CFA 3 Checkpoint C (after lesson 10)
 EoU: Assessment A

Pacing for Unit

12 Days
 Lessons to Add/Review
 • None
 Lessons to Remove/Modify
 • Move to outside of class 8.6.11, culminating lesson incorporating work from the unit

Family Overview

<https://accessim.org/6-8/grade-8/unit-6?a=family>

Integration of Technology:

- Desmos Online Graphing Calculator
- Pear Assessment (Edulastic)
- iM v.360 Digital Applets (see below)

Unit-specific Vocabulary:

Lesson	New Terminology <i>Bold = brand new, use glossary</i>	
	receptive	productive
8.6.1	scatter plot	
8.6.2	data display attribute	numerical data categorical data
8.6.4	outlier predict overpredict underpredict linear model	
8.6.5	positive association negative association linear association	
8.6.6	nonlinear association no association fitted line	

Aligned Unit Materials, Resources, and Technology

- Digital Applets
- 8.6.2 Digital Applet: Scatter Plots
- Provide access as needed throughout the unit:
- Books
 - Colored pencils
 - Copies of blackline masters
 - Dried linguine pasta (We specified linguine since it is flatter and less likely to roll around than spaghetti.)
 - Math Community Chart
 - Measuring tapes
 - Pre-printed cards, cut from copies of the blackline master
 - Rulers marked with centimeters
 - Scale 1A digital scale that can show measurements in grams, kilograms, ounces, or pounds)
 - Stopwatches
 - Straightedges

8.6.7	cluster		Lesson	Materials to Gather	Materials to Copy
8.6.8		independent variable dependent variable positive association negative association linear association	1	Copies of blackline masters: Activity 2	Tables and Their Scatter Plots Handout (1 copy for every 2 students): Activity 2
8.6.9	segmented bar graph relative frequency two-way (frequency) table		2	<ul style="list-style-type: none"> Books: Activity 1 Measuring tapes: Activity 1 Rulers marked with centimeters: Activity 1 Scale: Activity 1 	
8.6.11		scatter plot	5	<ul style="list-style-type: none"> Dried linguine pasta: Activity 1 Straightedges: Activity 1 	
			7	Pre-printed cards, cut from copies of the blackline master: Activity 1	Scatterplot City Cards (1 copy for every 1 student): Activity 1
			8	<ul style="list-style-type: none"> Dried linguine pasta: Activity 1 Straightedges: Activity 1 	
			9	Pre-printed cards, cut from copies of the blackline master: Activity 1	Matching Representations Cards (1 copy for every 2 students): Activity 1
			10	<ul style="list-style-type: none"> Colored pencils: Activity 2 Straightedges: Activity 2 	
			11	Stopwatches: Activity 1	
Opportunities for Interdisciplinary Connections:			Anticipated misconceptions:		
<ul style="list-style-type: none"> Geology: Students analyze data from Old Faithful to look for patterns in geyser eruption times. Zoology: Scatter plots model the relationship between a dog's stride length and its speed 			<p>Relative Frequency Errors: Students may divide by the table total instead of the row or column total when looking for associations</p> <p>See teacher's guide for specific misconceptions aligned to each lesson.</p>		
Connections to Prior Units:			Connections to Future Units:		
<p>Essential prior concepts to engage with this unit:</p> <ul style="list-style-type: none"> Interpreting dot plots, histograms, and box plots Identifying independent and dependent variables Analyzing positive and negative slopes <p>Relevant Unit(s)/Lesson(s) to Review:</p> <ul style="list-style-type: none"> Grade 6 Unit 8: Data Sets and Distributions Grade 7 Unit 8: Probability and Sampling 			Lays the foundation for statistics and bivariate data analysis in high school courses.		
Differentiation through <i>Universal Design for Learning</i>					
Engagement:					
<ul style="list-style-type: none"> Develop effort and persistence by providing sentence frames for comparing different scatter plots (Lesson 7, Activity 2 Launch) 					
Representation:					
<ul style="list-style-type: none"> Internalize comprehension through the use of multiple examples and non-examples of non-similar polygons (Lesson 7, Activity 2 Launch) 					
Action & Expression:					
<ul style="list-style-type: none"> Support organizational skills by chunking data tasks to check for understanding within the first few minutes (Lesson 1, Activity 2 Launch) 					
Related <i>CELP standards</i> aligned to Learning Targets:					
Math Language Routines					
The Illustrative Mathematics curriculum incorporates eight Mathematical Language Routines (MLRs) that support English					

Language Learners:

- MLR1: *Stronger and Clearer Each Time* - Students revise and refine their mathematical language through multiple drafts
- MLR2: *Collect and Display* - Students capture and organize language in visual displays
- MLR3: *Clarify, Critique, Correct* - Students analyze mathematical writing/talk
- MLR4: *Information Gap* - Students share information to solve problems
- MLR5: *Co-Craft Questions* - Students create and improve questions
- MLR6: *Three Reads* - Students analyze complex mathematical text
- MLR7: *Compare and Connect* - Students connect different mathematical representations
- MLR8: *Discussion Supports* - Students participate in mathematical discussions

In this unit:

- MLR2: Collect and Display (Lessons 7, 9, 10)
- MLR7: Compare and Connect (Lessons 2, 6, 7, 8, 9)
- MLR8: Discussion Supports (Lessons 1, 4, 5, 8, 10)

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes such as explaining, representing, and interpreting. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Explain

- how to estimate using available data (Lesson 1)
- how to use tables and scatter plots to make estimates and predictions (Lesson 3)
- the meaning of slope for a situation (Lesson 6)
- how to use lines to show associations, identify outliers, and answer questions (Lesson 8)

Represent

- data in organized ways (Lesson 1)
- data using two-way tables, bar graphs, and segmented bar graphs (Lessons 9 and 10)
- data using scatter plots (Lesson 11)

Interpret

- situations and graphs involving bivariate data (Lesson 2)
- tables and scatter plots of bivariate data (Lesson 3)
- tables, scatter plots, equations, and situations involving bivariate data (Lesson 4)

Sentence Frames and Stems

Section A

- Each point on the scatter plot represents _____ and _____.
- The data in this scatter plot represent ...
- I can locate a specific data point on a scatter plot by ...

