



Calculus

Content Standards Revisions

2022

Course Title: Calculus
Course/Unit Credit: 1
Course Number: 434010
Teacher Licensure: Please refer to the Course Code Management System (<https://adedata.arkansas.gov/ccms/>) for the most current licensure codes.
Grades: 9-12
Prerequisites: Students must have successfully completed coursework for Algebra I, Geometry, and Algebra II or Pre-Calculus.

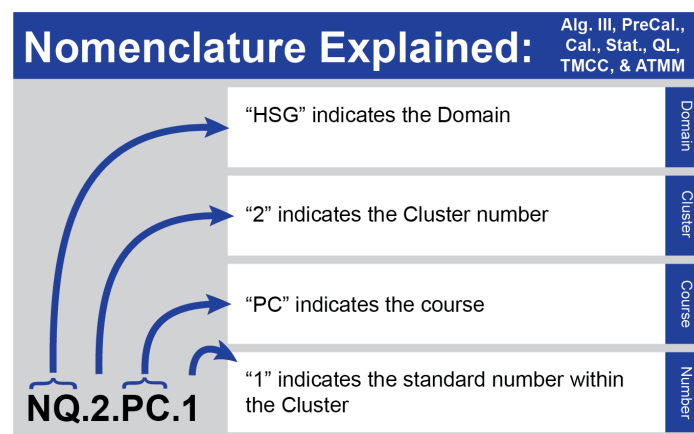
Course Description: Calculus is a two-semester survey course designed to provide students with experience in the methods and applications of calculus and to develop an understanding of its concepts. This course emphasizes a multi-representational approach to Calculus, with concepts, results, and problems being expressed graphically, numerically, symbolically, analytically, and verbally through the use of unifying themes of derivatives, integrals, limits, application and modeling, and approximation. An important distinction between this course and the AP Calculus courses is that this course does not require some topics such as: the exploration of trigonometric functions, solids of revolution, and/or separable differential equations. Calculus does not require Arkansas Department of Education approval.

Introduction to Secondary Arkansas Mathematics Standards

When the Division of Elementary and Secondary Education (DESE) began the process of revising math standards, a diverse group of qualified educators from across the state came together to craft Arkansas standards specific to the schools and students in the state. The result of this work, the Arkansas Mathematics Standards, is contained in this document. These standards reflect what educators across the state know to be best for Arkansas students.

Standards Organization: The revision committee maintained the organizational structure and nomenclature of the previous standards. Secondary Arkansas Mathematics Standards are categorized into domains, clusters, and standards.

- **Domains** represent the big ideas to be studied in each course and sometimes across grade bands. These big ideas support educators in determining the proper amount of focus and instructional time to be given to each of these topics.
- **Clusters** represent collections of standards grouped to help educators understand the building blocks of rich and meaningful instructional units. These units help students make connections within clusters and avoid seeing mathematics as a discrete list of skills they must master.
- **Standards** represent the foundational building blocks of math instruction. The standards outlined in this document work together to ensure that students are college and career ready and on track for success.



Standards Support: The revision of the Arkansas Mathematics Standards represent the work of the committee to provide greater clarity, strength, and support of the standards. Additionally, the revised mathematics standards are designed to help educators better understand the areas of emphasis and the focus within the standards. Educators should address the bulleted content as more than a checklist of items that they must teach individually. Content is bulleted to provide specificity of learning expectations included within some extensive standards. In some instances, the standard document includes Arkansas examples, teacher notes, specifications, and italicized words to assist educators with planning, teaching, and student learning.

- **Examples** included in the original standards were either changed for clarity or separated from the body of the actual standard. The examples included in the body of the standards document in no way reflect all of the possible examples. Likewise, these examples do not mandate curriculum or problem types. Local districts are free to select the high-quality curricula and instructional methods that best meet the needs of their students.
- **Teacher notes** offer clarification of the standards. These notes are intended to clarify, for teachers, what the expectations are for the learner. Likewise, these notes provide instructional guidance and limitations so that educators can better understand the scope of the standard. This will help with determining what is developmentally appropriate for students when working with specific standards.
- **Standard specifications** are to strengthen standards. The specifications are precise statements highlighting the need for mastery or function-type parameters for specific standards. This will assist educators in pinpointing the best opportunities for students to gain and master the knowledge and skills needed to succeed in a progression.
- **Asterisks (*)** are denoted to represent the modeling component of the standards. These standards should be presented in a modeling context which allows students to engage in the modeling process that is outlined in the Standards for Mathematical Process. (See Appendix A)
- **Italicized words** are defined in the glossary.

