Algebra II
Content Standards Revisions
2022

Course Title:	Algebra II
Course/Unit Credit:	1
Course Number:	432000
Teacher Licensure:	Please refer to the Course Code Management System (https://adedata.arkansas.gov/ccms/) for the most current licensure codes.
Grades:	9-12
Prerequisite:	Algebra I or Algebra A/B

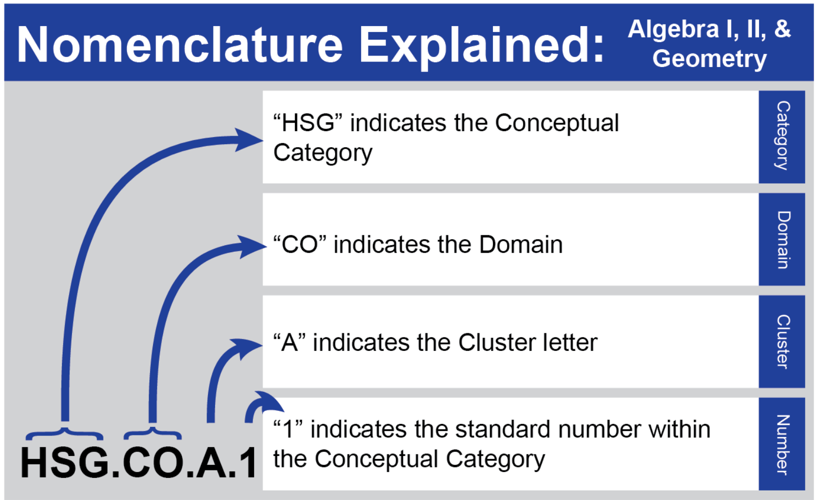
Course Description: Building on their work with linear, quadratic, and exponential functions, students extend their repertoire of functions to include polynomial, rational, and radical functions. Students work closely with the expressions that define the functions, and continue to expand and hone their abilities to model situations and to solve equations, including solving quadratic equations over the set of complex numbers and solving exponential equations using the properties of logarithms.

Introduction to the Algebra I, Algebra II, and Geometry Arkansas Mathematics Standards

When the Division of Elementary and Secondary Education (DESE) began the process of revising math standards, a diverse group of qualified educators from across the state came together to craft Arkansas standards specific to the schools and students in the state. The result of this work, the Arkansas Mathematics Standards, is contained in this document. These standards reflect what educators across the state know to be best for Arkansas students.

Standards Organization: The revision committee maintained the organizational structure and nomenclature of the previous standards. Algebra I, Algebra II, and Geometry Arkansas Mathematics Standards are categorized into conceptual categories, domains, clusters, and standards.

- **Conceptual category** represent the big picture across the high school grades.
- **Domains** represent the big ideas to be studied in each course. These big ideas support educators in determining the proper amount of focus and instructional time to be given to each of these topics.
- **Clusters** represent collections of standards grouped to help educators understand the building blocks of rich and meaningful instructional units. These units help students make connections within clusters and avoid seeing mathematics as a discrete list of skills they must master.
- **Standards** represent the foundational building blocks of math instruction. The standards outlined in this document work together to ensure that students are college and career ready and on track for success.



Standards Support: The revision of the Arkansas Mathematics Standards represent the work of the committee to provide greater clarity, strength, and support of the standards. Additionally, the revised mathematics standards are designed to help educators better understand the areas of emphasis and the focus within the standards. Educators should address the bulleted content as more than a checklist of items that they must teach individually. Content is bulleted to provide specificity of learning expectations included within some extensive standards. In some instances, the standard document includes Arkansas examples, teacher notes, specifications, and italicized words to assist educators with planning, teaching, and student learning.