Section B

- The association between _____ and _____ is _____ because ...
- The outlier in the data set is _____ because ...
- The linear equation _____ would be a good fit for this data because ...
- If the _____ increases by 1 _____, the model predicts that _____ increases/decreases by _____.
- To draw a line that best fits the data, first I _____, then I ...

Section C

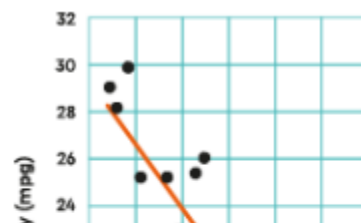
- Using the data shown in the _____, I can determine that ...
- To find the missing values in the table, first I _____, then I ...
- The percentage of _____ that _____ is _____ because ...
- I created a _____ to show the association between _____ and _____ is _____.

Section D

- Comparing Time 1 to Time 2, there is a _____ association because ...
- The _____ was most helpful to determine if there was an association because ...

Unit Outline

In this unit, students analyze bivariate data. They will use scatter plots and fitted lines to analyze numerical data, and two-way tables, bar graphs, and segmented bar graphs to analyze categorical data. Students advance their understanding of lines by examining slopes in the context of data. They will revisit these data analysis topics in a later course in more depth. At this level, students should be able to construct and interpret points on a scatter plot, informally fit linear models to data, interpret a given linear model in the context of data, and generally recognize patterns of association using relative frequencies in a two-way table.



In prior grades, students analyzed data collected about one variable using dot plots, histograms, and box plots. This unit expands on that by considering the possible influence of a second variable on measurements about individuals. In the first section, students are introduced to scatter plots and are reminded how to interpret points on a graph using a context. They also begin to recognize general trends in data.

In the second section, students look more closely at associations in data by informally drawing lines that model the general trend of the data. They also classify associations as positive, negative, linear, and non-linear by looking at the shape of the data in a scatter plot.

In the third section, students look at categorical data using two-way tables and relative frequencies. They then informally look at the relative frequencies to notice whether the variables are associated or not.

The unit ends with a lesson in which students collect and analyze numerical data using a scatter plot, then categorize the data based on a threshold and analyze the categories based on a two-way table.

Lesson Sequence	Learning Target(s)	Success Criteria/Assessment
<p>Section A Does This Predict That? (Lessons 1-3)</p>	<p>Learning Target #1 Create a scatter plot from a table of data, and describe the trend of the data.</p> <p>Learning Target #2 Interpret a point on a scatter plot in context.</p>	<p>Lesson 1 Organizing Data</p> <ul style="list-style-type: none"> I can organize data to see patterns more clearly. <p>Lesson 2 Plotting Data</p> <ul style="list-style-type: none"> I can draw a scatter plot to show data that has two paired variables. <p>Lesson 3 What a Point in a Scatter Plot Means</p> <ul style="list-style-type: none"> I can describe the meaning of a point in a scatter plot in context.
<p>Checkpoint A</p>	<p>Responding to Student Thinking More Chances: Students will have more opportunities to develop this understanding in later lessons. There is no need to slow down or add additional work to review this concept at this time.</p>	
<p>Section B Associations in Numerical Data (Lessons 4-8)</p>	<p>Learning Target #3 Describe the relationship between two variables using a line fit to data on a scatter plot.</p> <p>Learning Target #4 Draw a linear model to fit data in a scatter plot, and describe features of a line that fits data well.</p> <p>Learning Target #5 Interpret features of data on a scatter plot, including linear and non-linear association, outliers, slope of a linear model, and clustering.</p>	<p>Lesson 4 Fitting a Line to Data</p> <ul style="list-style-type: none"> I can pick out outliers on a scatter plot. I can use a model to predict values for data. <p>Lesson 5 Describing Trends of Scatter Plots</p> <ul style="list-style-type: none"> I can draw a line to fit data in a scatter plot. I can say whether data in a scatter plot has a positive or negative association (or neither). <p>Lesson 6 The Slope of a Fitted Line</p> <ul style="list-style-type: none"> I can use the slope of a line fit to data in a scatter plot to say how variables are connected in real-world situations. <p>Lesson 7 Observing More Patterns in Scatter Plots</p> <ul style="list-style-type: none"> I can pick out clusters in data from a scatter plot. I can use a scatter plot to decide if two variables have a linear association. <p>Lesson 8 Analyzing Bivariate Data</p> <ul style="list-style-type: none"> I can analyze a set of data to determine associations between two variables.
<p>Checkpoint B</p>	<p>Responding to Student Thinking</p> <ul style="list-style-type: none"> Problem 1: Press Pause: By this point in the unit, there should be some student mastery of slope and its meaning. If students struggle, make time to revisit related work in the referenced lesson. See the Course Guide for ideas to help students re-engage with earlier work. For example, revisit contexts from linear situations throughout this course and ask students to determine the slope and interpret its meaning. Problem 2: More Chances: Students will have more opportunities to understand the mathematical idea addressed here. There is no need to slow down or add additional work to the next lessons. 	
<p>Section C Associations in Categorical Data (Lessons 9-10)</p>	<p>Learning Target #6 Calculate relative frequencies, and describe associations between variables using a relative frequency table.</p> <p>Learning Target #7 Create a two-way table and a segmented bar graph that represent relative frequencies,</p>	<p>Lesson 9 Looking for Associations</p> <ul style="list-style-type: none"> I can identify the same data represented in a bar graph, a segmented bar graph, and a two-way table. I can use a two-way frequency table or relative frequency table to find associations among variables. <p>Lesson 10 Using Data Displays to Find Associations</p> <ul style="list-style-type: none"> I can create relative frequency tables, bar graphs, and segmented bar graphs from frequency tables to find associations among variables.