Finally, the Arkansas Mathematics Standards will be a living document. As these standards are implemented across schools in the state, DESE welcomes further suggestions related to notes of clarification, examples, professional development needs, and future revisions of the standards.

K - 12 Standards for Mathematical Practices

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| <ol style="list-style-type: none"> 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics.* | <ol style="list-style-type: none"> 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning. |
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Calculus Standards: Overview

Abbreviations: The following abbreviations are for the domains for the Arkansas Mathematics Standards.

Limits and Continuity - LC	
	1. Students will determine the limit of a function at a value numerically, graphically, and analytically.
Derivatives - D	
	2. Students will use derivatives to solve problems both theoretically and in real-world context.
Integrals - I	
	3. Students will apply the techniques of integration to solve problems both theoretically and in contextual models that represent real-world phenomena.

Limits and Continuity

Students will determine the limit of a function at a value numerically, graphically, and analytically.

LC.1.C.1	Identify vertical asymptotes in rational and logarithmic functions by identifying locations where the function value approaches infinity; estimate <i>limits</i> numerically and graphically; calculate <i>limits</i> analytically: • algebraic simplification • direct substitution • one-sided limits • rationalization
LC.1.C.2	Calculate <i>infinite limits</i> and use the result to identify vertical asymptotes in rational and logarithmic functions.
LC.1.C.3	Calculate <i>limits</i> at infinity and use the result to identify horizontal asymptotes in rational and exponential functions.
LC.1.C.4	Calculate <i>limits</i> at infinity and use the result to identify unbounded behavior in rational, exponential, and logarithmic functions.
LC.1.C.5	Identify and classify graphically, algebraically, and numerically if a discontinuity is removable or non-removable; identify the three conditions that must exist in order for a function to be continuous at $x = a$: • $f(a)$ is defined • the <i>limit</i> as x approaches a of $f(x)$ equals $f(a)$ • the <i>limit</i> as x approaches a of $f(x)$ exists
LC.1.C.6	Apply the <i>Intermediate Value Theorem</i> for <i>continuous functions</i> .

Derivatives

Students will use derivatives to solve problems both theoretically and in real-world context.

D.2.C.1	Approximate the <i>derivative</i> : - graphically by finding the slope of a <i>tangent line</i> drawn to a curve at a given point. - numerically by using the <i>difference quotient</i>.
D.2.C.2	Find the equation of the <i>tangent line</i> using the definition of <i>derivative</i> .
D.2.C.3	Establish and apply that <i>differentiability</i> implies continuity, but continuity does not necessarily imply <i>differentiability</i> .
D.2.C.4	Compare the characteristic of graphs of f and f' : - Generate the graph of f given the graph of f' and vice versa. - Establish the relationship between the increasing and decreasing behavior of f and the sign of f'. - Identify maxima and minima as points where increasing and decreasing behavior change.
D.2.C.5	Apply the <i>Mean Value Theorem</i> on a given interval.
D.2.C.6	Compare the characteristic of graphs of f , f' , and f'' : - Generate the graphs of f and f' given the graph of f'' and vice versa. - Establish the relationship between the <i>concavity</i> of f and the sign of f''. - Identify points of inflection as points where <i>concavity</i> changes.
D.2.C.7	Find <i>derivatives</i> of functions using: <i>Power rule</i>. <i>Product rule</i>. <i>Quotient rule</i>.
D.2.C.8	Find <i>derivatives</i> of: - An implicitly defined equation. - Composite functions using <i>chain rule</i>. - Exponential and logarithmic functions. - Functions requiring the use of more than one differentiation rule.
D.2.C.9	Find the equation of: - A line tangent to the graph of a function at a point. - A <i>normal line</i> to the graph of a function at a point.
D.2.C.10	Solve application problems involving: - Optimization. - Related rates.
D.2.C.11	Interpret the <i>derivative</i> as a rate of change and varied applied contexts, including velocity, speed, and <i>acceleration</i> .

Integrals

Students will apply the techniques of integration to solve problems, both theoretically and in contextual models that represent real-world phenomena.