- **Examples** included in the original standards were either changed for clarity or separated from the body of the actual standard. The examples included in the body of the standards document in no way reflect all of the possible examples. Likewise, these examples do not mandate curriculum or problem types. Local districts are free to select the high-quality curricula and instructional methods that best meet the needs of their students.
- **Teacher notes** offer clarification of the standards. These notes are intended to clarify, for teachers, what the expectations are for the learner. Likewise, these notes provide instructional guidance and limitations so that educators can better understand the scope of the standard. This will help with determining what is developmentally appropriate for students when working with specific standards.
- **Standard specifications** are to strengthen standards. The specifications are precise statements highlighting the need for mastery or function-type parameters for specific standards. This will assist educators in pinpointing the best opportunities for students to gain and master the knowledge and skills needed to succeed in a progression.
- **Asterisks (*)** are denoted to represent the modeling component of the standards. These standards should be presented in a modeling context which allows students to engage in the modeling process that is outlined in the Standards for Mathematical Process. (See Appendix A)
- **Italicized words** are defined in the glossary.

Finally, the Arkansas Mathematics Standards will be a living document. As these standards are implemented across schools in the state, DESE welcomes further suggestions related to notes of clarification, examples, professional development needs, and future revisions of the standards.

K - 12 Standards for Mathematical Practices	
1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics.	5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Algebra II Standards: Overview

Abbreviations: The following abbreviations are for the conceptual categories and domains for the Arkansas Mathematics Standards. For example, the standard HSN.RN.B.3 is in the High School Number and Quantity conceptual category and in The Real Number System domain.

High School Number and Quantity – HSN

- The Real Number System – RN
  - Extend the properties of exponents to rational exponents
  - Use properties of rational and irrational numbers
- Quantities – Q
  - Reason quantitatively and use units to solve problems
- The Complex Number System – CN
  - Perform arithmetic operations with complex numbers
  - Use complex numbers in polynomial identities and equations

High School Algebra – HSA

- Seeing Structure in Expressions – SSE
  - Interpret the structure of expressions
  - Write expressions in equivalent forms to solve problems
- Arithmetic with Polynomials and Rational Expressions – APR
  - Understand the relationship between zeros and factors of polynomials
  - Use polynomial identities to solve problems
  - Rewrite rational expressions
- Creating Equations – CED
  - Create equations that describe numbers or relationships
- Reasoning with Equations and Inequalities – REI
  - Understand solving equations as a process of reasoning and explain the reasoning
  - Solve equations and inequalities in one variable
  - Solve systems of equations
  - Represent and solve equations and inequalities graphically

High School Functions – HSF

- Interpreting Functions – IF

- Interpret functions that arise in applications in terms of the context
    - Analyze functions using different representations
- Building Functions – BF
    - Build a function that models a relationship between two quantities
    - Build new functions from existing functions
- Linear, Quadratic and Exponential Models – LE
    - Construct and compare linear, quadratic, and exponential models and solve problems

High School Statistics and Probability – HSS

- Interpreting Categorical and Quantitative Data – ID
    - Summarize, represent, and interpret data on a single count or measurement variable
    - Summarize, represent, and interpret data on two categorical and quantitative variables
- Making Inferences and Justifying Conclusions – IC
    - Understand and evaluate random processes underlying statistical experiments
    - Make inferences and justify conclusions from sample surveys, experiments, and observational studies
    -

The Real Number System	
Cluster A: Extend the properties of exponents to rational exponents.	
HSN.RN.A.1	Explore how extending the properties of integer exponents to rational exponents provides an alternative notation for radicals. Teacher Note: Example: We define $5^{\frac{4}{3}}$ to be the cube root of 5^4 so that $\left(5^{\frac{4}{3}}\right)^{\frac{3}{4}} = 5^1 = 5$ holds true.
HSN.RN.A.2	Rewrite expressions involving radicals and rational exponents using the properties of exponents. Teacher Note: All radical expressions involving variables assume the variables are representing positive numbers.
Cluster B: Use properties of rational and irrational numbers.	
HSN.RN.B.4	Use properties of rational and irrational numbers to: ● Simplify *radical expressions.* ● Perform operations (add, subtract, multiply, and divide) with *radical expressions.* ● Rationalize denominators. Teacher Note: ● This is a shared standard with Algebra I. ● Algebra II extends the work of Algebra I to include higher power *radical expressions*, variables, and rationalizing denominators with binomials. All *radical expressions* involving variables assume the variables are representing positive numbers.