	and interpret the frequencies in context.	
Checkpoint C	Responding to Student Thinking <ul style="list-style-type: none"> • More Chances: Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons. 	
Section D Let's Put it to Work (Lessons 11)	Learning Target #8 Compare and contrast (orally) representations of bivariate data, including scatter plots, two-way tables, segmented bar graphs, and relative frequency tables. Learning Target #9 Describe (orally and in writing) associations in bivariate data using different representations of the same data.	Lesson 11 Gone in 30 Seconds <ul style="list-style-type: none"> • I can collect data and analyze it for associations using scatter plots, two-way tables, and segmented bar graphs.
End of Unit Assessment		

Unit Title:

Unit 7: Exponents and Scientific Notation

Relevant Standards: Bold indicates priority

Lesson	Standards	Lesson	Standards
Lesson 1		Lesson 9	8.EE.A.3
Lesson 2	8.EE.A.1	Lesson 10	8.EE.A.3 8.EE.A.4
Lesson 3	8.EE.A.1	Lesson 11	8.EE.A.1 8.EE.A.3 8.EE.A.4
Lesson 4	8.EE.A.1	Lesson 12	8.EE.A.3 8.EE.A.4
Lesson 5	8.EE.A.1	Lesson 13	8.EE.A.4
Lesson 6	8.EE.A.1	Lesson 14	8.EE.A.1 8.EE.A.3 8.EE.A.4
Lesson 7	8.EE.A.1	Lesson 15	8.EE.A.4
Lesson 8	8.EE.A.1	Lesson 16	8.EE.A.3 8.EE.A.4

Essential Question(s):	Enduring Understanding(s):
<ul style="list-style-type: none"> How do exponent rules simplify expressions with powers of 10? What do zero and negative exponents represent? How does scientific notation help in calculating with very large or very small numbers? 	<ul style="list-style-type: none"> Exponent rules for multiplication, division, and powers of powers provide efficient ways to generate equivalent numerical expressions 10^0 is defined as 1, and 10^{-n} represents $1/10^n$, allowing the decimal place value system and very small numbers to be represented using powers of 10 Scientific notation expresses numbers as a product of a power of 10 and a factor between 1 and 10, making it easier to estimate, compare, and perform arithmetic with quantities of vastly different magnitudes

Demonstration of Learning:	Pacing for Unit
CFA 1: Checkpoint A (after lesson 5) CFA 2: Checkpoint B (after lesson 8) CFA 3: Checkpoint C (after lesson 12) CFA 4: Checkpoint D (after lesson 15) EoU: Assessment A	16 Days Lesson to Review/Add <ul style="list-style-type: none"> None Lesson to Modify/Remove <ul style="list-style-type: none"> Move to outside of class 8.7.16: culminating lesson incorporating work from the unit

Family Overview	Integration of Technology:
https://accessim.org/6-8/grade-8/unit-7?a=family	<ul style="list-style-type: none"> Desmos Online Graphing Calculator Pear Assessment (Edulastic) iM v.360 Digital Applets (see below)

Unit-specific Vocabulary:	Aligned Unit Materials, Resources, and Technology																																
<table border="1"> <thead> <tr> <th rowspan="2">Lesson</th> <th colspan="2">New Terminology <i>Bold = brand new, use glossary</i></th> </tr> <tr> <th>receptive</th> <th>productive</th> </tr> </thead> <tbody> <tr> <td>8.7.1</td> <td>exponent power factor reciprocal</td> <td>repeated multiplication</td> </tr> <tr> <td>8.7.2</td> <td>powers of 10</td> <td></td> </tr> <tr> <td>8.7.3</td> <td>base (of an exponent) power of powers</td> <td></td> </tr> <tr> <td>8.7.4</td> <td>expanded positive exponent zero exponent</td> <td></td> </tr> <tr> <td>8.7.5</td> <td>negative exponent</td> <td>positive exponent</td> </tr> <tr> <td>8.7.6</td> <td></td> <td>exponent base (of an exponent)</td> </tr> </tbody> </table>	Lesson	New Terminology <i>Bold = brand new, use glossary</i>		receptive	productive	8.7.1	exponent power factor reciprocal	repeated multiplication	8.7.2	powers of 10		8.7.3	base (of an exponent) power of powers		8.7.4	expanded positive exponent zero exponent		8.7.5	negative exponent	positive exponent	8.7.6		exponent base (of an exponent)	Digital Applets <ul style="list-style-type: none"> 8.7.1 Activity 2: Doubling Coins 8.7.2 Activity 2 Applet: Zoom 8.7.10 Activity 1 & 2 Applets: Comparing Large Numbers with a Number Line, The Speeds of Light Provide access as needed throughout the unit: <ul style="list-style-type: none"> Blank paper Math Community Chart Tools for creating a visual display <table border="1"> <thead> <tr> <th>Lesson</th> <th>Materials to Gather</th> <th>Materials to Copy</th> </tr> </thead> <tbody> <tr> <td>8</td> <td>Tools for creating a visual display: Activity 2</td> <td></td> </tr> <tr> <td>9</td> <td></td> <td>Using Powers of 10 to Describe Cards (1 copy for every 6 students): Activity 2</td> </tr> </tbody> </table>	Lesson	Materials to Gather	Materials to Copy	8	Tools for creating a visual display: Activity 2		9		Using Powers of 10 to Describe Cards (1 copy for every 6 students): Activity 2
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		power zero exponent	12	Math Community Chart: Activity 1	
8.7.7	reciprocal evaluate	factor power of powers negative exponent	13	Blank paper	Scientific Notation Matching Cards (1 copy for every 4 students): Activity 2
8.7.8	square (of a number)		14		Distances in the Solar System Cards (1 copy for every 4 students): Activity 2
8.7.9	billion trillion multiple of		16		Old Hardware New Hardware Handout (1 copy for every 2 students): Activity 1
8.7.10	integer				
8.7.12		multiple of			
8.7.13	scientific notation	integer			
8.7.14		powers of 10 billion trillion			
8.7.15		scientific notation			
Opportunities for Interdisciplinary Connections:			Anticipated misconceptions:		
Epidemiology: Students model the spread of information or viruses in "Going Viral"			Negative Exponent Meaning: Students often think 10^{-3} is a negative number rather than a small positive fraction (1/1,000) See teacher's guide for specific misconceptions aligned to each lesson.		
Connections to Prior Units:			Connections to Future Units:		
Essential prior concepts to engage with this unit: <ul style="list-style-type: none"> expressions that have positive whole-number exponents and whole-number, fraction, or variable bases Relevant Unit(s)/Lesson(s) to Review: <ul style="list-style-type: none"> Grade 6 Unit 6: Expressions and Exponents 			Preparation for exponential growth and decay and logarithmic functions.		
Differentiation through Universal Design for Learning					
Engagement: <ul style="list-style-type: none"> Provide tools like blank tables or graphing software to calculate interest compounding (Unit 7 BPS modification - context of Lesson 16) Representation: <ul style="list-style-type: none"> Support structure by color coding connections between expressions, expanded forms, and single powers (Lesson 4, Activity 1 Launch) Action & Expression: <ul style="list-style-type: none"> Support memory and conceptual processing through a whole-class "think-aloud" demonstration of card descriptions (Lesson 9, Activity 2 Launch) 					
Related CELP standards aligned to Learning Targets:					
Math Language Routines					
The Illustrative Mathematics curriculum incorporates eight Mathematical Language Routines (MLRs) that support English Language Learners:					
MLR1: <i>Stronger and Clearer Each Time</i> - Students revise and refine their mathematical language through multiple drafts					
MLR2: <i>Collect and Display</i> - Students capture and organize language in visual displays					
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MLR6: <i>Three Reads</i> - Students analyze complex mathematical text					
MLR7: <i>Compare and Connect</i> - Students connect different mathematical representations					
MLR8: <i>Discussion Supports</i> - Students participate in mathematical discussions					
In this unit:					
<ul style="list-style-type: none"> MLR2: Collect and Display (Lessons 3, 6, 9) MLR8: Discussion Supports (Lessons 2, 3, 5, 6, 9, 10, 12, 13, 14, 15) MLR1: Stronger and Clearer Each Time (Lessons 3, 8, 12) 					
Progression of Disciplinary Language					