I.3.C.1	Define the definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval. Teacher note: If f is a real, <i>continuous function</i> defined on $[a, b]$ and F is an antiderivative of f in $[a, b]$, then $\int_a^b f(x)dx = F(b) - F(a)$
I.3.C.2	Determine the area between two curves and identify the definite integral as the area of the region bounded by two curves.
I.3.C.3	Apply the <i>Fundamental Theorem of Calculus</i> to solve contextual models that represent real-world phenomena.
I.3.C.4	Find the general solution to <i>indefinite integrals</i> .
I.3.C.5	Determine the antiderivative of a function using rules of basic differentiation, and solve problems using the techniques of antidifferentiation including but not limited to <i>power rule</i> and <i>u</i> -substitution.
I.3.C.6	Estimate definite integrals by using Riemann sums (left, right, midpoint, and trapezoidal) and identify the definite integral as a <i>limit</i> of Riemann sums.
I.3.C.7	Explore applications of integration.

Glossary

Acceleration	The rate of change of velocity over time
Chain Rule	A method for finding the derivative of a composition of functions; the formula is $\frac{d}{dx} f(g(x)) = f'(g(x))g'(x)$
Concavity	If a curve is concave up (convex), the graph of the curve is bent upward, like an upright bowl. If a curve is concave down (or simply concave), then the graph of the curve is bent down, like a bridge. For a function $f(x)$ where $f(x)$ and $f'(x)$ are both differentiable, $f(x)$ is concave up if $f''(x) \geq 0$ and concave down if $f''(x) \leq 0$. If $f''(x) = 0$, then x is an inflection point, where the graph changes direction of concavity
Continuous function(s)	A function is continuous at $x = a$ if 1. $f(x)$ exists 2. $f(a)$ exists 3. $f(x) = f(a)$
Derivative(s)	A function which gives the slope of a curve; that is, the slope of the line tangent to a function; the derivative of a function f at a point x is commonly written $f'(x)$
Difference quotient	For a function f , the formula $\frac{(x+h)-f(x)}{h}$; this formula computes the slope of the secant line through two points on the graph of f ; these are the points with x -coordinates x and $x + h$; the difference quotient is used in the definition the derivative
Differentiability	A curve that is smooth and contains no discontinuities or cusps; formally, a curve is differentiable at all values of the domain variable(s) for which the derivative exists
Fundamental Theorem of Calculus	The theorem that establishes the connection between derivatives, antiderivatives, and definite integrals Evaluation part of the FTC: If f is continuous on $[a, b]$, and F is any antiderivative of f , then $\int_a^b f(x)dx = F(b) - F(a)$ Antiderivative part of the FTC: If f is continuous on $[a, b]$, then $\frac{d}{dx} \int_a^x f(t)dt = f(x)$ for every x in $[a, b]$
Indefinite integral(s)	The family of functions that have a given function as a common derivative; the indefinite integral of $f(x)$ is written $\int f(x)dx$ [e.g., $\int x^2 dx = \frac{1}{3}x^3 + C$]

Infinite limits	A limit that has an infinite result (either ∞ or $-\infty$), or a limit taken as the variable approaches ∞ (infinity) or $-\infty$ (negative infinity); the limit can be one-sided
Intermediate Value Theorem	If f is a function that is continuous over the domain $[a, b]$ and if m is a number between $f(a)$ and $f(b)$, then there is some number c between a and b such that $f(c) = m$
Limit(s)	The value that a function or expression approaches as the domain variable(s) approach a specific value; limits are written in the form $f(x)$ [e.g., the limit of $f(x) = \frac{1}{x}$ as x approaches 3 is $\frac{1}{3}$; this is written $\frac{1}{x} = \frac{1}{3}$]
Mean Value Theorem	If function f is continuous on $[a, b]$ and differentiable on (a, b) , then there exists a number c in (a, b) such that $f'(c) = \frac{f(b)-f(a)}{b-a}$
Normal line	The line which is perpendicular to the tangent line at the point where the tangent line intersects the curve.
Power Rule	A formula for finding the derivative of a power of a variable; $\frac{d}{dx}(x^n) = nx^{n-1}$
Product Rule	A formula for the derivative of the product of two functions; $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$ or $(uv)' = u'v + uv'$
Quotient Rule	A formula for the derivative of the quotient of two functions; $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ or $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$
Tangent line	A line that touches a curve at a point without crossing over; formally, it is a line which intersects a differentiable curve at a point where the slope of the curve equals the slope of the line