Quantities	
Cluster A: Reason quantitatively and use units to solve problems.	
HSN.Q.A.2*	Define appropriate quantities including units of measure for descriptive modeling of real-world problems (e.g., variables to consider, relevant quantities and information, describe mathematically different methods for solving). Teacher Note: ● This is a shared standard with Algebra I. ● This standard should be addressed in all Algebra II units of study through modeling tasks.

The Complex Number System	
Cluster A: Perform arithmetic operations with complex numbers.	
HSN.CN.A.1	Understand there is a *complex number i* such that $i^2 = -1$, and every *complex number* has the form $a + bi$ where *a* and *b* are real numbers. Teacher Note: Students should describe the context from which *complex numbers* can arise.
HSN.CN.A.2	Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply *complex numbers.*
HSN.CN.A.3	Find the conjugate of a *complex number.*

Cluster C: Use complex numbers in polynomial identities and equations.	
HSN.CN.C.7	Solve quadratic equations with real coefficients that have real or *complex solutions.* Teacher Note: ● This standard is connected to HSA.REI.B.4. ● Complex solutions come in *conjugate pairs*.
HSN.CN.C.8	Extend *polynomial identities* to the *complex numbers*. Teacher Note: Example: Rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$.
HSN.CN.C.9	Apply the *Fundamental Theorem of Algebra* to determine the number and potential types of solutions for polynomial functions and show that it is true for quadratic polynomials. Teacher Note: This standard fits in with the *complex number* system as every polynomial equation of degree n with *complex number* coefficients has n roots (up to multiplicity) in the *complex numbers*.

Seeing Structure in Expressions	
Cluster A: Interpret the structure of expressions.	
HSA.SSE.A.2	Use the structure of an *expression* to identify ways to rewrite it. Teacher Note: ● This is a shared standard with Algebra I. ● Algebra II extends the work of Algebra I to include polynomial expressions with more than one variable. Example: ● See that $(2x + 3y)(2x + 3y)$ is the same as $(2x + 3y)^2$. ● See $16x^4 - 25y^2$ *as* $\left(4x^2\right)^2 - (5y)^2$, thus recognizing it as a difference of squares that can be factored as $(4x^2 - 5y)(4x^2 + 5y)$. ● See an opportunity to rewrite $6a^2 + 7ab + 2b^2$ as $(2a + b)(3a + 2b)$.
Cluster B: Write expressions in equivalent forms to solve problems.	
HSA.SSE.B.3*	Choose and produce an equivalent form of a *quadratic* and *exponential expression* to reveal and explain properties of the quantity represented by the *expression* in a real-world context. ● Factor a quadratic expression to reveal the *zeros* of the function it defines. ● Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. ● Recognize that each form of a quadratic (*standard, factored*, and *vertex* form) has its appropriate place to analyze the quantities represented by the expression in context. ● Use the properties of exponents to transform expressions for *exponential functions*. (Refer to example). Teacher Note: ● This is a shared standard with Algebra I.

	• Algebra II extends the work of Algebra I to include exponential expressions and quadratic expressions where $a > 1$ and b has no limitations. Example: The expression can be rewritten as $\left(1.15^{\frac{1}{12}}\right)^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.