In this unit, teachers can anticipate students using language for mathematical purposes such as critiquing, representing, and justifying. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Critique

- reasoning about powers of powers (Lesson 3)
- reasoning about zero exponents (Lesson 4)
- applications of exponent rules (Lesson 7)
- reasoning about scientific notation (Lesson 15)

Represent

- situations using exponents (Lesson 1)
- large and small numbers using number lines, exponents, and decimals (Lesson 9–11)
- situations comparing quantities expressed in scientific notation (Lesson 14)

Justify

- reasoning about multiplying powers of 10 (Lesson 2)
- reasoning about powers of powers (Lesson 3)
- reasoning about dividing powers of 10 (Lesson 4)
- whether or not expressions are equivalent to exponential expressions (Lesson 6)
- reasoning about situations comparing powers of 10 (Lesson 12)

Sentence Frames and Stems

Section A

- The expression _____ is equivalent to _____ because ...
- Using the rule for _____ powers, I know that _____ is equal to _____.
- The rule for _____ powers makes sense because ...

Section B

- The expression _____ is equivalent to _____ because ...
- Using the rule for _____ powers, I know that _____ is equal to _____.
- To simplify the expression with powers _____, first I _____, then I ...

Section C

- Using scientific notation, the value _____ can be represented as _____.
- It's helpful to be able to write values like _____ as _____ using scientific notation because ...
- Using a number line to plot the value _____ given in scientific notation, I must first _____, then I ...
- I used the expression _____ to represent this scenario and found the value of _____ to be _____.

Section D

- I know the value _____ is/is not written in scientific notation because ...
- I used the expression _____ to represent this scenario and found the value of _____ to be _____.

Section E

- I know _____ can store _____ times as much as Apollo because ...
- The _____ processor is _____ times as fast as the Apollo.
- _____ has _____ times the memory of the Apollo.

Unit Outline

In this unit, students deepen their understanding of exponents, powers of 10, and place value before being introduced to scientific notation. They build on work done in a previous course where students focused on whole-number exponents with whole-number, fraction, decimal, or variable bases, but did not formulate rules regarding the use of exponents.

Students begin this unit by identifying patterns that emerge when multiplying and dividing powers of 10, and when raising powers of 10 to another power. Students generalize these patterns to develop exponent rules. They extend these rules to see why 10^0 must be equal to 1 and to understand what negative exponents mean.

Next, students determine that the rules developed for powers of 10 also work with other bases, as long as the bases in both expressions are the same. They observe a new rule that applies when multiplying bases that are different if the exponents are the same.

In the next section, students return to working with powers of 10 as they use multiples of powers of 10 to describe magnitudes of very large and very small quantities, such as the distance from Earth to the sun in kilometers or the mass of a proton in grams. Students plot these large and small values on number lines labeled using exponents and see how these numbers can be expressed in different ways — for example as $75 \cdot 10^5$ or $7.5 \cdot 10^6$.

$a^n \cdot a^m = a^{n+m}$	$(a^n)^m = a^{n \cdot m}$
$\frac{a^n}{a^m} = a^{n-m}$	$a^0 = 1$
$a^{-n} = \frac{1}{a^n}$	$a^n \cdot a^m = (a \cdot b)^n$

After building a foundation connecting powers of 10 with place value, students are finally introduced to scientific notation as a specific and useful way of writing numbers as a power of 10. They compute sums, differences, products, and quotients of numbers written in scientific notation to make additive and multiplicative comparisons, estimate quantities, and make measurement conversions.