Arithmetic with Polynomials and Rational Expressions	
Cluster B: Understand the relationship between zeros and factors of polynomials.	
HSA.APR.B.2	Know and apply the Factor and Remainder Theorems: for a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$. Teacher Note: Example: $\frac{(x^3+4x^2+7x+6)}{(x+2)} = x^2 + 2x + 3$ (remainder of 0) and $f(x) = (x + 2)(x^2 + 2x + 3) = x^3 + 4x^2 + 7x + 6$, evaluated at $f(-2) = 0$.
HSA.APR.B.3	Identify *zeros* of polynomials when suitable factorizations are available; use the *zeros* to construct a rough graph of the function defined by the polynomial.
Cluster C: Use polynomial identities to solve problems.	
HSA.APR.C.4	Verify polynomial identities and use them to describe numerical relationships. Teacher Note: See appendix for a list of common polynomial identities.
Cluster D: Rewrite rational expressions.	
HSA.APR.D.6	Rewrite simple *rational expressions* in different forms; write $\frac{a(x)}{b(x)}$ in the form $q(x) + \frac{r(x)}{b(x)}$, where $a(x)$ is the dividend, $b(x)$ is the divisor, $q(x)$ is the quotient, and $r(x)$ is the remainder and all are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, *synthetic division* (when possible), or a computer algebra system for the more complicated examples. Teacher Note: Students should understand that long division of polynomials can be used for any polynomial expression, but that *synthetic division* should only be used when the divisor is a first-degree polynomial. Example: $\frac{3x^3-5x^2+10x-3}{3x+1} = x^2 - 2x + 4 + \frac{-7}{3x+1}$
HSA.APR.D.7	Add, subtract, multiply, and divide by nonzero *rational expressions*. Understand that *rational expressions*, like the integers, are *closed* under addition, subtraction, and multiplication.

Creating Equations	
Cluster A: Create equations that describe numbers or relationships.	
HSA.CED.A.1*	Create equations (arising from *quadratic,* simple *rational, exponential, square root, and absolute value functions)* and inequalities in one variable; use them to solve problems. Teacher Note: This is a shared standard with Algebra I.
HSA.CED.A.2*	Create equations in two variables to represent relationships between quantities and graph on a coordinate plane using appropriate labels and scales with the option to graph using technology when appropriate. Teacher Note: ● This is a shared standard with Algebra I. ● Algebra II extends the work of Algebra I to include all function types with no limitations.
HSA.CED.A.3*	Create *equations* and *inequalities* to: ● Represent and interpret constraints by equations or inequalities and by *systems of equations* and/or inequalities. ● Interpret solutions as viable or nonviable options in a modeling context. Teacher Note: ● This is a shared standard with Algebra I. ● While functions will often be *linear*, *exponential*, or *quadratic*, the types of problems should draw from increasingly complicated situations.
HSA.CED.A.4*	Rearrange formulas to isolate a quantity of interest, using the same reasoning as in solving equations (e.g., rearrange $A = \frac{1}{2}h(b_1 + b_2)$ to isolate b_2). Teacher Note: ● This is a shared standard with Algebra I. ● Algebra II extends the work of Algebra I to include all types of formulas.

Reasoning with Equations and Inequalities	
Cluster A: Understand solving equations as a process of reasoning and explain the reasoning.	
HSA.REI.A.2	Solve simple rational and radical equations in one *variable*; give examples and explain how *extraneous solutions* may arise. Teacher Note: Students are not required to use only one procedure to solve problems nor are they required to show each step of the process. Examples (include but are not limited to): ● $\frac{2}{x-2} = \frac{x}{x-2} - \frac{3}{x}$ ● $\sqrt{x-1} + 4 = x - 3$

Cluster B: Solve equations and inequalities in one variable.	
HSA.REI.B.4	Solve quadratic equations in one variable with real and/or *complex number* solutions using appropriate methods depending on the form of the quadratic (factored, standard, or vertex) by: ● Inspecting a graph. ● Taking square roots. ● Completing the square. ● Using the quadratic formula. ● Factoring the equation. Teacher Note: ● This standard is connected to HSF.BF.B.3 and is a shared standard with Algebra I. ● Algebra II extends the work of Algebra I to include quadratic expressions where $a > 1$ and *b* has no limitations.
Cluster C: Solve systems of equations.	
HSA.REI.C.7*	Solve *systems of equations* consisting of linear equations and nonlinear equations in two variables algebraically (using substitution and elimination) and graphically. Teacher Note:This is a shared standard with Algebra I. Example: Find the points of intersection between $y = (-3x)$ and $y = x^2 + 2$.
Cluster D: Represent and solve equations and inequalities graphically.	
HSA.REI.D.11*	Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$. ● Find the solutions approximately using technology to graph the functions, making tables of values, or finding *successive approximations.* ● Include cases where $f(x)$ and/or $g(x)$ are *linear, polynomial, rational, exponential,* or *logarithmic functions* and where at least one of the functions is not *linear*: Teacher note: ● This is a shared standard with Algebra I. ● Algebra II extends the work of Algebra I to include the listed function types with no limitations on the functions.

Interpreting Functions	
Cluster B: Interpret functions that arise in applications in terms of the context.	
HSF.IF.B.4*	For a function that models a relationship between two quantities: ● Interpret key features of graphs and tables in terms of the quantities, including *intercepts*, *zeros;* intervals where the *function* is increasing, decreasing, positive, or negative; *maxima*, *minima*; symmetries; *end behavior*; and periodicity. ● Sketch graphs showing key features given a verbal description of the relationship. ● Identify transformations from a parent function. Teacher Note: ● This is a shared standard with Algebra I. ● Algebra II tasks have a real-world context and include the following functions: *quadratic*, polynomial, and *exponential* without limitations.

HSF.IF.B.6*	In a real-world context: ● Calculate and interpret the average rate of change of a function (presented algebraically or as a table) over a specified interval. ● Estimate the rate of change from a graph. Teacher Note: ● This is a shared standard with Algebra I. ● Function models include *quadratic*, polynomial, and *exponential functions* with no limitations. Examples could include problems regarding cell phone battery life, bacteria growth/decay, and stock market changes.
Cluster C: Analyze functions using different representations.	
HSF.IF.C.7*	Graph functions expressed algebraically and show key features of the graph, with or without technology. ● Graph *polynomial functions;* identify *zeros* when suitable factorizations are available, and show *end behavior.* ● Graph *rational functions,* identify *zeros* and asymptotes when suitable factorizations are available, and show *end behavior.* ● Graph *exponential* and *logarithmic functions* and show intercepts and *end behavior.* ● Graph *square-root functions* and identify intercepts, *end behavior*, and domain. ● Graph *piecewise-defined function* to examine domain restrictions. Teacher Note: This is a shared standard with Algebra I.
HSF.IF.C.8	Write expressions for functions in different but equivalent forms to reveal key features of the function. ● Use the properties of exponents to interpret expressions for *exponential functions*. Teacher Note:This standard is an extension of Algebra I and connected to HSA.SSE.B.3 in Algebra I. Example: $y = a\cdot b^x = a(1 \pm r)^x$, where *b* is the base and *r* is the rate of growth or decay.
HSF.IF.C.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or verbally described). Teacher Note: ● This is a shared standard with Algebra I. ● Algebra II extends the work of Algebra I to include *square root*, *cube root*, *piecewise-defined* (including step functions), and *exponential functions* with no limitation.

Building Functions

Cluster A: Build a function that models a relationship between two quantities.	
HSF.BF.A.1*	Write a function that describes a relationship between two quantities. ● Combine standard function types using arithmetic operations (e.g., given that *f(x)* and *g(x)* are functions developed from a context, find $(f+g)(x)$, $(f-g)(x)$, $(fg)(x)$, $(\frac{f}{g})(x)$, and any combination thereof, given $g(x) \neq 0$). ● Compose functions. Teacher Note: This standard is an split between Algebra I and II.