Lesson Sequence	Learning Target(s)	Success Criteria/Assessment
<p>Section A Exponent Rules (Lessons 1-5)</p>	<p>Learning Target #1 Use exponent rules to generate equivalent numerical expressions for powers of 10.</p>	<p>Lesson 1 Exponent Review</p> <ul style="list-style-type: none"> I can use exponents to describe repeated multiplication. I understand the meaning of a term with an exponent. <p>Lesson 2 Multiplying Powers of Ten</p> <ul style="list-style-type: none"> I can explain and use a rule for multiplying powers of 10. <p>Lesson 3 Powers of Powers of Ten</p> <ul style="list-style-type: none"> I can explain and use a rule for raising a power of 10 to a power. <p>Lesson 4 Dividing Powers of Ten</p> <ul style="list-style-type: none"> I can evaluate 10^0 and explain why it makes sense. I can explain and use a rule for dividing powers of 10. <p>Lesson 5 Negative Exponents of Powers of 10</p> <ul style="list-style-type: none"> I can use the exponent rules with negative exponents. I know what it means if 10 is raised to a negative power.
<p>Checkpoint A</p>	<p>Responding to Student Thinking Problem 1 & 2: More Chances: Students will have more opportunities to develop this understanding in later lessons. There is no need to slow down or add additional work to review this concept at this time.</p>	
<p>Section B More Exponent Rules (Lessons 6-8)</p>	<p>Learning Target #2 Use exponent rules to generate equivalent numerical expressions for expressions with different bases and bases other than 10.</p>	<p>Lesson 6 What About Other Bases</p> <ul style="list-style-type: none"> I can use the exponent rules for bases other than 10. <p>Lesson 7 Practice with Rational Bases</p> <ul style="list-style-type: none"> I can change an expression with a negative exponent into an equivalent expression with a positive exponent. I can choose an appropriate exponent rule to rewrite an expression to have a single exponent <p>Lesson 8 Combining Bases</p> <ul style="list-style-type: none"> I can use and explain a rule for multiplying terms that have different bases but the same exponent.
<p>Checkpoint B</p>	<p>Responding to Student Thinking</p> <ul style="list-style-type: none"> Problem 1: Points to Emphasize: If most students struggle with identifying equivalent exponential expressions, revisit the exponent rules in this practice problem: <ul style="list-style-type: none"> Grade 8, Unit 7, Lesson 13, Practice Problem 5 Problem 2: Points to Emphasize: If most students struggle with multiplying expressions with different bases, revisit this concept in this practice problem: <ul style="list-style-type: none"> Grade 8, Unit 7, Lesson 10, Practice Problem 6 Problem 3: Points to Emphasize: If most students struggle with explaining why expressions with different bases can not be multiplied if they do not have the same exponent, revisit this concept in this practice problem: <ul style="list-style-type: none"> Grade 8, Unit 7, Lesson 14, Practice Problem 5 	
<p>Section C Large and Small Numbers (Lessons 9-12)</p>	<p>Learning Target #3 Compare very large or very small quantities expressed as a multiple of a power of 10.</p> <p>Learning Target #4 Use exponent rules and powers of 10 to solve problems in context.</p>	<p>Lesson 9 Describing Large & Small Numbers Using Powers of Ten</p> <ul style="list-style-type: none"> Given a very large or very small number, I can write an expression equal to it using a power of 10. <p>Lesson 10 Representing Large Numbers on a Number Line</p> <ul style="list-style-type: none"> I can plot a multiple of a power of 10 on such a number line. I can subdivide and label a number line between 0 and a power of 10 with a positive exponent into 10 equal intervals. I can write a large number as a multiple of a power of 10. <p>Lesson 11 Representing Small Numbers on a Number Line</p> <ul style="list-style-type: none"> I can plot a multiple of a power of 10 on such a number line. I can subdivide and label a number line between 0 and a power of 10 with a negative exponent into 10 equal intervals. I can write a small number as a multiple of a power of 10. <p>Lesson 12 Applications with Arithmetic With Powers of Ten</p> <ul style="list-style-type: none"> I can apply what I learned about powers of 10 to answer questions about real-world situations.
<p>Checkpoint C</p>	<p>Responding to Student Thinking</p> <ul style="list-style-type: none"> Problem 1 & 2: More Chances: Students will have more opportunities to develop this understanding in later lessons. There is no need to slow down or add additional work to review this concept at this time. 	

<p>Section D Scientific Notation (Lessons 13-15)</p>	<p>Learning Target #5 Calculate with numbers in scientific notation and interpret them in context.</p> <p>Learning Target #6 Identify numbers written in scientific notation, including scientific notation that has been generated by technology.</p>	<p>Lesson 13 Definition of Scientific Notation</p> <ul style="list-style-type: none"> I can tell whether or not a number is written in scientific notation. <p>Lesson 14 Estimating with Scientific Notation</p> <ul style="list-style-type: none"> I can multiply and divide numbers given in scientific notation. I can use scientific notation and estimation to compare very large very small numbers. <p>Lesson 15 Adding and Subtracting with Scientific Notation</p> <ul style="list-style-type: none"> I can add and subtract numbers given in scientific notation.
<p>Checkpoint D</p>	<p>Responding to Student Thinking</p> <ul style="list-style-type: none"> Problem 1: Press Pause: If most students struggle with identifying numbers written in scientific notation, make time to revisit related work in the lesson referred to here. See the Course Guide for ideas to help students re-engage with earlier work. <ul style="list-style-type: none"> Grade 8, Unit 7, Lesson 13 Definition of Scientific Notation Problem 2: Points to Emphasize If students struggle to find the surface area, emphasize ways to find the area, struggle with arithmetic with numbers written in scientific notation, make time to revisit related work in the lesson referred to here. See the Course Guide for ideas to help students re-engage with earlier work. <ul style="list-style-type: none"> Grade 8, Unit 7, Lesson 15 Adding and Subtracting with Scientific Notation 	
<p>Section E Let's Put it To Work (Lessons 16)</p>	<p>Learning Target #7 Use scientific notation to compare quantities in context, and describe (orally) how using scientific notation helps with making comparisons between very large and very small quantities.</p>	<p>Lesson 16 Is a Smart Phone Enough to Get to the Moon</p> <ul style="list-style-type: none"> I can use scientific notation to compare different amounts and answer questions about real-world situations.
<p>End of Unit Assessment</p>		

Unit Title:

Unit 8: Pythagorean Theorem and Irrational Numbers

Relevant Standards: Bold indicates priority

Lesson	Standards	Lesson	Standards
Lesson 1		Lesson 10	8.G.B 8.G.B.6
Lesson 2	8.NS.A 8.NS.A.2	Lesson 11	8.EE.A.2 8.G.B.7 8.NS.A
Lesson 3	8.EE.A.2 8.F.B	Lesson 12	8.G.B 8.G.B.7
Lesson 4	8.EE.A.2 8.NS.A	Lesson 13	8.G.B.8
Lesson 5	8.EE.A.2 8.NS.A.2	Lesson 14	8.EE.A.2 8.NS.A.2
Lesson 6	8.EE.A.2 8.NS.A.2	Lesson 15	8.EE.A.2 8.NS.A.2
Lesson 7	8.G.B 8.G.B.7	Lesson 16	8.EE.A 8.NS.A 8.NS.A.1
Lesson 8	8.G.B 8.G.B.6 8.G.B.7	Lesson 17	8.NS.A.1
Lesson 9	8.G.B.7	Lesson 18	8.G.B.7

Essential Question(s):

- How are square and cube roots defined geometrically?
- How can we estimate the value of an irrational square root?
- How does the Pythagorean Theorem relate the side lengths of right triangles?

Enduring Understanding(s):

- The square root of n is the side length of a square with area n , and the cube root of n is the edge length of a cube with volume n
- Irrational numbers cannot be written as fractions; their values can be approximated on a number line by identifying the two closest whole number squares
- In any right triangle with legs a and b and hypotenuse c , the relationship $a^2+b^2=c^2$ holds; this can be used to find unknown lengths and distances in 2D and 3D contexts

Demonstration of Learning:

CFA 1: Checkpoint A (after lesson 6)
 CFA 2: Checkpoint B (after lesson 13)
 CFA 3: Checkpoint C (after lesson 15)
 CFA 4: Checkpoint D (after lesson 17)
 EoU: Assessment A

Pacing for Unit

17 Days
 Lesson to Review/Add

- None

 Lesson to Remove/Modify

- Move to outside of class 8.8.16: culminating lesson incorporating work from the unit

Family Overview

<https://accessim.org/6-8/grade-8/unit-8?a=family>

Integration of Technology:

- Desmos Online Graphing Calculator
- Pear Assessment (Edulastic)
- iM v.360 Digital Applets (see below)

Unit-specific Vocabulary:

Lesson	New Terminology <i>Bold = brand new, use glossary</i>	
	receptive	productive
8.8.2	square root	square (of a number)
8.8.3	irrational number square root symbol	rational number
8.8.4	diagonal decimal approximation	
8.8.5		square root square root symbol
8.8.6	Pythagorean Theorem hypotenuse legs	right triangle
8.8.9	converse of the Pythagorean Theorem	Pythagorean Theorem
8.8.10	edge length	hypotenuse

Aligned Unit Materials, Resources, and Technology

- Digital Applets
- 8.8.1 Digital Applet: Making Squares
 - 8.8.2 Digital Applet: The Sides and Areas of Tilted Squares
 - 8.8.8 Digital Applet: A Transformational Proof
- Provide access as needed throughout the unit:
- Blank paper
 - Compasses
 - Four-function calculators
 - Geometry toolkits
 - Math Community Chart
 - Scientific calculators
 - Scissors
 - Tracing paper

Lesson	Materials to Gather	Materials to Copy

		legs	1		Making Squares Cutouts (1 copy for every 2 students): Activity 2
8.8.12	cube root		2	<ul style="list-style-type: none"> Geometry toolkits: Activity 1, Activity 2 Tracing paper: Activity 1, Activity 2 	
8.8.13		cube root edge length	3	<ul style="list-style-type: none"> Geometry toolkits: Activity 1, Activity 3 Tracing paper: Activity 1 Four-function calculators: Activity 2 	
8.8.14	repeating decimal decimal representation finite decimal expansion		4	<ul style="list-style-type: none"> Geometry toolkits: Activity 2 Tracing paper: Activity 2 Four-function calculators: Activity 4 	
8.8.15	infinite decimal expansion	irrational number repeating decimal	5	<ul style="list-style-type: none"> Compasses: Activity 2 Geometry toolkits: Activity 2 Four-function calculators: Activity 3 	
			8	<ul style="list-style-type: none"> Blank paper: Lesson Scissors: Activity 3 	A Transformational Proof Cutouts (1 copy for every 2 students): Activity 3
			9	Blank paper: Lesson	
			12	Math Community Chart: Activity 2	Pythagorean Theorem Cards (1 copy for every 2 students): Activity 2
			14	Math Community Chart: Activity 3	Rooted in the Number Line Cards (1 copy for every 2 students): Activity 3
			17		Some Numbers Are Rational Cards (1 copy for every 2 students): Activity 2
			18	Scientific calculators: Activity 2, Activity 3	

Opportunities for Interdisciplinary Connections:	Anticipated misconceptions:
Civil Engineering: The geometry of the Golden Gate Bridge is used to introduce the practical application of square roots and the Pythagorean Theorem	Distance Calculation: Students may make sign errors when calculating distances on a coordinate plane, such as subtracting a negative coordinate incorrectly See teacher's guide for specific misconceptions aligned to each lesson.
Connections to Prior Units:	Connections to Future Units:
Essential prior concepts to engage with this unit: <ul style="list-style-type: none"> expressions that have positive whole-number exponents and whole-number, fraction, or variable bases Relevant Unit(s)/Lesson(s) to Review: <ul style="list-style-type: none"> Grade 6 Unit 6: Expressions and Equations 	Essential for high school geometry and understanding the real-number system.
Differentiation through <i>Universal Design for Learning</i>	

Engagement:

- Develop effort and persistence by reviewing previous successes with area tables to provide an entry point for cube roots (Lesson 14, Activity 1 Launch)

Representation:

- Support language and symbols through a kinesthetic representation of the number line using a clothesline and cards (Lesson 15, Activity 2 Synth)

Action & Expression:

- Support organizational skills by providing a blank two-column table to list x- and y-values for perimeter sets (Lesson 12, Activity 1 Synth)

Related **CELP standards** aligned to Learning Targets:

Math Language Routines

The Illustrative Mathematics curriculum incorporates eight Mathematical Language Routines (MLRs) that support English Language Learners:

MLR1: *Stronger and Clearer Each Time* - Students revise and refine their mathematical language through multiple drafts

MLR2: *Collect and Display* - Students capture and organize language in visual displays

MLR3: *Clarify, Critique, Correct* - Students analyze mathematical writing/talk

MLR4: *Information Gap* - Students share information to solve problems

MLR5: *Co-Craft Questions* - Students create and improve questions

MLR6: *Three Reads* - Students analyze complex mathematical text

MLR7: *Compare and Connect* - Students connect different mathematical representations

MLR8: *Discussion Supports* - Students participate in mathematical discussions

In this unit:

- MLR1: Stronger and Clearer Each Time (Lessons 5, 8, 13, 14, 17, 18)
- MLR2: Collect and Display (Lessons 2, 9, 11, 14, 15, 16, 17, 21)
- MLR7: Compare and Connect (Lessons 2, 4, 5, 10, 16, 17, 18)

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes such as explaining, justifying, and comparing. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Explain

- strategies for finding area (Lesson 1)
- strategies for approximating and finding square roots (Lesson 4)
- strategies for finding triangle side lengths (Lesson 6)
- predictions about situations involving right triangles and strategies to verify (Lesson 10)
- strategies for finding distances between points on a coordinate plane (Lesson 11)
- strategies for approximating the value of cube roots (Lesson 13)

Justify

- which squares have side lengths in a given range (Lesson 1)
- ordering of irrational numbers (Lesson 5)
- ordering of hypotenuse lengths (Lesson 9)

Compare

- rational and irrational numbers (Lesson 3)
- lengths of diagonals in rectangular prisms (Lesson 10)
- strategies for approximating irrational numbers (Lesson 15)

Sentence Frames and Stems

Section A

- _____ is an irrational number because ...
- The square root of _____ must be _____ because ...
- The solution to the equation _____ is _____ because ...
- To place the square root of _____ on a number line, first I _____, then I ...

Section B

- The distance between point _____ and point _____ is _____ because ...
- If the leg lengths of the right triangle are _____ and _____, then the hypotenuse must be _____ because ...
- The side lengths _____, _____, and _____ make a right triangle because ...
- I used the equation _____ to find the missing side length of the right triangle. The missing side length is _____.

Section C

- If the volume of a cube is _____, the edge length must be _____ because ...
- The cube root of _____ must be _____ because ...
- To place the cube root of _____ on a number line, first I _____, then I ...

- The solution to the equation _____ is _____ because ...

Section D

- _____ is a rational number because ...
- The number _____ is rational because it can be written as the fraction _____.
- _____ is an irrational number because ...
- To place _____ on a number line, first I _____, then I ...

Section E

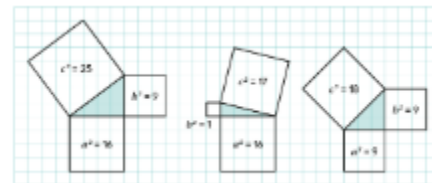
- The equation _____ represents this situation because ...
- The missing length in this situation is _____ because ...

Unit Outline

This unit introduces students to irrational numbers with a focus on connecting geometric and algebraic representations of square roots, cube roots, and the Pythagorean Theorem.

In the first section, students extend work from grade 6, composing and decomposing shapes to find the areas of tilted squares. They see “square root of n ” and \sqrt{n} to mean the side length of a square with area n square units, and understand that finding the solution to equations of the form $x^2=n$ means determining which values of x make the equation true. Students learn and use definitions for “rational number” and “irrational number,” learn (without proof) that $\sqrt{2}$ is irrational, and plot square roots on the number line.

In the second section, students continue using tilted squares as they investigate relationships between side lengths of right and non-right triangles. Students are encouraged to notice patterns among the triangles before being shown geometric and algebraic proofs of the Pythagorean Theorem. They use the Pythagorean Theorem and its converse to solve problems in two and three dimensions, for example, to determine lengths of diagonals of rectangles and right rectangular prisms, and to estimate distances between points in the coordinate plane.



In the third section, students see that “cube root of n ” and $\sqrt[3]{n}$ mean the side length of a cube with volume n cubic units. They also represent a cube root as a decimal approximation and as a point on the number line.

In the fourth section, students consider the decimal expansions of rational and irrational numbers. They learn how to rewrite fractions as a repeating decimal, how to rewrite a repeating decimal as a fraction, and reinforce their understanding that irrational numbers have a place on the number line even if they cannot be written as a fraction of integers.

Lesson Sequence	Learning Target(s)	Success Criteria/Assessment
Section A Side Lengths and Areas of Squares (Lessons 1-6)	Learning Target #1 Comprehend the term “irrational number” to mean a number that is not rational and that $\sqrt{2}$ is an example of an irrational number. Learning Target #2 Comprehend the term “square root of n ” and the notation $\sqrt{2}$ to mean the side length of a square whose area is n square units. Learning Target #3 Use the square root symbol to represent solutions to equations of the form $x^2=n$ and represent the square root as a point on the number line.	Lesson 1 Areas of Squares <ul style="list-style-type: none"> • I can find the area of a tilted square on a grid by using methods like “decompose and rearrange” and “surround and subtract.” • I can find the area of a triangle. Lesson 2 Side Lengths and Areas <ul style="list-style-type: none"> • I can explain what a square root is. • If I know the area of a square, I can express its side length using square root notation. Lesson 3 Square Roots <ul style="list-style-type: none"> • I can find exact and approximate side lengths of squares. • I know what an irrational number is and can give an example. Lesson 4 Rational and Irrational Numbers <ul style="list-style-type: none"> • I know what a rational number is and can give an example. • I can find a decimal approximation for square roots. Lesson 5 Square Roots on the Number Line <ul style="list-style-type: none"> • I can plot square roots on the number line. • When I have a square root, I can reason about which two whole numbers it is between. Lesson 6 Reasoning About Square Roots <ul style="list-style-type: none"> • When I have a square root, I can reason about which two whole numbers it is between.
Checkpoint A	Responding to Student Thinking Problem 1, 2, & 3: More Chances: Students will have more opportunities to develop this understanding in later lessons. There is no need to slow down or add additional work to review this concept at this time.	
Section B The Pythagorean Theorem (Lessons 7-13)	Learning Target #4 Calculate the distance between two points in the coordinate plane	Lesson 7 Finding Side Lengths of Triangles <ul style="list-style-type: none"> • I can explain what the Pythagorean Theorem says. Lesson 8 A Proof of the Pythagorean Theorem <ul style="list-style-type: none"> • I can explain why the Pythagorean Theorem is true.