HSF.BF.A.2	For arithmetic and geometric sequences, build a function to model a relationship. ● Write *arithmetic* and *geometric sequences* both *recursively* and with an explicit formula; translate between the two forms given as: o a graph, o a description of a relationship, or o two input-output pairs (include reading these from a table). ● Use *arithmetic* and *geometric sequences* to model situations.
Cluster B: Build new functions from existing functions.	
HSF.BF.B.3	Demonstrate function understanding. ● Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$ and $f(x + k)$ for specific values of k (k is a constant and both positive and negative). ● Find the value of k given the graphs of the transformed functions. ● Experiment with multiple transformations and illustrate an explanation of the effects on the graph with or without technology. ● Recognize *even* and *odd functions* from their graphs and algebraic expressions. Teacher Note: ● This is a shared standard with Algebra I. ● Algebra II tasks will use written descriptions and/or the function. Function models include *linear*, *quadratic*, *cubic*, *absolute value*, *exponential*, *square root*, *reciprocal,* and *logarithmic*.
HSF.BF.B.4	Find *inverse* functions. ● Solve an equation of the form $y = f(x)$ for a simple function *f* that has an *inverse;* write an expression for the *inverse (e.g.,* $f(x) = 2x^2$ *or* $f(x) = (x + 1)(x - 1)$ for $x \neq 1$). ● Verify by *composition* that one function is the *inverse* of another. ● Read values of an *inverse* function from a graph or a table, given that the function has an *inverse.* ● Produce an invertible function from a non-invertible *quadratic function* by restricting the domain.
HSF.BF.B.5	Understand the *inverse* relationship between exponents and logarithms; use this relationship to solve problems involving logarithms and exponents.

Linear, Quadratic, and Exponential Models	
Cluster A: Construct and compare linear, quadratic, and exponential models and solve problems.	
HSF.LE.A.4*	Construct and compare exponential models to solve problems. ● Express exponential models as logarithms. ● Express logarithmic models as exponentials. ● Use properties of logarithms to simplify and evaluate logarithmic expressions (expanding and/or condensing logarithms as appropriate). ● Evaluate logarithms with or without technology. Teacher Note: ● This standard is connected to HSF.BF.B.5. ● For exponential models, express the solution to $ab^{ct} = d$ where a, c, and d are constants and b is the base (Including, but not limited to: 2, 10, or *e*) as a logarithm; then evaluate the logarithm with or without technology.

<table>
<tr><th colspan="2">Interpreting Categorical and Quantitative Data</th></tr>
<tr><th colspan="2">Summarize, represent, and interpret data on a single count or measurement variable.</th></tr>
<tr><td>HSS.ID.A.4</td><td>Emphasize the use of the normal distribution to obtain the probability or likelihood of certain events.
● Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages.
● Recognize that there are data sets for which such a procedure is not appropriate.
● Use calculators and/or spreadsheets to estimate areas under the normal curve.

Teacher Note: Limit area under the curve to the empirical rule (68-95-99.7) to estimate the percent of a normal population that falls within 1, 2, or 3 standard deviations of the mean. Also, recognize that normal distributions are only appropriate for unimodal and symmetric shapes.</td></tr>
<tr><th colspan="2">Cluster B: Summarize, represent, and interpret data on two categorical and quantitative variables.</th></tr>
<tr><td>HSS.ID.B.6</td><td>Represent data on two quantitative variables on a scatter plot; describe how the variables are related.
● Fit a function to the data; use functions fitted to data to solve problems in the context of the data.

Teacher Note:
● This is a shared standard with Algebra I.
● Use given functions or choose a function suggested by the context. The focus of Algebra II should be on quadratic and more complex exponential models.</td></tr>
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<tr><th colspan="2">Making Inferences and Justifying Conclusions</th></tr>
<tr><th colspan="2">Cluster A: Understand and evaluate random processes underlying statistical experiments.</th></tr>
<tr><td>HSS.IC.A.1</td><td>Recognize statistics as a process for making inferences about population parameters based on a random sample from that population.</td></tr>
<tr><td>HSS.IC.A.2</td><td>Compare theoretical and empirical probabilities using simulations (e.g., such as flipping a coin, rolling a number cube, spinning a spinner, technology).</td></tr>
<tr><th colspan="2">Cluster B: Make inferences and justify conclusions from sample surveys, experiments and observational studies.</th></tr>
<tr><td>HSS.IC.B.3</td><td>Recognize the purposes of and differences among sample surveys, experiments, and observational studies. Explain how randomization relates to sample surveys, experiments, and observational studies.</td></tr>
<tr><td>HSS.IC.B.6</td><td>Read and explain, in context, the validity of data from outside reports by:
● Identifying the variables as quantitative or categorical.
● Describing how the data was collected.
● Indicating any potential biases or flaws.
● Identifying inferences the author of the report made from sample data.

Teacher Note: As a strategy, students could collect reports published in the media and ask students to consider the source of the data, the design of the study, and the way the data are analyzed and displayed.</td></tr>
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Glossary

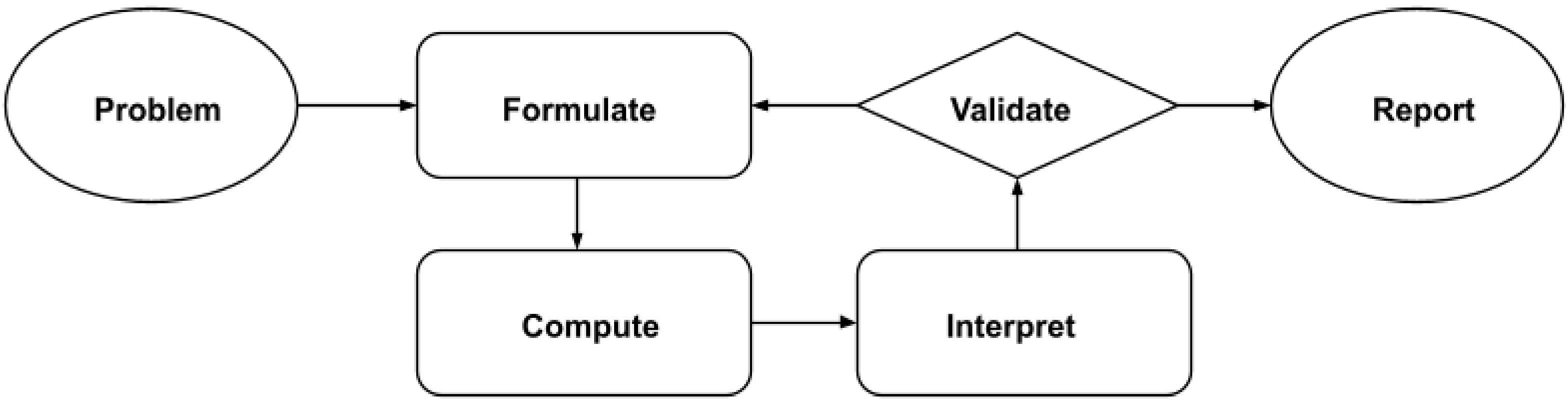
Absolute value function	Any function with an algebraic expression contained in absolute value symbols, in the family with parent function $f(x) = \|x\|$.
Bias	When certain responses are systematically favored over others.
Binomial	A polynomial with exactly two terms.
Closed	When an operation is performed on a set, the result is within the same set. For example: When you add, subtract, or multiply rational expressions, the result will be rational.
Complex number	A number with a real part and an imaginary part; *i* is the imaginary unit, Example: $3 + 2i$ where 3 is the real part and $2i$ is the imaginary part.
Conjugate(s)	The result of writing the sum of two terms as a difference or vice-versa.
Conjugate pair	A pair of binomials with identical terms and opposite operations. ● Complex numbers of the form $a + bi$ and $a - bi$, where *a* and *b* are real numbers. ○ For example: If $x = 2i$ is a solution, then $x = -2i$ is also a solution. Therefore, $(x - 2i)(x + 2i)$ are factors. ● Radical expressions of the form $a + \sqrt{b}$ and $a - \sqrt{b}$, where a and b are real numbers. ○ For example: Use the conjugate $2 - \sqrt{3}$ to rationalize the denominator in $\frac{2}{2+\sqrt{3}}$. The binomials $2 - \sqrt{3}$ and $2 + \sqrt{3}$ are a conjugate pair.
Cubic Function	Any function in the family with parent function $f(x) = x^3$
End behavior	The behavior of a graph of $f(x)$ as x approaches positive or negative infinity.
Exponential function	A function in which a variable appears in the exponent; $f(x) = 2^x$.
Extraneous solution	A solution that emerges from the process of solving an equation, but is not a valid solution to the original problem.
Factored Form	A quadratic function in the form $f(x) = a(x - r_1)(x - r_2)$.
Fundamental Theorem of Algebra	The degree of a polynomial identifies the number of solutions, such that a polynomial of degree *n* has *n* number of solutions.
Geometric sequence	A sequence such as $2, 6, 18, 54, 162,$ ... or $3, 1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81},$ ... which has a constant (common) ratio between terms.
Inverse	The relationship that reverses the independent and dependent variables of a relation.
Linear function	A function characterized by a constant rate of change (slope).
Logarithmic function	A function in the family with parent function $y = log_b x$
Normal Distribution	A distribution where the graph is unimodal and symmetric in shape (i.e. symmetrical bell-shaped graph).
Odd function	A function symmetric with respect to the origin; $f(-x) = -f(x)$
Piecewise-defined function	A function with the domain divided into parts and each part can be defined by a different rule.
Polynomial function	A function which is a sum of terms that have positive integer exponents.