	<p>by using the Pythagorean Theorem.</p> <p>Learning Target #5 Explain an area-based algebraic proof of the Pythagorean Theorem.</p> <p>Learning Target #6 Use the Pythagorean Theorem to calculate unknown side lengths of right triangles and to solve problems within a context.</p>	<p>Lesson 9 Finding Unknown Side Lengths</p> <ul style="list-style-type: none"> If I know the lengths of two sides, I can find the length of the third side in a right triangle. When I have a right triangle, I can identify which side is the hypotenuse and which sides are the legs. <p>Lesson 10 The Converse</p> <ul style="list-style-type: none"> I can explain why it is true that if the side lengths of a triangle satisfy the equation $a^2+b^2=c^2$ then it must be a right triangle. If I know the side lengths of a triangle, I can determine if it is a right triangle or not. <p>Lesson 11 Applications of the Pythagorean Theorem</p> <ul style="list-style-type: none"> I can use the Pythagorean Theorem to solve problems. <p>Lesson 12 More Applications of the Pythagorean Theorem</p> <ul style="list-style-type: none"> I can recognize situations where the Pythagorean Theorem can be used to solve a problem. <p>Lesson 13 Finding Distances in the Coordinate Plane</p> <ul style="list-style-type: none"> I can find the distance between two points in the coordinate plane. I can find the length of a diagonal line segment in the coordinate plane.
Checkpoint B	<p>Responding to Student Thinking</p> <ul style="list-style-type: none"> Problem 1: Press Pause: By this point in the unit, there should be some student mastery with using the Pythagorean Theorem. If most students struggle, revisit related work. For example, in the practice problem referred to here, review how to use the Pythagorean Theorem when finding a leg versus when finding the hypotenuse. See the Course Guide for ideas to help students re-engage with earlier work. <ul style="list-style-type: none"> Grade 8, Unit 8, Lesson 15, Practice Problem 6 Problem 2: Points to Emphasize: If most students struggle with finding the distance between two points, revisit how constructing a right triangle allows for the use of the Pythagorean Theorem. For example, in the practice problem referred to here, ask students to sketch a graph of the pairs of points when finding the distance between them. <ul style="list-style-type: none"> Grade 8, Unit 8, Lesson 14, Practice Problem 3 Problem 3: More Chances: Students will have more opportunities to understand the mathematical idea addressed here. There is no need to slow down or add additional work to the next lessons. 	
Section C Side Lengths and Volumes of Cubes (Lessons 14-15)	<p>Learning Target #7 Coordinate representations of a cube root, including cube root notation, decimal representation, the edge length of a cube of given volume, and a point on the number line.</p>	<p>Lesson 14 Edge Lengths and Volumes</p> <ul style="list-style-type: none"> I can approximate cube roots. I know what a cube root is. I understand the meaning of expressions like $\sqrt[3]{5}$. <p>Lesson 15 Cube Roots</p> <ul style="list-style-type: none"> When I have a cube root, I can reason about which two whole numbers it is between.
Checkpoint C	<p>Responding to Student Thinking</p> <ul style="list-style-type: none"> Problem 1: Points to Emphasize: If most students struggle with determining the side length of a cube given its volume, use the practice problem referred to here to emphasize the relationship between edge length and cube volume. <ul style="list-style-type: none"> Grade 8, Unit 8, Lesson 16, Practice Problem 6 Problem 2: Points to Emphasize: If most students struggle with estimating the value of a cube root, revisit strategies for making approximations when working with this in the next section. For example, in the Activity Synthesis of the activity referred to here, ask students to approximate the location of additional cube root values and place them on the number line. <ul style="list-style-type: none"> Grade 8, Unit 8, Lesson 16, Activity 3 Rational Numbers as Decimals 	
Section D Decimal Representation of Rational and Irrational Numbers (Lessons 16-17)	<p>Learning Target #8 Represent rational numbers as equivalent decimals and fractions.</p>	<p>Lesson 16 Decimal Representations of Rational Numbers</p> <ul style="list-style-type: none"> I can write a fraction as a repeating decimal. I understand that every number has a decimal expansion. <p>Lesson 17 Infinite Decimal Expansions</p> <ul style="list-style-type: none"> I can write a repeating decimal as a fraction. I understand that every number has a decimal expansion.
Checkpoint D	<p>Responding to Student Thinking</p> <ul style="list-style-type: none"> Problem 1: Press Pause: If most students struggle with representing a fraction as a decimal, make time to review long division when reviewing the practice problem referred to here. The Course Guide provides additional ideas for revisiting earlier work. <ul style="list-style-type: none"> Grade 8, Unit 8, Lesson 17, Practice Problem 2 Problem 2: Press Pause: If most students struggle with representing a repeating decimal as a fraction, make time to review the steps for converting when reviewing the practice problem referred to here. The Course Guide provides additional ideas for revisiting earlier work. 	

○ Grade 8, Unit 8, Lesson 17, Practice Problem 3		
<p>Section E Let's Put it To Work (Lesson 18)</p>	<p>Learning Target #9 Apply ratios and the Pythagorean Theorem to solve a problem involving the aspect ratio of screens or photos, and explain (orally) the reasoning.</p> <p>Learning Target #10 Describe (in writing and using other representations) characteristics of rectangles with the same aspect ratio or with different aspect ratios.</p>	<p>Lesson 18 When is the Same Size Not the Same Size?</p> <ul style="list-style-type: none"> • I can apply what I have learned about the Pythagorean Theorem to solve a more complicated problem. • I can decide what information I need to know to be able to solve a real-world problem using the Pythagorean Theorem.
End of Unit Assessment		