Polynomial identity	Equations that involve polynomials and are true for all possible values of a variable. See the appendix for examples.
Quadratic function	Any function in the family with parent function $f(x) = x^2$
Quantitative variable	Variables that are numerical and represent a measurable quantity
Radical expression	An expression containing a root symbol; $\sqrt{\ }$ (Examples: square root, cubic root, etc.)
Rational Exponent (Fractional Exponents)	An expression with a rational number as an exponent, Example: $a^{\frac{1}{n}} = \sqrt[n]{a}$ and $a^{\frac{m}{n}} = (\sqrt[n]{a})^m$ or $\sqrt[n]{a^m}$
Rational expression	A ratio of two polynomial expressions with a non-zero denominator. Example: $\frac{3x+1}{x+2}$, where $x \neq -2$.
Rational function	A function that can be written as a quotient, $f(x) = \frac{p(x)}{q(x)}$, where *p(x)* and *q(x)* are polynomial expressions and *q(x)* is a degree of 1 or higher.
Reciprocal function	Any function in the family with parent function $f(x) = \frac{1}{x}$
Standard deviation	A numerical value used to indicate how widely individuals in a group vary
Standard form	A quadratic Function in the form $f(x) = ax^2 + bx + c$.
Square root function	Any function in the family with parent function $f(x) = \sqrt{x}$
Successive Approximations	A method of finding a solution by using a sequence of approximations which converge to the solution.
Synthetic division	A shortcut method of dividing two polynomials when dividing by a linear factor.
Systems of Equations	A set of two or more equations with the same variables. To solve a system is to find all common solutions or points that satisfy all equations.
Theoretical probability	The number of favorable outcomes divided by the number of possible outcomes.
Unimodal	A set of data containing a unique mode and having a graph with one main peak.
Vertex Form	A quadratic function in the form $f(x) = a(x - h)^2 + k$.
Zeros	The values of the independent (*x*-value) that make the corresponding values of the function equal to zero.

Appendix A.

Mathematical Modeling Cycle

The basic modeling cycle is summarized in this diagram. It involves: (1) identifying variables in the situation and selecting those that represent essential features; (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables; (3) analyzing and performing operations on these relationships to draw conclusions; (4) interpreting the results of the mathematics in terms of the original situation; (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable; (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.



Appendix B.

Table 1: Common Polynomial Identities

$x^2 - y^2 = (x - y)(x + y)$
$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$
$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$
$(x + y)^2 = x^2 + 2xy + y$
$(x + a)(x + b) = x^2 + (a + b)x + ab$
$(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2xz$
$(x + y)^3 = x^3 + y^3 - 3xy(x + y)$
$(x - y)^3 = x^3 - y^3 - 3xy(x - y)$
$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$
$(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples
$ax^2 + bx + c = 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$